Section 3: Math Test – No Calculator

QUESTION 1

Choice B is correct. Multiplying both sides of the first equation in the system by 2 yields $4x - 2y = 16$. Adding $4x - 2y = 16$ to the second equation in the system yields $5x = 20$. Dividing both sides of $5x = 20$ by 5 yields $x = 4$. Substituting 4 for x in $x + 2y = 4$ yields $4 + 2y = 4$. Subtracting 4 from both sides of $4 + 2y = 4$ yields $2y = 0$. Dividing both sides of this equation by 2 yields $y = 0$. Substituting 4 for x and 0 for y in the expression $x + y$ yields $4 + 0 = 4$.

Choices A, C, and D are incorrect and may result from various computation errors.

QUESTION 2

Choice A is correct. Since $(x^2 - x)$ is a common term in the original expression, like terms can be added: $2(x^2 - x) + 3(x^2 - x) = 5(x^2 - x)$. Distributing the constant term 5 yields $5x^2 - 5x$.

Choice B is incorrect and may result from not distributing the negative signs in the expressions within the parentheses. Choice C is incorrect and may result from not distributing the negative signs in the expressions within the parentheses and from incorrectly eliminating the x^2-term. Choice D is incorrect and may result from incorrectly eliminating the x-term.

QUESTION 3

Choice D is correct. To find the slope and y-intercept, the given equation can be rewritten in slope-intercept form $y = mx + b$, where m represents the slope of the line and b represents the y-intercept. The given equation $2y - 3x = -4$ can be rewritten in slope-intercept form by first adding $3x$ to both sides of the equation, which yields $2y = 3x - 4$. Then, dividing both sides of the equation by 2 results in the equation $y = \frac{3}{2}x - 2$. The coefficient of x, $\frac{3}{2}$, is the slope of the graph and is positive, and the constant term, -2, is the y-intercept of the graph and is negative. Thus, the graph of the equation $2y - 3x = -4$ has a positive slope and a negative y-intercept.

Choice A is incorrect and may result from reversing the values of the slope and the y-intercept. Choices B and C are incorrect and may result from errors in calculation when determining the slope and y-intercept values.

QUESTION 4

Choice A is correct. It's given that the front of the roller-coaster car starts rising when it's 15 feet above the ground. This initial height of 15 feet can be represented by a constant term, 15, in an equation. Each second, the front of the roller-coaster car rises 8 feet, which can be represented by $8s$. Thus, the equation $h = 8s + 15$ gives the height, in feet, of the front of the roller-coaster car s seconds after it starts up the hill.

Choices B and C are incorrect and may result from conceptual errors in creating a linear equation. Choice D is incorrect and may result from switching the rate at which the roller-coaster car rises with its initial height.

QUESTION 5

Choice C is correct. Since the variable h represents the number of hours a job took, the coefficient of h, 75, represents the electrician's charge per hour, in dollars, after an initial fixed charge of \$125. It's given that the electrician worked 2 hours longer on Ms. Sanchez's job than on Mr. Roland's job; therefore, the additional charge for Ms. Sanchez's job is $\$75 \times 2 = \$150$.

Alternate approach: The amounts the electrician charged for Mr. Roland's job and Ms. Sanchez's job can be expressed in terms of t. If Mr. Roland's job took t hours, then it cost $75t + 125$ dollars. Ms. Sanchez's job must then have taken $t + 2$ hours, so it cost $75(t + 2) + 125 = 75t + 275$ dollars. The difference between the two costs is $(75t + 275) - (75t + 125) = \$150$.

Choice A is incorrect. This is the electrician's charge per hour, not the difference between what Ms. Sanchez was charged and what Mr. Roland was charged. Choice B is incorrect. This is the fixed charge for each job, not the difference between the two. Choice D is incorrect and may result from finding the total charge for a 2-hour job.

QUESTION 6

Choice B is correct. The ratio of the lengths of two arcs of a circle is equal to the ratio of the measures of the central angles that subtend the arcs. It's given that arc $\overparen{ADC}$ is subtended by a central angle with measure 100°. Since the sum of the measures of the angles about a point is 360°, it follows that arc $\overparen{ABC}$ is subtended by a central angle with measure $360° - 100° = 260°$. If s is the length of arc $\overparen{ABC}$, then s must satisfy the ratio $\frac{s}{5\pi} = \frac{260}{100}$. Reducing the fraction $\frac{260}{100}$ to its simplest form gives $\frac{13}{5}$. Therefore, $\frac{s}{5\pi} = \frac{13}{5}$. Multiplying both sides of $\frac{s}{5\pi} = \frac{13}{5}$ by 5π yields $s = 13\pi$.

Choice A is incorrect. This is the length of an arc consisting of exactly half of the circle, but arc $\overparen{ABC}$ is greater than half of the circle. Choice C is incorrect. This is the total circumference of the circle. Choice D is incorrect. This is half the length of arc $\overparen{ABC}$, not its full length.

QUESTION 7

Choice D is correct. Multiplying both sides of the given equation by x yields $160x = 8$. Dividing both sides of the equation $160x = 8$ by 160 results in $x = \frac{8}{160}$. Reducing $\frac{8}{160}$ to its simplest form gives $x = \frac{1}{20}$, or its decimal equivalent 0.05.

Choice A is incorrect and may result from multiplying, instead of dividing, the left-hand side of the given equation by 160. Choice B is incorrect and may result from a computational error. Choice C is incorrect. This is the value of $\frac{1}{x}$.

QUESTION 8

Choice C is correct. Applying the distributive property of multiplication to the right-hand side of the given equation gives $(3x + 15) + (5x - 5)$, or $8x + 10$. An equation in the form $cx + d = rx + s$ will have no solutions if $c = r$ and $d \neq s$. Therefore, it follows that the equation $2ax - 15 = 8x + 10$ will have no solutions if $2a = 8$, or $a = 4$.

