87. $12ax^2 - 23axy + 10ay^2$

88. $6ax^2 - 19axy - 20ay^2$

89. $3bx^3 + 4bx^2 - 3bx - 4b$

90. $2x^6 - 2$

91. $72bx^2 + 24bxy + 2by^2$

92. $64y^3 - 16y^2z + yz^2$

93. $(w - 5)^3 + 8$

94. $5xy + 20y - 15x - 60$

95. $x^2 + 6xy + 9y^2 - 1$

96. $4y^2 - 4yz + z^2 - 9$

97. $8x^2 + 3x - 4$

98. $16x^2 + 81$

99. $5x(2x - 5)^2 - (2x - 5)^3$

100. $6x(3x + 1)^3 - (3x + 1)^4$

101. $4x^2 + 2x - y - y^2$

102. $a^2 + a + b - b^2$

Enrichment Exercises

In Exercises 103 and 106, find all values of k such that the trinomial is a perfect-square trinomial.

103. $x^2 + kx + 16$

104. $36x^2 + kxy + 100y^2$

105. $x^2 + 16x + k$

106. $x^2 - 14xy + ky^2$

In Exercises 107 to 110, find all integer values of k for which the resulting trinomial will factor over the integers.

107. $x^2 + kx - 6$

108. $x^2 + kx + 12$

109. $2x^2 + kx - 10$

110. $3x^2 + kx + 5$

SECTION P.5

Rational Expressions

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A3.

PS1. Simplify: $1 + \dfrac{1}{2 - \dfrac{1}{3}}$ [P.1]

PS2. Simplify: $\left(\dfrac{w}{x}\right)^{-1}\left(\dfrac{y}{z}\right)^{-1}$ [P.2]

PS3. What is the common binomial factor of $x^2 + 2x - 3$ and $x^2 + 7x + 12$? [P.4]

In Exercises PS4 to PS6, factor completely over the integers.

PS4. $(2x - 3)(3x + 2) - (2x - 3)(x + 2)$ [P.4]

PS5. $x^2 - 5x - 6$ [P.4]

PS6. $x^3 - 64$ [P.4]

Math Matters

Evidence from work left by early Egyptians more than 3600 years ago shows that they used, with one exception, unit fractions—that is, fractions whose numerators are 1. The one exception was 2/3. A unit fraction was represented by placing an oval over the symbol for the number in the denominator. For instance, 1/4 = .

A **rational expression** is a fraction in which the numerator and denominator are polynomials. For example, the expressions below are rational expressions.

$$\frac{3}{x + 1} \quad \text{and} \quad \frac{x^2 - 4x - 21}{x^2 - 9}$$

The **domain of a rational expression** is the set of all real numbers that can be used as replacements for the variable. Any value of the variable that causes division by zero is excluded from the domain of the rational expression. For example, the domain of

$$\frac{x + 3}{x^2 - 5x}, \quad x \neq 0, x \neq 5$$

is the set of all real numbers except 0 and 5. Both 0 and 5 are excluded values because the denominator $x^2 - 5x$ equals zero when $x = 0$ and also when $x = 5$. Sometimes the excluded values are specified to the right of a rational expression, as shown here. However, a rational expression is meaningful only for those real numbers that are not excluded values, regardless of whether the excluded values are specifically stated.

Question • What value of x must be excluded from the domain of $\frac{x-2}{x+1}$?

Rational expressions have properties similar to the properties of rational numbers.

Properties of Rational Expressions

For all rational expressions $\frac{P}{Q}$ and $\frac{R}{S}$, where $Q \neq 0$ and $S \neq 0$,

Equality	$\frac{P}{Q} = \frac{R}{S}$ if and only if $PS = QR$
Equivalent expressions	$\frac{P}{Q} = \frac{PR}{QR}$, $R \neq 0$
Sign	$-\frac{P}{Q} = \frac{-P}{Q} = \frac{P}{-Q}$

Simplifying Rational Expressions

To **simplify a rational expression,** factor the numerator and denominator. Then use the equivalent expressions property to eliminate factors common to both the numerator and the denominator. A rational expression is *simplified* when 1 is the only common factor of both the numerator and the denominator.

EXAMPLE 1 Simplify a Rational Expression

Simplify: $\frac{7 + 20x - 3x^2}{2x^2 - 11x - 21}$

Solution

$$\frac{7 + 20x - 3x^2}{2x^2 - 11x - 21} = \frac{(7 - x)(1 + 3x)}{(x - 7)(2x + 3)} \qquad \text{• Factor.}$$

$$= \frac{-(x - 7)(1 + 3x)}{(x - 7)(2x + 3)} \qquad \text{• Use } (7 - x) = -(x - 7).$$

$$= \frac{-\cancel{(x - 7)}(1 + 3x)}{\cancel{(x - 7)}(2x + 3)} \qquad \text{• } x \neq 7.$$

$$= \frac{-(1 + 3x)}{2x + 3}$$

$$= -\frac{3x + 1}{2x + 3}, \quad x \neq -\frac{3}{2}$$

Try Exercise 10, page 56

Answer • When $x = -1$, $x + 1 = 0$. Therefore, -1 must be excluded from the domain. When $x = 2$, the value of $\frac{x-2}{x+1}$ is $\frac{2-2}{2+1} = \frac{0}{3} = 0$. The value of the numerator can equal zero; the value of the denominator cannot equal zero.

Operations on Rational Expressions

Arithmetic operations are defined on rational expressions in the same way as they are on rational numbers.

Definitions of Arithmetic Operations for Rational Expressions

For all rational expressions $\frac{P}{Q}$, $\frac{R}{Q}$, and $\frac{R}{S}$, where $Q \neq 0$ and $S \neq 0$,

Addition $\quad \frac{P}{Q} + \frac{R}{Q} = \frac{P + R}{Q}$

Subtraction $\quad \frac{P}{Q} - \frac{R}{Q} = \frac{P - R}{Q}$

Multiplication $\quad \frac{P}{Q} \cdot \frac{R}{S} = \frac{PR}{QS}$

Division $\quad \frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R} = \frac{PS}{QR}, \quad R \neq 0$

Factoring and the equivalent expressions property of rational expressions are used in the multiplication and division of rational expressions.

EXAMPLE 2 Multiply Rational Expressions

Multiply: $\frac{4 - x^2}{x^2 + 2x - 8} \cdot \frac{x^2 - 11x + 28}{x^2 - 5x - 14}$

Solution

$$\frac{4 - x^2}{x^2 + 2x - 8} \cdot \frac{x^2 - 11x + 28}{x^2 - 5x - 14}$$

$$= \frac{(2 - x)(2 + x)}{(x - 2)(x + 4)} \cdot \frac{(x - 4)(x - 7)}{(x + 2)(x - 7)}$$

• Factor.

$$= \frac{-(x - 2)(2 + x)}{(x - 2)(x + 4)} \cdot \frac{(x - 4)(x - 7)}{(x + 2)(x - 7)}$$

• $2 - x = -(x - 2)$.

$$= \frac{-\cancel{(x - 2)}\cancel{(2 + x)}(x - 4)\cancel{(x - 7)}}{\cancel{(x - 2)}(x + 4)\cancel{(x + 2)}\cancel{(x - 7)}}$$

• Simplify.

$$= \frac{-(x - 4)}{x + 4} = -\frac{x - 4}{x + 4}$$

Try Exercise 24, page 57

EXAMPLE 3 Divide Rational Expressions

Divide: $\frac{x^2 + 6x + 9}{x^3 + 27} \div \frac{x^2 + 7x + 12}{x^3 - 3x^2 + 9x}$

(continued)

TO REVIEW

Factors of the Sum or Difference of Two Perfect Cubes
See page 44.