Choice A is incorrect. If $a = 1$, then the given equation could be written as $2x - 15 = 8x + 10$. Since $2 \neq 8$, this equation has exactly one solution. Choice B is incorrect. If $a = 2$, then the given equation could be written as $4x - 15 = 8x + 10$. Since $4 \neq 8$, this equation has exactly one solution. Choice D is incorrect. If $a = 8$, then the given equation could be written as $16x - 15 = 8x + 10$. Since $16 \neq 8$, this equation has exactly one solution.

QUESTION 9

Choice B is correct. A solution to the system of three equations is any ordered pair (x, y) that is a solution to each of the three equations. Such an ordered pair (x, y) must lie on the graph of each equation in the xy-plane; in other words, it must be a point where all three graphs intersect. The graphs of all three equations intersect at exactly one point, $(-1, 3)$. Therefore, the system of equations has one solution.

Choice A is incorrect. A system of equations has no solutions when there is no point at which all the graphs intersect. Because the graphs of all three equations intersect at the point $(-1, 3)$, there is a solution. Choice C is incorrect. The graphs of all three equations intersect at only one point, $(-1, 3)$. Since there is no other such point, there cannot be two solutions. Choice D is incorrect and may result from counting the number of points of intersection of the graphs of any two equations, including the point of intersection of all three equations.

QUESTION 10

Choice C is correct. If the equation is true for all x, then the expressions on both sides of the equation will be equivalent. Multiplying the polynomials on the left-hand side of the equation gives $5ax^3 - abx^2 + 4ax + 15x^2 - 3bx + 12$. On the right-hand side of the equation, the only x^2-term is $-9x^2$. Since the expressions on both sides of the equation are equivalent, it follows that $-abx^2 + 15x^2 = -9x^2$, which can be rewritten as $(-ab + 15)x^2 = -9x^2$. Therefore, $-ab + 15 = -9$, which gives $ab = 24$.

Choice A is incorrect. If $ab = 18$, then the coefficient of x^2 on the left-hand side of the equation would be $-18 + 15 = -3$, which doesn't equal the coefficient of x^2, -9, on the right-hand side. Choice B is incorrect. If $ab = 20$, then the coefficient of x^2 on the left-hand side of the equation would be $-20 + 15 = -5$, which doesn't equal the coefficient of x^2, -9, on the right-hand side. Choice D is incorrect. If $ab = 40$, then the coefficient of x^2 on the left-hand side of the equation would be $-40 + 15 = -25$, which doesn't equal the coefficient of x^2, -9, on the right-hand side.

QUESTION 11

Choice B is correct. The right-hand side of the given equation, $\frac{2x}{2}$, can be rewritten as x. Multiplying both sides of the equation $\frac{x}{x-3} = x$ by $x - 3$ yields $x = x(x - 3)$. Applying the distributive property of multiplication to the right-hand side of the equation $x = x(x - 3)$ yields $x = x^2 - 3x$. Subtracting x from both sides of this equation yields $0 = x^2 - 4x$. Factoring x from both terms of $x^2 - 4x$ yields $0 = x(x - 4)$. By the zero product property, the solutions to the equation $0 = x(x - 4)$ are $x = 0$ and $x - 4 = 0$, or $x = 4$. Substituting 0 and 4 for x in the given equation yields $0 = 0$ and $4 = 4$, respectively. Since both are true statements, both 0 and 4 are solutions to the given equation.

Choice A is incorrect and may result from a sign error. Choice C is incorrect and may result from an error in factoring. Choice D is incorrect and may result from not considering 0 as a possible solution.

QUESTION 12

Choice D is correct. The original expression can be combined into one rational expression by multiplying the numerator and denominator of the second term by the denominator of the first term: $\frac{1}{2x+1} + 5\left(\frac{2x+1}{2x+1}\right)$, which can be rewritten as $\frac{1}{2x+1} + \frac{10x+5}{2x+1}$. This expression is now the sum of two rational expressions with a common denominator, and it can be rewritten as $\frac{1}{2x+1} + \frac{10x+5}{2x+1} = \frac{10x+6}{2x+1}$.

Choice A is incorrect and may result from a calculation error. Choice B is incorrect and may be the result of adding the denominator of the first term to the second term rather than multiplying the first term by the numerator and denominator of the second term. Choice C is incorrect and may result from not adding the numerator of $\frac{1}{2x+1}$ to the numerator of $\frac{10x+5}{2x+1}$.

QUESTION 13

Choice A is correct. The equation of a parabola in vertex form is $f(x) = a(x - h)^2 + k$, where the point (h, k) is the vertex of the parabola and a is a constant. The graph shows that the coordinates of the vertex are (3, 1), so $h = 3$ and $k = 1$. Therefore, an equation that defines f can be written as $f(x) = a(x - 3)^2 + 1$. To find a, substitute a value for x and its corresponding value for y, or $f(x)$. For example, (4, 5) is a point on the graph of f. So a must satisfy the equation $5 = a(4 - 3)^2 + 1$, which can be rewritten as $4 = a(1)^2$, or $a = 4$. An equation that defines f is therefore $f(x) = 4(x - 3)^2 + 1$.

Choice B is incorrect and may result from a sign error when writing the equation of the parabola in vertex form. Choice C is incorrect and may result from omitting the constant a from the vertex form of the equation of the parabola. Choice D is incorrect and may result from a sign error when writing the equation of the parabola in vertex form as well as by miscalculating the value of a.