Solution

$$\frac{x^2 + 6x + 9}{x^3 + 27} \div \frac{x^2 + 7x + 12}{x^3 - 3x^2 + 9x}$$

$$= \frac{(x + 3)^2}{(x + 3)(x^2 - 3x + 9)} \div \frac{(x + 4)(x + 3)}{x(x^2 - 3x + 9)}$$ • Factor.

$$= \frac{(x + 3)^2}{(x + 3)(x^2 - 3x + 9)} \cdot \frac{x(x^2 - 3x + 9)}{(x + 4)(x + 3)}$$ • Multiply by the reciprocal.

$$= \frac{\cancel{(x + 3)^2}x\cancel{(x^2 - 3x + 9)}}{\cancel{(x + 3)}\cancel{(x^2 - 3x + 9)}(x + 4)\cancel{(x + 3)}}$$ • Simplify.

$$= \frac{x}{x + 4}$$

Try Exercise 30, page 57

Addition of rational expressions with a **common denominator** is accomplished by writing the sum of the numerators over the common denominator. For example,

$$\frac{5x}{18} + \frac{x}{18} = \frac{5x + x}{18} = \frac{6x}{18} = \frac{\cancel{6}x}{\cancel{6} \cdot 3} = \frac{x}{3}$$

If the rational expressions do not have a common denominator, then they can be written as equivalent expressions that have a common denominator by multiplying the numerator and denominator of each of the rational expressions by the required polynomials. The following procedure can be used to determine the least common denominator (LCD) of rational expressions. It is similar to the process used to find the LCD of rational numbers.

Determining the LCD of Rational Expressions

1. Factor each denominator completely and express repeated factors using exponential notation.
2. Identify the largest power of each factor in any single factorization. The LCD is the product of each factor raised to its largest power.

For example, the rational expressions

$$\frac{1}{x + 3} \quad \text{and} \quad \frac{5}{2x - 1}$$

have an LCD of $(x + 3)(2x - 1)$. The rational expressions

$$\frac{5x}{(x + 5)(x - 7)^3} \quad \text{and} \quad \frac{7}{x(x + 5)^2(x - 7)}$$

have an LCD of $x(x + 5)^2(x - 7)^3$.

EXAMPLE 4 **Add and Subtract Rational Expressions**

Perform the indicated operation and then simplify, if possible.

a. $\dfrac{2x + 1}{x - 3} + \dfrac{x + 2}{x + 5}$

b. $\dfrac{39x + 36}{x^2 - 3x - 10} - \dfrac{23x - 16}{x^2 - 7x + 10}$

Solution

a. The LCD is $(x - 3)(x + 5)$. Write equivalent fractions in terms of the LCD, and then add.

$$\frac{2x+1}{x-3} + \frac{x+2}{x+5} = \frac{2x+1}{x-3} \cdot \frac{x+5}{x+5} + \frac{x+2}{x+5} \cdot \frac{x-3}{x-3}$$

$$= \frac{2x^2+11x+5}{(x-3)(x+5)} + \frac{x^2-x-6}{(x-3)(x+5)}$$

$$= \frac{(2x^2+11x+5)+(x^2-x-6)}{(x-3)(x+5)}$$ • Add.

$$= \frac{3x^2+10x-1}{(x-3)(x+5)}$$ • Simplify.

b. Factor the denominators:

$x^2 - 3x - 10 = (x - 5)(x + 2)$

$x^2 - 7x + 10 = (x - 5)(x - 2)$

The LCD is $(x - 5)(x + 2)(x - 2)$. Write equivalent fractions in terms of the LCD, and then subtract.

$$\frac{39x+36}{x^2-3x-10} - \frac{23x-16}{x^2-7x+10}$$

$$= \frac{39x+36}{(x-5)(x+2)} \cdot \frac{x-2}{x-2} - \frac{23x-16}{(x-5)(x-2)} \cdot \frac{x+2}{x+2}$$

$$= \frac{39x^2-42x-72}{(x-5)(x+2)(x-2)} - \frac{23x^2+30x-32}{(x-5)(x+2)(x-2)}$$

$$= \frac{(39x^2-42x-72)-(23x^2+30x-32)}{(x-5)(x+2)(x-2)}$$

$$= \frac{16x^2-72x-40}{(x-5)(x+2)(x-2)} = \frac{8(2x^2-9x-5)}{(x-5)(x+2)(x-2)}$$

$$= \frac{8(2x+1)(x-5)}{(x-5)(x+2)(x-2)} = \frac{8(2x+1)}{(x+2)(x-2)}$$

▶ **Try Exercise 38, page 57**

EXAMPLE 5 Use the Order of Operations Agreement with Rational Expressions

Simplify: $\dfrac{x+3}{x-2} - \dfrac{x+4}{x-1} \div \dfrac{x^2+5x+4}{x^2+4x-5}$

Solution

The Order of Operations Agreement requires that division be completed before subtraction. To divide fractions, multiply by the reciprocal as shown below.

$$\frac{x+3}{x-2} - \frac{x+4}{x-1} \div \frac{x^2+5x+4}{x^2+4x-5}$$

$$= \frac{x+3}{x-2} - \frac{x+4}{x-1} \cdot \frac{x^2+4x-5}{x^2+5x+4}$$ • Multiply by the reciprocal.

(continued)

$$= \frac{x+3}{x-2} - \frac{x+4}{x-1} \cdot \frac{(x-1)(x+5)}{(x+1)(x+4)}$$ • Factor the trinomials.

$$= \frac{x+3}{x-2} - \frac{(x+4)(x-1)(x+5)}{(x-1)(x+1)(x+4)}$$ • Multiply.

$$= \frac{x+3}{x-2} - \frac{x+5}{x+1}$$ • Simplify.

$$= \frac{x+3}{x-2} \cdot \frac{x+1}{x+1} - \frac{x+5}{x+1} \cdot \frac{x-2}{x-2}$$ • Subtract. The LCD is $(x-2)(x+1)$.

$$= \frac{(x^2+4x+3) - (x^2+3x-10)}{(x-2)(x+1)}$$

$$= \frac{x+13}{(x-2)(x+1)}$$

Try Exercise 42, page 57

Complex Fractions

A **complex fraction** is a fraction whose numerator or denominator contains one or more fractions. Simplify complex fractions using one of the following methods.

Methods for Simplifying Complex Fractions

Method 1: Multiply by 1 in the form $\frac{LCD}{LCD}$.

1. Determine the LCD of all fractions in the complex fraction.
2. Multiply both the numerator and the denominator of the complex fraction by the LCD.
3. If possible, simplify the resulting rational expression.

Method 2: Multiply the numerator by the reciprocal of the denominator.

1. Simplify the numerator to a single fraction and the denominator to a single fraction.
2. Using the definition for dividing fractions, multiply the numerator by the reciprocal of the denominator.
3. If possible, simplify the resulting rational expression.

EXAMPLE 6 Simplify Complex Fractions

Simplify.

a. $\dfrac{\dfrac{2}{x-2} + \dfrac{1}{x}}{\dfrac{3x}{x-5} - \dfrac{2}{x-5}}$ **b.** $4 - \dfrac{2x}{2 - \dfrac{x-2}{x}}$

Solution

a. Simplify the numerator to a single fraction and the denominator to a single fraction.

$$\frac{\dfrac{2}{x-2} + \dfrac{1}{x}}{\dfrac{3x}{x-5} - \dfrac{2}{x-5}} = \frac{\dfrac{2 \cdot x}{(x-2) \cdot x} + \dfrac{1 \cdot (x-2)}{x \cdot (x-2)}}{\dfrac{3x-2}{x-5}}$$ • Simplify the numerator and denominator.

$$= \frac{\dfrac{2x + (x-2)}{x(x-2)}}{\dfrac{3x-2}{x-5}} = \frac{\dfrac{3x-2}{x(x-2)}}{\dfrac{3x-2}{x-5}}$$

$$= \frac{3x-2}{x(x-2)} \cdot \frac{x-5}{3x-2}$$ • Multiply the numerator by the reciprocal of the denominator.

$$= \frac{x-5}{x(x-2)}$$

b. $$4 - \frac{2x}{2 - \dfrac{x-2}{x}} = 4 - \frac{2x}{2 - \dfrac{x-2}{x}} \cdot \frac{x}{x}$$ • Multiply the numerator and denominator by the LCD of all the fractions.

$$= 4 - \frac{2x^2}{2x - (x-2)}$$

$$= 4 - \frac{2x^2}{x+2}$$ • Simplify.

$$= \frac{4}{1} \cdot \frac{x+2}{x+2} - \frac{2x^2}{x+2}$$ • Subtract. The LCD is $x + 2$.

$$= \frac{4x+8}{x+2} - \frac{2x^2}{x+2}$$

$$= \frac{-2x^2 + 4x + 8}{x+2}$$

▶ Try Exercise 62, page 58

EXAMPLE 7 Simplify a Fraction

Simplify the fraction $\dfrac{c^{-1}}{a^{-1} + b^{-1}}$.