QUESTION 14

Choice B is correct. The solutions of the first inequality, $y \geq x + 2$, lie on or above the line $y = x + 2$, which is the line that passes through (−2, 0) and (0, 2). The second inequality can be rewritten in slope-intercept form by dividing the second inequality, $2x + 3y \leq 6$, by 3 on both sides, which yields $\frac{2}{3}x + y \leq 2$, and then subtracting $\frac{2}{3}x$ from both sides, which yields $y \leq -\frac{2}{3}x + 2$. The solutions to this inequality lie on or below the line $y = -\frac{2}{3}x + 2$, which is the line that passes through (0, 2) and (3, 0). The only graph in which the shaded region meets these criteria is choice B.

Choice A is incorrect and may result from reversing the inequality sign in the first inequality. Choice C is incorrect and may result from reversing the inequality sign in the second inequality. Choice D is incorrect and may result from reversing the inequality signs in both inequalities.

QUESTION 15

Choice B is correct. Squaring both sides of the given equation yields $x + 2 = x^2$. Subtracting x and 2 from both sides of $x + 2 = x^2$ yields $x^2 - x - 2 = 0$. Factoring the left-hand side of this equation yields $(x - 2)(x + 1) = 0$. Applying the zero product property, the solutions to $(x - 2)(x + 1) = 0$ are $x - 2 = 0$, or $x = 2$ and $x + 1 = 0$, or $x = -1$. Substituting $x = 2$ in the given equation gives $\sqrt{4} = -2$, which is false because $\sqrt{4} = 2$ by the definition of a principal square root. So, $x = 2$ isn't a solution. Substituting $x = -1$ into the given equation gives $\sqrt{1} = -(-1)$, which is true because $-(-1) = 1$. So $x = -1$ is the only solution.

Choices A and C are incorrect. The square root symbol represents the principal, or nonnegative, square root. Therefore, in the equation $\sqrt{x + 2} = -x$, the value of $-x$ must be zero or positive. If $x = 2$, then $-x = -2$, which is negative, so 2 can't be in the set of solutions. Choice D is incorrect and may result from incorrectly reasoning that $-x$ always has a negative value and therefore can't be equal to a value of a principal square root, which cannot be negative.

QUESTION 16

The correct answer is 360. The volume of a right rectangular prism is calculated by multiplying its dimensions: length, width, and height. Multiplying the values given for these dimensions yields a volume of $(4)(9)(10) = 360$ cubic centimeters.

QUESTION 17

The correct answer is 2. The left-hand side of the given equation contains a common factor of 2 and can be rewritten as $2(2x + 1)$. Dividing both sides of this equation by 2 yields $2x + 1 = 2$. Therefore, the value of $2x + 1$ is 2.

Alternate approach: Subtracting 2 from both sides of the given equation yields $4x = 2$. Dividing both sides of this equation by 4 yields $x = \frac{1}{2}$. Substituting $\frac{1}{2}$ for x in the expression $2x + 1$ yields $2\left(\frac{1}{2}\right) + 1 = 2$.

QUESTION 18

The correct answer is 8. The graph shows that the maximum value of $f(x)$ is 2. Since $g(x) = f(x) + 6$, the graph of g is the graph of f shifted up by 6 units. Therefore, the maximum value of $g(x)$ is $2 + 6 = 8$.

QUESTION 19

The correct answer is $\frac{3}{4}$, or .75. By definition of the sine ratio, since $\sin R = \frac{4}{5}$, $\frac{PQ}{PR} = \frac{4}{5}$. Therefore, if $PQ = 4n$, then $PR = 5n$, where n is a positive constant. Then $QR = kn$, where k is another positive constant. Applying the Pythagorean theorem, the following relationship holds: $(kn)^2 + (4n)^2 = (5n)^2$, or $k^2n^2 + 16n^2 = 25n^2$. Subtracting $16n^2$ from both sides of this equation yields $k^2n^2 = 9n^2$. Taking the square root of both sides of $k^2n^2 = 9n^2$ yields $kn = 3n$. It follows that $k = 3$. Therefore, if $PQ = 4n$ and $PR = 5n$, then $QR = 3n$, and by definition of the tangent ratio, $\tan P = \frac{3n}{4n}$, or $\frac{3}{4}$. Either 3/4 or .75 may be entered as the correct answer.

QUESTION 20

The correct answer is 2.5. The graph of the linear function f passes through the points (0, 3) and (1, 1). The slope of the graph of the function f is therefore $\frac{1-3}{1-0} = -2$. It's given that the graph of the linear function g is perpendicular to the graph of the function f. Therefore, the slope of the graph of the function g is the negative reciprocal of -2, which is $-\frac{1}{-2} = \frac{1}{2}$, and an equation that defines the function g is $g(x) = \frac{1}{2}x + b$, where b is a constant. Since it's given that the graph of the function g passes through the point (1, 3), the value of b can be found using the equation $3 = \frac{1}{2}(1) + b$. Solving this equation for b yields $b = \frac{5}{2}$, so an equation that defines the function g is $g(x) = \frac{1}{2}x + \frac{5}{2}$. Finding the value of $g(0)$ by substituting 0 for x into this equation yields $g(0) = \frac{1}{2}(0) + \frac{5}{2}$, or $\frac{5}{2}$. Either 2.5 or 5/2 may be entered as the correct answer.

Section 4: Math Test – Calculator

QUESTION 1

Choice B is correct. Subtracting 3 from both sides of the equation yields $3x = 24$. Dividing both sides of this equation by 3 yields $x = 8$.

Choice A is incorrect and may result from finding a common factor among the three given terms instead of finding x. Choice C is incorrect and may result from incorrectly adding 3 to, instead of subtracting 3 from, the right-hand side of the equation. Choice D is incorrect. This is the value of $3x + 3$, not the value of x.

QUESTION 2

Choice D is correct. Since 1 cubit is equivalent to 7 palms, 140 cubits are equivalent to 140(7) palms, or 980 palms.