Solution

The fraction written without negative exponents becomes

$$\frac{c^{-1}}{a^{-1} + b^{-1}} = \frac{\dfrac{1}{c}}{\dfrac{1}{a} + \dfrac{1}{b}}$$ • Use $x^{-n} = \dfrac{1}{x^n}$.

$$= \frac{\dfrac{1}{c} \cdot abc}{\left(\dfrac{1}{a} + \dfrac{1}{b}\right)abc}$$ • Multiply the numerator and denominator by abc, which is the LCD of the fraction in the numerator and the two fractions in the denominator.

$$= \frac{ab}{bc + ac}$$

▶ Try Exercise 68, page 58

Application of Rational Expressions

EXAMPLE 8 Solve an Application

The *average speed* for a round-trip is given by the complex fraction

$$\frac{2}{\frac{1}{v_1}+\frac{1}{v_2}}$$

where v_1 is the average speed on the way to your destination and v_2 is the average speed on your return trip. Find the average speed for a round-trip if $v_1 = 50$ mph and $v_2 = 40$ mph.

Solution

Evaluate the complex fraction with $v_1 = 50$ and $v_2 = 40$.

$$\frac{2}{\frac{1}{v_1}+\frac{1}{v_2}} = \frac{2}{\frac{1}{50}+\frac{1}{40}} = \frac{2}{\frac{1\cdot 4}{50\cdot 4}+\frac{1\cdot 5}{40\cdot 5}}$$

• Substitute the given values for v_1 and v_2. Then simplify the denominator.

$$= \frac{2}{\frac{4}{200}+\frac{5}{200}} = \frac{2}{\frac{9}{200}}$$

$$= 2\cdot\frac{200}{9} = \frac{400}{9} = 44\frac{4}{9}$$

The average speed for the round-trip is $44\frac{4}{9}$ mph.

Try Exercise 72, page 58

Question • In Example 8, why is the average speed for the round-trip *not* the average of v_1 and v_2?

Answer • Because you were traveling more slowly on the return trip, the return trip took longer than the trip to your destination. More time was spent traveling at the slower speed. Thus the average speed is less than the average of v_1 and v_2.

EXERCISE SET P.5

Concept Check

In Exercises 1 to 4, determine whether the given value of *x* is in the domain of the rational expression.

1. $\frac{x+5}{x^2-9}$; $x = -3$

2. $\frac{2x+4}{x^2+x-6}$; $x = -2$

3. $\frac{3x-7}{x^3+x-2}$; $x = 1$

4. $\frac{x^2-4}{x^2-6x+8}$; $x = 2$

In Exercises 5 to 8, find the LCD of the fractions.

5. $\frac{3}{xy}$ and $\frac{4}{xz^2}$

6. $\frac{x}{3x+9}$ and $\frac{5x+1}{2x+6}$

7. $\frac{2x-1}{x+5}$ and $\frac{8}{x-4}$

8. $\frac{3x-5}{x^2-16}$ and $\frac{x+7}{x^2-3x-4}$

In Exercises 9 to 18, simplify each rational expression.

9. $\frac{x^2-x-20}{3x-15}$

10. $\frac{2x^2-5x-12}{2x^2+5x+3}$

11. $\frac{x^3-9x}{x^3+x^2-6x}$

12. $\frac{x^3+125}{2x^3-50x}$

13. $\frac{a^3+8}{a^2-4}$

14. $\frac{y^3-27}{-y^2+11y-24}$

Indicates Try It Exercises

15. $\dfrac{x^2 + 3x - 40}{-x^2 + 3x + 10}$

16. $\dfrac{2x^3 - 6x^2 + 5x - 15}{9 - x^2}$

17. $\dfrac{4y^3 - 8y^2 + 7y - 14}{-y^2 - 5y + 14}$

18. $\dfrac{x^3 - x^2 + x}{x^3 + 1}$

In Exercises 19 to 48, simplify each expression.