Choice A is incorrect and may result from dividing 7 by 140. Choice B is incorrect and may result from dividing 140 by 7. Choice C is incorrect. This is the length of the Great Sphinx statue in cubits, not palms.

QUESTION 3

Choice B is correct. Multiplying both sides of the given equation by 5 yields $2n = 50$. Substituting 50 for $2n$ in the expression $2n - 1$ yields $50 - 1 = 49$.

Alternate approach: Dividing both sides of $2n = 50$ by 2 yields $n = 25$. Evaluating the expression $2n - 1$ for $n = 25$ yields $2(25) - 1 = 49$.

Choice A is incorrect and may result from finding the value of $n - 1$ instead of $2n - 1$. Choice C is incorrect and may result from finding the value of $2n$ instead of $2n - 1$. Choice D is incorrect and may result from finding the value of $4n - 1$ instead of $2n - 1$.

QUESTION 4

Choice A is correct. The square root symbol represents the principal, or nonnegative, square root. Therefore, the equation $\sqrt{x^2} = x$ is only true for values of x greater than or equal to 0. Thus, -4 isn't a solution to the given equation.

Choices B, C, and D are incorrect because these values of x are solutions to the equation $\sqrt{x^2} = x$. Choosing one of these as a value of x that isn't a solution may result from incorrectly using the rules of exponents or incorrectly evaluating these values in the given equation.

QUESTION 5

Choice D is correct. The x-axis of the graph represents the time, in minutes, after the coffee was removed from the heat source, and the y-axis of the graph represents the temperature, in degrees Fahrenheit, of the coffee. The coffee was first removed from the heat source when $x = 0$. The graph shows that when $x = 0$, the y-value was a little less than 200°F. Of the answer choices given, 195 is the best approximation.

Choice A is incorrect and may result from finding the temperature after 140 minutes. Choice B is incorrect and may result from finding the temperature after 50 minutes. Choice C is incorrect and may result from finding the temperature after 10 minutes.

QUESTION 6

Choice A is correct. The average rate of change in temperature of the coffee in degrees Fahrenheit per minute is calculated by dividing the difference between two recorded temperatures by the number of minutes in the corresponding interval of time. Since the time intervals given are all 10 minutes, the average rate of change is greatest for the points with the greatest difference in temperature. Of the choices, the greatest difference in temperature occurs between 0 and 10 minutes.

Choices B, C, and D are incorrect and may result from misinterpreting the average rate of change from the graph.

QUESTION 7

Choice C is correct. It's given that $x = 100$; therefore, substituting 100 for x in triangle ABC gives two known angle measures for this triangle. The sum of the measures of the interior angles of any triangle equals 180°. Subtracting the two known angle measures of triangle ABC from 180° gives the third angle measure: $180° - 100° - 20° = 60°$. This is the measure of angle BCA. Since vertical angles are congruent, the measure of angle DCE is also 60°. Subtracting the two known angle measures of triangle CDE from 180° gives the third angle measure: $180° - 60° - 40° = 80°$. Therefore, the value of y is 80.

Choice A is incorrect and may result from a calculation error. Choice B is incorrect and may result from classifying angle CDE as a right angle. Choice D is incorrect and may result from finding the measure of angle BCA or DCE instead of the measure of angle CDE.

QUESTION 8

Choice A is correct. The cost of each additional mile traveled is represented by the slope of the given line. The slope of the line can be calculated by identifying two points on the line and then calculating the ratio of the change in y to the change in x between the two points. Using the points (1, 5) and (2, 7), the slope is equal to $\frac{7-5}{2-1}$, or 2. Therefore, the cost for each additional mile traveled of the cab ride is \$2.00.

Choice B is incorrect and may result from calculating the slope of the line that passes through the points (5, 13) and (0, 0). However, (0, 0) does not lie on the line shown. Choice C is incorrect. This is the y-coordinate of the y-intercept of the graph and represents the flat fee for a cab ride before the charge for any miles traveled is added. Choice D is incorrect. This value represents the total cost of a 1-mile cab ride.

QUESTION 9

Choice D is correct. The total number of gas station customers on Tuesday was 135. The table shows that the number of customers who did not purchase gasoline was 50. Finding the ratio of the number of customers who did not purchase gasoline to the total number of customers gives the probability that a customer selected at random on that day did not purchase gasoline, which is $\frac{50}{135}$.

Choice A is incorrect and may result from finding the probability that a customer did not purchase a beverage, given that the customer did not purchase gasoline. Choice B is incorrect and may result from finding the probability that a customer did not purchase gasoline, given that the customer did not purchase a beverage. Choice C is incorrect and may result from finding the probability that a customer did purchase a beverage, given that the customer did not purchase gasoline.

QUESTION 10

Choice D is correct. It is given that the number of students surveyed was 336. Finding $\frac{1}{4}$ of 336 yields $\left(\frac{1}{4}\right)(336) = 84$, the number of freshmen, and finding $\frac{1}{3}$ of 336 yields $\left(\frac{1}{3}\right)(336) = 112$, the number of sophomores. Subtracting these numbers from the total number of selected students results in $336 - 84 - 112 = 140$, the number of juniors and seniors combined. Finding half of this total yields $\left(\frac{1}{2}\right)(140) = 70$, the number of juniors. Subtracting this number from the number of juniors and seniors combined yields $140 - 70 = 70$, the number of seniors.

Choices A and C are incorrect and may result from calculation errors. Choice B is incorrect. This is the total number of juniors and seniors.