19. $\left(-\dfrac{4a}{3b^2}\right)\left(\dfrac{6b}{a^4}\right)$

20. $\left(\dfrac{12x^2y}{5z^4}\right)\left(-\dfrac{25x^2z^3}{15y^2}\right)$

21. $\left(\dfrac{6p^2}{5q^2}\right)^{-1}\left(\dfrac{2p}{3q^2}\right)^2$

22. $\left(\dfrac{4r^2s}{3t^3}\right)^{-1}\left(\dfrac{6rs^3}{5t^2}\right)$

23. $\dfrac{x^2 + x}{2x + 3} \cdot \dfrac{3x^2 + 19x + 28}{x^2 + 5x + 4}$

24. $\dfrac{x^2 - 16}{x^2 + 7x + 12} \cdot \dfrac{x^2 - 4x - 21}{x^2 - 4x}$

25. $\dfrac{3x - 15}{2x^2 - 50} \cdot \dfrac{2x^2 + 16x + 30}{6x + 9}$

26. $\dfrac{y^3 - 8}{y^2 + y - 6} \cdot \dfrac{y^2 + 3y}{y^3 + 2y^2 + 4y}$

27. $\dfrac{12y^2 + 28y + 15}{6y^2 + 35y + 25} \div \dfrac{2y^2 - y - 3}{3y^2 + 11y - 20}$

28. $\dfrac{z^2 - 81}{z^2 - 16} \div \dfrac{z^2 - z - 20}{z^2 + 5z - 36}$

29. $\dfrac{a^2 + 9}{a^2 - 64} \div \dfrac{a^3 - 3a^2 + 9a - 27}{a^2 + 5a - 24}$

30. $\dfrac{6x^2 + 13xy + 6y^2}{4x^2 - 9y^2} \div \dfrac{3x^2 - xy - 2y^2}{2x^2 + xy - 3y^2}$

31. $\dfrac{p + 5}{r} + \dfrac{2p - 7}{r}$

32. $\dfrac{2s + 5t}{4t} + \dfrac{-2s + 3t}{4t}$

33. $\dfrac{5y - 7}{y + 4} - \dfrac{2y - 3}{y + 4}$

34. $\dfrac{6x - 5}{x - 3} - \dfrac{3x - 8}{x - 3}$

35. $\dfrac{x}{x - 5} + \dfrac{7x}{x + 3}$

36. $\dfrac{2x}{3x + 1} + \dfrac{5x}{x - 7}$

37. $\dfrac{4z}{2z - 3} + \dfrac{5z}{z - 5}$

38. $\dfrac{3y - 1}{3y + 1} - \dfrac{2y - 5}{y - 3}$

39. $\dfrac{x}{x^2 - 9} - \dfrac{3x - 1}{x^2 + 7x + 12}$

40. $\dfrac{m - n}{m^2 - mn - 6n^2} + \dfrac{3m - 5n}{m^2 + mn - 2n^2}$

41. $\dfrac{1}{x} + \dfrac{2}{3x - 1} \cdot \dfrac{3x^2 + 11x - 4}{x - 5}$

42. $\dfrac{2}{y} - \dfrac{3}{y + 1} \cdot \dfrac{y^2 - 1}{y + 4}$

43. $\dfrac{q + 1}{q - 3} - \dfrac{2q}{q - 3} \div \dfrac{q + 5}{q - 3}$

44. $\dfrac{p}{p + 5} + \dfrac{p}{p - 4} \div \dfrac{p + 2}{p^2 - p - 12}$

45. $\dfrac{1}{x^2 + 7x + 12} + \dfrac{1}{x^2 - 9} + \dfrac{1}{x^2 - 16}$

46. $\dfrac{2}{a^2 - 3a + 2} + \dfrac{3}{a^2 - 1} - \dfrac{5}{a^2 + 3a - 10}$

47. $\left(1 + \dfrac{2}{x}\right)\left(3 - \dfrac{1}{x}\right)$

48. $\left(4 - \dfrac{1}{z}\right)\left(4 + \dfrac{2}{z}\right)$

In Exercises 49 to 66, simplify each complex fraction.