QUESTION 11

Choice A is correct. It's given that the ratio of the heights of Plant A to Plant B is 20 to 12 and that the height of Plant C is 54 centimeters. Let x be the height of Plant D. The proportion $\frac{20}{12} = \frac{54}{x}$ can be used to solve for the value of x. Multiplying both sides of this equation by x yields $\frac{20x}{12} = 54$ and then multiplying both sides of this equation by 12 yields $20x = 648$. Dividing both sides of this equation by 20 yields $x = 32.4$ centimeters.

Choice B is incorrect and may result from a calculation error. Choice C is incorrect and may result from finding the difference in heights between Plant A and Plant B and then adding that to the height of Plant C. Choice D is incorrect and may result from using the ratio 12 to 20 rather than 20 to 12.

QUESTION 12

Choice D is correct. It's given that 1 kilometer is approximately equivalent to 0.6214 miles. Let x be the number of kilometers equivalent to 3.1 miles. The proportion $\frac{1 \text{ kilometer}}{0.6214 \text{ miles}} = \frac{x \text{ kilometers}}{3.1 \text{ miles}}$ can be used to solve for the value of x. Multiplying both sides of this equation by 3.1 yields $\frac{3.1}{0.6214} = x$, or $x \approx 4.99$. This is approximately 5 kilometers.

Choice A is incorrect and may result from misidentifying the ratio of kilometers to miles as miles to kilometers. Choice B is incorrect and may result from calculation errors. Choice C is incorrect and may result from calculation and rounding errors.

QUESTION 13

Choice C is correct. Let a equal the number of 120-pound packages, and let b equal the number of 100-pound packages. It's given that the total weight of the packages can be at most 1,100 pounds: the inequality $120a + 100b \leq 1{,}100$ represents this situation. It's also given that the helicopter must carry at least 10 packages: the inequality $a + b \geq 10$ represents this situation. Values of a and b that satisfy these two inequalities represent the allowable numbers of 120-pound packages and 100-pound packages the helicopter can transport. To maximize the number of 120-pound packages, a, in the helicopter, the number of 100-pound packages, b, in the helicopter needs to be minimized. Expressing b in terms of a in the second inequality yields $b \geq 10 - a$, so the minimum value of b is equal to $10 - a$. Substituting $10 - a$ for b in the first inequality results in $120a + 100(10 - a) \leq 1{,}100$. Using the distributive property to rewrite this inequality yields $120a + 1{,}000 - 100a \leq 1{,}100$, or $20a + 1{,}000 \leq 1{,}100$. Subtracting 1,000 from both sides of this inequality yields $20a \leq 100$. Dividing both sides of this inequality by 20 results in $a \leq 5$. This means that the maximum number of 120-pound packages that the helicopter can carry per trip is 5.

Choices A, B, and D are incorrect and may result from incorrectly creating or solving the system of inequalities.

QUESTION 14

Choice B is correct. The difference between the machine's starting value and its value after 10 years can be found by subtracting \$30,000 from \$120,000: $120{,}000 - 30{,}000 = 90{,}000$. It's given that the value of the machine depreciates by the same amount each year for 10 years. Dividing \$90,000 by 10 gives \$9,000, which is the amount by which the value depreciates each year. Therefore, over a period of t years, the value of the machine depreciates by a total of 9,000t dollars. The value v of the machine, in dollars, t years after it was purchased is the starting value minus the amount of depreciation after t years, or $v = 120{,}000 - 9{,}000t$.

Choice A is incorrect and may result from using the value of the machine after 10 years as the machine's starting value. Choice C is incorrect. This equation shows the amount the machine's value changes each year being added to, rather than subtracted from, the starting value. Choice D is incorrect and may result from multiplying the machine's value after 10 years by t instead of multiplying the amount the machine depreciates each year by t.

QUESTION 15

Choice D is correct. The slope-intercept form of a linear equation is $y = ax + b$, where a is the slope of the graph of the equation and b is the y-coordinate of the y-intercept of the graph. Two ordered pairs (x_1, y_1) and (x_2, y_2) can be used to compute the slope of the line with the formula $a = \frac{y_2 - y_1}{x_2 - x_1}$. Substituting the two ordered pairs (2, 4) and (0, 1) into this formula gives $a = \frac{4 - 1}{2 - 0}$, which simplifies to $\frac{3}{2}$. Substituting this value for a in the slope-intercept form of the equation yields $y = \frac{3}{2}x + b$. Substituting values from the ordered pair (0, 1) into this equation yields $1 = \frac{3}{2}(0) + b$, so $b = 1$. Substituting this value for b in the slope-intercept equation yields $y = \frac{3}{2}x + 1$.

Choice A is incorrect. This may result from misinterpreting the change in x-values as the slope and misinterpreting the change in y-values as the y-coordinate of the y-intercept of the graph. Choice B is incorrect and may result from using the x- and y-values of one of the given points as the slope and y-coordinate of the y-intercept, respectively. Choice C is incorrect. This equation has the correct slope but the incorrect y-coordinate of the y-intercept.

QUESTION 16

Choice B is correct. Multiplying the binomials in the given expression results in $4ax^2 + 4ax - 4x - 4 - x^2 + 4$. Combining like terms yields $4ax^2 + 4ax - 4 - x^2$. Grouping by powers of x and factoring out their greatest common factors yields $(4a - 1)x^2 + (4a - 4)x$. It's given that this expression is equivalent to bx, so $(4a - 1)x^2 + (4a - 4)x = bx$. Since the right-hand side of the equation has no x^2 term, the coefficient of the x^2 term on the left-hand side must be 0. This gives $4a - 1 = 0$ and $4a - 4 = b$. Since $4a - 1 = 0$, $4a = 1$. Substituting the value of $4a$ into the second equation gives $1 - 4 = b$, so $b = -3$.

Choices A, C, and D are incorrect and may result from a calculation error.

QUESTION 17

Choice C is correct. Multiplying both sides of $2w + 4t = 14$ by 2 yields $4w + 8t = 28$. Subtracting the second given equation from $4w + 8t = 28$ yields $(4w - 4w) + (8t - 5t) = (28 - 25)$ or $3t = 3$. Dividing both sides of this equation by 3 yields $t = 1$. Substituting 1 for t in the equation $2w + 4t = 14$ yields $2w + 4(1) = 14$, or $2w + 4 = 14$. Subtracting 4 from both sides of this equation yields $2w = 10$, and dividing both sides of this equation by 2 yields $w = 5$. Substituting 5 for w and 1 for t in the expression $2w + 3t$ yields $2(5) + 3(1) = 13$.

Choices A, B, and D are incorrect and may result from incorrectly calculating the values of w and t, or from correctly calculating the values of w and t but finding the value of an expression other than $2w + 3t$. For instance, choice A is the value of $w + t$, choice B is the value of $2w$, and choice D is the value of $2t + 3w$.

QUESTION 18

Choice B is correct. It's given that each serving of Crunchy Grain cereal provides 5% of an adult's daily allowance of potassium, so x servings would provide x times 5%. The percentage of an adult's daily allowance of potassium, p, is 5 times the number of servings, x. Therefore, the percentage of an adult's daily allowance of potassium can be expressed as $p = 5x$.

Choices A, C, and D are incorrect and may result from incorrectly converting 5% to its decimal equivalent, which isn't necessary since p is expressed as a percentage. Additionally, choices C and D are incorrect because the context should be represented by a linear relationship, not by an exponential relationship.

QUESTION 19

Choice B is correct. It's given that a $\frac{3}{4}$-cup serving of Crunchy Grain cereal provides 210 calories. The total number of calories per cup can be found by dividing 210 by $\frac{3}{4}$, which gives $210 \div \frac{3}{4} = 280$ calories per cup. Let c be the number of cups of Crunchy Grain cereal and s be the number of cups of Super Grain cereal. The expression $280c$ represents the number of calories in c cups of Crunchy Grain cereal, and $240s$ represents the number of calories in s cups of Super Grain cereal. The equation $280c + 240s = 270$ gives the total number of calories in one cup of the mixture. Since $c + s = 1$ cup, $c = 1 - s$. Substituting $1 - s$ for c in the equation $280c + 240s = 270$ yields $280(1 - s) + 240s = 270$, or $280 - 280s + 240s = 270$. Simplifying this equation yields $280 - 40s = 270$. Subtracting 280 from both sides results in $-40s = -10$. Dividing both sides of the equation by -40 results in $s = \frac{1}{4}$, so there is $\frac{1}{4}$ cup of Super Grain cereal in one cup of the mixture.

Choices A, C, and D are incorrect and may result from incorrectly creating or solving the system of equations.

QUESTION 20

Choice A is correct. There are 0 calories in 0 servings of Crunchy Grain cereal so the line must begin at the point (0, 0). Point (0, 0) is the origin, labeled O. Additionally, each serving increases the calories by 250. Therefore, the number of calories increases as the number of servings increases, so the line must have a positive slope. Of the choices, only choice A shows a graph with a line that begins at the origin and has a positive slope.

Choices B, C, and D are incorrect. These graphs don't show a line that passes through the origin. Additionally, choices C and D may result from misidentifying the slope of the graph.

QUESTION 21

Choice D is correct. Since the function h is exponential, it can be written as $h(x) = ab^x$, where a is the y-coordinate of the y-intercept and b is the growth rate. Since it's given that the y-coordinate of the y-intercept is d, the exponential function can be written as $h(x) = db^x$. These conditions are only met by the equation in choice D.

Choice A is incorrect. For this function, the value of $h(x)$ when $x = 0$ is -3, not d. Choice B is incorrect. This function is a linear function, not an exponential function. Choice C is incorrect. This function is a polynomial function, not an exponential function.

QUESTION 22

Choice B is correct. The median weight is found by ordering the horses' weights from least to greatest and then determining the middle value from this list of weights. Decreasing the value for the horse with the lowest weight doesn't affect the median since it's still the lowest value.

Choice A is incorrect. The mean is calculated by finding the sum of all the weights of the horses and then dividing by the number of horses. Decreasing one of the weights would decrease the sum and therefore decrease the mean. Choice C is incorrect. Range is the difference between the highest and lowest weights, so decreasing the lowest weight would increase the range. Choice D is incorrect. Standard deviation is calculated based on the mean weight of the horses. Decreasing one of the weights decreases the mean and therefore would affect the standard deviation.

QUESTION 23

Choice B is correct. In order for the poll results from a sample of a population to represent the entire population, the sample must be representative of the population. A sample that is randomly selected from a population is more likely than a sample of the type described to represent the population. In this case, the people who responded were people with access to cable television and websites, which aren't accessible to the entire population. Moreover, the people who responded also chose to watch the show and respond to the poll. The people who made these choices aren't representative of the entire population of the United States because they were not a random sample of the population of the United States.

Choices A, C, and D are incorrect because they present reasons unrelated to whether the sample is representative of the population of the United States.

QUESTION 24

Choice C is correct. Substituting $x + a$ for x in $f(x) = 5x^2 - 3$ yields $f(x + a) = 5(x + a)^2 - 3$. Expanding the expression $5(x + a)^2$ by multiplication yields $5x^2 + 10ax + 5a^2$, and thus $f(x + a) = 5x^2 + 10ax + 5a^2 - 3$. Setting the expression on the right-hand side of this equation equal to the given expression for $f(x + a)$ yields $5x^2 + 30x + 42 = 5x^2 + 10ax + 5a^2 - 3$. Because this equality must be true for all values of x, the coefficients of each power of x are equal. Setting the coefficients of x equal to each other gives $10a = 30$. Dividing each side of this equation by 10 yields $a = 3$.