49. $\dfrac{4 + \dfrac{1}{x}}{1 - \dfrac{1}{x}}$

50. $\dfrac{3 - \dfrac{2}{a}}{5 + \dfrac{3}{a}}$

51. $\dfrac{\dfrac{x}{y} - 2}{y - x}$

52. $\dfrac{3+\dfrac{2}{x-3}}{4+\dfrac{1}{2+\dfrac{1}{x}}}$

53. $\dfrac{5-\dfrac{1}{x+2}}{1+\dfrac{3}{1+\dfrac{3}{x}}}$

54. $\dfrac{\dfrac{1}{(x+h)^2}-1}{h}$

55. $\dfrac{1+\dfrac{1}{b-2}}{1-\dfrac{1}{b+3}}$

56. $r-\dfrac{r}{r+\dfrac{1}{3}}$

57. $\dfrac{1-\dfrac{1}{x^2}}{1+\dfrac{1}{x}}$

58. $\dfrac{1}{\dfrac{1}{a}+\dfrac{1}{b}}$

59. $2-\dfrac{m}{1-\dfrac{1-m}{-m}}$

60. $\dfrac{\dfrac{x+h+1}{x+h}-\dfrac{x}{x+1}}{h}$

61. $\dfrac{\dfrac{1}{x}-\dfrac{x-4}{x+1}}{\dfrac{x}{x+1}}$

62. $\dfrac{\dfrac{2}{y}-\dfrac{3y-2}{y-1}}{\dfrac{y}{y-1}}$

63. $\dfrac{\dfrac{1}{x+3}-\dfrac{2}{x-1}}{\dfrac{x}{x-1}+\dfrac{3}{x+3}}$

64. $\dfrac{\dfrac{x+2}{x^2-1}+\dfrac{1}{x+1}}{\dfrac{x}{2x^2-x-1}+\dfrac{1}{x-1}}$

65. $\dfrac{\dfrac{x^2+3x-10}{x^2+x-6}}{\dfrac{x^2-x-30}{2x^2-15x+18}}$

66. $\dfrac{\dfrac{2y^2+11y+15}{y^2-4y-21}}{\dfrac{6y^2+11y-10}{3y^2-23y+14}}$

In Exercises 67 to 70, simplify each algebraic fraction. Write all answers with positive exponents.

67. $\dfrac{a^{-1}+b^{-1}}{a-b}$

68. $\dfrac{e^{-2}-f^{-1}}{ef}$

69. $\dfrac{a^{-1}b-ab^{-1}}{a^2+b^2}$

70. $(a+b^{-2})^{-1}$

71. Average Speed According to Example 8, the average speed for a round-trip in which the average speed on the way to your destination is v_1 and the average speed on your return is v_2 is given by the complex fraction

$$\frac{2}{\dfrac{1}{v_1}+\dfrac{1}{v_2}}$$

a. Find the average speed for a round-trip by helicopter with $v_1 = 180$ mph and $v_2 = 110$ mph.

b. Simplify the complex fraction.

Enrichment Exercises

72. Relativity Theory Using Einstein's Theory of Relativity, the "sum" of the two speeds v_1 and v_2 is given by the complex fraction

$$\frac{v_1+v_2}{1+\dfrac{v_1v_2}{c^2}}$$

where c is the speed of light.

a. Evaluate this complex fraction with $v_1 = 1.2 \times 10^8$ mph, $v_2 = 2.4 \times 10^8$ mph, and $c = 6.7 \times 10^8$ mph.

b. Simplify the complex fraction.

73. An expression such as the one at the right is called a **continued fraction.** The pattern of the fraction *continues* on forever. A **convergent** of the continued fraction is an approximation that is found by stopping the process at some step. The first four convergents are shown at the right.

$$3+\cfrac{1}{6+\cfrac{3^2}{6+\cfrac{5^2}{6+\cfrac{7^2}{6+\cfrac{9^2}{6+\cfrac{11^2}{6+\dots}}}}}}$$

(i) 3 (ii) $3+\dfrac{1}{6}$

(iii) $3+\cfrac{1}{6+\cfrac{3^2}{6}}$

(iv) $3+\cfrac{1}{6+\cfrac{3^2}{6+\cfrac{5^2}{6}}}$

a. Write the expression for the fifth convergent.

b. What is the value of the sixth convergent? Round to the nearest ten-thousandth.

Note: It can be shown that as more and more convergents are evaluated, the values of the convergents become closer to π.