Choices A, B, and D are incorrect and may result from a calculation error.

QUESTION 25

Choice C is correct. The sine of an angle is equal to the cosine of the angle's complement. This relationship can be expressed by the equation $\sin x° = \cos (90° - x°)$. Therefore, if $\sin x° = a$, then $\cos (90° - x°)$ must also be equal to a.

Choices A and B are incorrect and may result from misunderstanding the relationship between the sine and cosine of complementary angles. Choice D is incorrect and may result from misinterpreting $\sin (x^2)°$ as $\sin^2 (x)°$.

QUESTION 26

Choice D is correct. The positive x-intercept of the graph of $y = h(x)$ is a point (x, y) for which $y = 0$. Since $y = h(x)$ models the height above the ground, in feet, of the projectile, a y-value of 0 must correspond to the height of the projectile when it is 0 feet above ground or, in other words, when the projectile is on the ground. Since x represents the time since the projectile was launched, it follows that the positive x-intercept, $(x, 0)$, represents the time at which the projectile hits the ground.

Choice A is incorrect and may result from misidentifying the y-intercept as a positive x-intercept. Choice B is incorrect and may result from misidentifying the y-value of the vertex of the graph of the function as an x-intercept. Choice C is incorrect and may result from misidentifying the x-value of the vertex of the graph of the function as an x-intercept.

QUESTION 27

Choice A is correct. Since $(a, 0)$ and $(b, 0)$ are the only two points where the graph of f crosses the x-axis, it must be true that $f(a) = 0$ and $f(b) = 0$ and that $f(x)$ is not equal to 0 for any other value of x. Of the given choices, choice A is the only function for which this is true. If $f(x) = (x - a)(x - b)$, then $f(a) = (a - a)(a - b)$, which can be rewritten as $f(a) = 0(a - b)$, or $f(a) = 0$. Also, $f(b) = (b - a)(b - b)$, which can be rewritten as $f(b) = (b - a)(0)$, or $f(b) = 0$. Furthermore, if $f(x) = (x - a)(x - b)$ is equal to 0, then it follows that either $x - a = 0$ or $x - b = 0$. Solving each of these equations by adding a to both sides of the first equation and adding b to both sides of the second equation yields $x = a$ or $x = b$. Therefore, the graph of $f(x) = (x - a)(x - b)$ crosses the x-axis at exactly two points, $(a, 0)$ and $(b, 0)$.

Choice B is incorrect because $f(a) = (2a)(a + b)$, which can't be 0 because it's given that a and b are positive. Choice C is incorrect because $f(b) = (b - a)(2b)$; its graph could only be 0 if $b = a$, but it would cross the x-axis at only one point, since $(a, 0)$ and $(b, 0)$ would be the same point. Choice D is incorrect because its graph crosses the x-axis at $(0, 0)$ as well as at $(a, 0)$ and $(b, 0)$.

QUESTION 28

Choice C is correct. Substituting 0 for x in the given equation yields $3(0)^2 + 6(0) + 2 = 2$. Therefore, the graph of the given equation passes through the point $(0, 2)$, which is the y-intercept of the graph. The right-hand side of the given equation, $y = 3x^2 + 6x + 2$, displays the constant 2, which directly corresponds to the y-coordinate of the y-intercept of the graph of this equation in the xy-plane.

Choice A is incorrect. The y-coordinate of the vertex of the graph is -1, not 3, 6, or 2. Choice B is incorrect. The x-coordinates of the x-intercepts of the graph are at approximately -1.577 and -0.423, not 3, 6, or 2. Choice D is incorrect. The x-coordinate of the x-intercept of the line of symmetry is at -1, not 3, 6, or 2.

QUESTION 29

Choice A is correct. The given equation is in slope-intercept form, or $y = mx + b$, where m is the value of the slope of the line of best fit. Therefore, the slope of the line of best fit is 0.096. From the definition of slope, it follows that an increase of 1 in the x-value corresponds to an increase of 0.096 in the y-value. Therefore, the line of best fit predicts that for each year between 1940 and 2010, the minimum wage will increase by 0.096 dollar per hour.

Choice B is incorrect and may result from using the y-coordinate of the y-intercept as the average increase, instead of the slope. Choice C is incorrect and may result from using the 10-year increments given on the x-axis to incorrectly interpret the slope of the line of best fit. Choice D is incorrect and may result from using the y-coordinate of the y-intercept as the average increase, instead of the slope, and from using the 10-year increments given on the x-axis to incorrectly interpret the slope of the line of best fit.

QUESTION 30

Choice D is correct. On the line of best fit, d increases from approximately 480 to 880 between $t = 12$ and $t = 24$. The slope of the line of best fit is the difference in d-values divided by the difference in t-values, which gives $\frac{880 - 480}{24 - 12} = \frac{400}{12}$, or approximately 33. Writing the equation of the line of best fit in slope-intercept form gives $d = 33t + b$, where b is the y-coordinate of the y-intercept. This equation is satisfied by all points on the line, so $d = 480$ when $t = 12$. Thus, $480 = 33(12) + b$, which is equivalent to $480 = 396 + b$. Subtracting 396 from both sides of this equation gives $b = 84$. Therefore, an equation for the line of best fit could be $d = 33t + 84$.

Choice A is incorrect and may result from an error in calculating the slope and misidentifying the y-coordinate of the y-intercept of the graph as the value of d at $t = 10$ rather than the value of d at $t = 0$. Choice B is incorrect and may result from using the smallest value of t on the graph as the slope and misidentifying the y-coordinate of the y-intercept of the graph as the value of d at $t = 10$ rather than the value of d at $t = 0$. Choice C is incorrect and may result from misidentifying the y-coordinate of the y-intercept as the smallest value of d on the graph.

QUESTION 31

The correct answer is 6. Circles are symmetric with respect to any given diameter through the center (h, k). One diameter of the circle is perpendicular to the x-axis. Therefore, the value of h is the mean of the x-coordinates of the circle's two x-intercepts: $h = \frac{20 + 4}{2} = 12$. The radius of the circle is given as 10, so the point (h, k) must be a distance of 10 units from any point on the circle. The equation of any circle can be written as $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle and r is the length of the radius of the circle. Substituting 12 for h and 10 for r into this equation gives $(x - 12)^2 + (y - k)^2 = 10^2$. Substituting the x-coordinate and y-coordinate of a point on the circle, $(4, 0)$, gives $(4 - 12)^2 + (0 - k)^2 = 10^2$, or $64 + k^2 = 100$. Subtracting 64 from both sides of this equation yields $k^2 = 36$. Therefore, $k = \pm\sqrt{36}$. Since the graph shows the point (h, k) in the first quadrant, k must be the positive square root of 36, so $k = 6$.

QUESTION 32

The correct answer is 2. It's given that line ℓ is perpendicular to the line with equation $y = -\frac{2}{3}x$. Since the equation $y = -\frac{2}{3}x$ is written in slope-intercept form, the slope of the line is $-\frac{2}{3}$. The slope of line ℓ must be the negative reciprocal of $-\frac{2}{3}$, which is $\frac{3}{2}$. It's also given that the y-coordinate of the y-intercept of line ℓ is −13, so the equation of line ℓ in slope-intercept form is $y = \frac{3}{2}x - 13$. If $y = b$ when $x = 10$, $b = \frac{3}{2}(10) - 13$, which is equivalent to $b = 15 - 13$, or $b = 2$.

QUESTION 33

The correct answer is 8. In this group, $\frac{1}{9}$th of the people who are rhesus negative have blood type B. The total number of people who are rhesus negative in the group is $7 + 2 + 1 + x$, and there are 2 people who are rhesus negative with blood type B. Therefore, $\frac{2}{(7 + 2 + 1 + x)} = \frac{1}{9}$. Combining like terms on the left-hand side of the equation yields $\frac{2}{(10 + x)} = \frac{1}{9}$. Multiplying both sides of this equation by 9 yields $\frac{18}{(10 + x)} = 1$, and multiplying both sides of this equation by $(10 + x)$ yields $18 = 10 + x$. Subtracting 10 from both sides of this equation yields $8 = x$.

QUESTION 34

The correct answer is 9. The median number of goals scored is found by ordering the number of goals scored from least to greatest and then determining the middle value in the list. If the number of goals scored in each of the 29 games were listed in order from least to greatest, the median would be the fifteenth number of goals. The graph shows there were 8 games with 1 goal scored and 9 games with 2 goals scored. Therefore, the fifteenth number, or the median number, of goals scored must be 2. According to the graph, the soccer team scored 2 goals in 9 of the games played.

QUESTION 35

The correct answer is 15. It's given that the deductions reduce the original amount of taxes owed by $2,325.00. Since the deductions reduce the original amount of taxes owed by d%, the equation $\frac{2,325}{15,500} = \frac{d}{100}$ can be used to find this percent decrease, d. Multiplying both sides of this equation by 100 yields $\frac{232,500}{15,500} = d$, or $15 = d$. Thus, the tax deductions reduce the original amount of taxes owed by 15%.

QUESTION 36

The correct answer is 1.5. It's given that the system of linear equations has no solutions. Therefore, the lines represented by the two equations are parallel. Each of the equations can be written in slope-intercept form, or $y = mx + b$, where m is the slope of the line and b is the y-coordinate of the line's y-intercept. Subtracting $\frac{3}{4}x$ from both sides of $\frac{3}{4}x - \frac{1}{2}y = 12$ yields $-\frac{1}{2}y = -\frac{3}{4}x + 12$. Dividing both sides of this equation by $-\frac{1}{2}$ yields $y = \frac{-\frac{3}{4}}{-\frac{1}{2}}x + \frac{12}{-\frac{1}{2}}$, or $y = \frac{3}{2}x - 24$. Therefore, the slope of the line represented by the first equation in the system is $\frac{3}{2}$. The second equation in the system can be put into slope-intercept form by first subtracting ax from both sides of $ax - by = 9$, then dividing both sides of the equation by $-b$, which yields $y = \frac{a}{b}x - \frac{9}{b}$. Therefore, the slope of the line represented by the second equation in the system is $\frac{a}{b}$. Parallel lines have equal slopes. Therefore, $\frac{a}{b} = \frac{3}{2}$. Either 3/2 or 1.5 may be entered as the correct answer.

QUESTION 37

The correct answer is 1.3. The median number of tourists is found by ordering the number of tourists from least to greatest and determining the middle value from this list. When the number of tourists in 2012 is ordered from least to greatest, the middle value, or the fifth number, is 46.4 million. When the number of tourists in 2013 is ordered from least to greatest, the middle value, or the fifth number, is 47.7 million. The difference between these two medians is 47.7 million − 46.4 million = 1.3 million.

QUESTION 38

The correct answer is 3. Let y be the number of international tourist arrivals in Russia in 2012, and let x be the number of these arrivals in 2011. It's given that y is 13.5% greater than x, or $y = 1.135x$. The table gives that $y = 24.7$, so $24.7 = 1.135x$. Dividing both sides of this equation by 1.135 yields $\frac{24.7}{1.135} = x$, or $x \approx 21.8$ million arrivals. The difference in the number of tourist arrivals between these two years is 24.7 million − 21.8 million = 2.9 million. Therefore, the value of k is 3 when rounded to the nearest integer.