Application of Rational Expressions

Alternative to Example 8
The total resistance R in an electric circuit consisting of three resistors in parallel is given by

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Find the total resistance when $R_1 = 2$ ohms, $R_2 = 4$ ohms, and $R_3 = 8$ ohms.

- $\frac{8}{7}$ ohms

EXAMPLE 8 Solve an Application

The *average speed* for a round-trip is given by the complex fraction

$$\frac{2}{\frac{1}{v_1} + \frac{1}{v_2}}$$

where v_1 is the average speed on the way to your destination and v_2 is the average speed on your return trip. Find the average speed for a round-trip if $v_1 = 50$ mph and $v_2 = 40$ mph.

Solution

Evaluate the complex fraction with $v_1 = 50$ and $v_2 = 40$.

$$\frac{2}{\frac{1}{v_1} + \frac{1}{v_2}} = \frac{2}{\frac{1}{50} + \frac{1}{40}} = \frac{2}{\frac{1 \cdot 4}{50 \cdot 4} + \frac{1 \cdot 5}{40 \cdot 5}}$$

• Substitute the given values for v_1 and v_2. Then simplify the denominator.

$$= \frac{2}{\frac{4}{200} + \frac{5}{200}} = \frac{2}{\frac{9}{200}}$$

$$= 2 \cdot \frac{200}{9} = \frac{400}{9} = 44\frac{4}{9}$$

The average speed for the round-trip is $44\frac{4}{9}$ mph.

Try Exercise 72, page 58

Question • In Example 8, why is the average speed for the round-trip *not* the average of v_1 and v_2?

Answer • Because you were traveling more slowly on the return trip, the return trip took longer than the trip to your destination. More time was spent traveling at the slower speed. Thus the average speed is less than the average of v_1 and v_2.

EXERCISE SET P.5

Concept Check

In Exercises 1 to 4, determine whether the given value of x is in the domain of the rational expression.

1. $\frac{x+5}{x^2-9}$; $x = -3$
   No
2. $\frac{2x+4}{x^2+x-6}$; $x = -2$
   Yes
3. $\frac{3x-7}{x^3+x-2}$; $x = 1$
   No
4. $\frac{x^2-4}{x^2-6x+8}$; $x = 2$
   No

In Exercises 5 to 8, find the LCD of the fractions.

5. $\frac{3}{xy}$ and $\frac{4}{xz^2}$
   xyz^2
6. $\frac{x}{3x+9}$ and $\frac{5x+1}{2x+6}$
   $6(x+3)$
7. $\frac{2x-1}{x+5}$ and $\frac{8}{x-4}$
   $(x+5)(x-4)$
8. $\frac{3x-5}{x^2-16}$ and $\frac{x+7}{x^2-3x-4}$
   $(x+4)(x-4)(x+1)$

In Exercises 9 to 18, simplify each rational expression.

9. $\frac{x^2-x-20}{3x-15}$ $\quad \frac{x+4}{3}$
10. $\frac{2x^2-5x-12}{2x^2+5x+3}$ $\quad \frac{x-4}{x+1}$
11. $\frac{x^3-9x}{x^3+x^2-6x}$ $\quad \frac{x-3}{x-2}$
12. $\frac{x^3+125}{2x^3-50x}$ $\quad \frac{x^2-5x+25}{2x(x-5)}$
13. $\frac{a^3+8}{a^2-4}$ $\quad \frac{a^2-2a+4}{a-2}$
14. $\frac{y^3-27}{-y^2+11y-24}$ $\quad -\frac{y^2+3y+9}{y-8}$

Indicates Try It Exercises

15. $\dfrac{x^2+3x-40}{-x^2+3x+10}$ $-\dfrac{x+8}{x+2}$

16. $\dfrac{2x^3-6x^2+5x-15}{9-x^2}$ $-\dfrac{2x^2+5}{x+3}$

17. $\dfrac{4y^3-8y^2+7y-14}{-y^2-5y+14}$ $-\dfrac{4y^2+7}{y+7}$

18. $\dfrac{x^3-x^2+x}{x^3+1}$ $\dfrac{x}{x+1}$

In Exercises 19 to 48, simplify each expression.

19. $\left(-\dfrac{4a}{3b^2}\right)\left(\dfrac{6b}{a^4}\right)$ $-\dfrac{8}{a^3b}$

20. $\left(\dfrac{12x^2y}{5z^4}\right)\left(-\dfrac{25x^2z^3}{15y^2}\right)$ $-\dfrac{4x^4}{yz}$

21. $\left(\dfrac{6p^2}{5q^2}\right)^{-1}\left(\dfrac{2p}{3q^2}\right)^2$ $\dfrac{10}{27q^2}$

22. $\left(\dfrac{4r^2s}{3t^3}\right)^{-1}\left(\dfrac{6rs^3}{5t^2}\right)$ $\dfrac{9ts^2}{10r}$

23. $\dfrac{x^2+x}{2x+3}\cdot\dfrac{3x^2+19x+28}{x^2+5x+4}$ $\dfrac{x(3x+7)}{2x+3}$

24. $\dfrac{x^2-16}{x^2+7x+12}\cdot\dfrac{x^2-4x-21}{x^2-4x}$ $\dfrac{x-7}{x}$

25. $\dfrac{3x-15}{2x^2-50}\cdot\dfrac{2x^2+16x+30}{6x+9}$ $\dfrac{x+3}{2x+3}$

26. $\dfrac{y^3-8}{y^2+y-6}\cdot\dfrac{y^2+3y}{y^3+2y^2+4y}$ 1

27. $\dfrac{12y^2+28y+15}{6y^2+35y+25}\div\dfrac{2y^2-y-3}{3y^2+11y-20}$ $\dfrac{(2y+3)(3y-4)}{(2y-3)(y+1)}$

28. $\dfrac{z^2-81}{z^2-16}\div\dfrac{z^2-z-20}{z^2+5z-36}$ $\dfrac{(z-9)(z+9)(z+9)}{(z+4)(z-5)(z+4)}$

29. $\dfrac{a^2+9}{a^2-64}\div\dfrac{a^3-3a^2+9a-27}{a^2+5a-24}$ $\dfrac{1}{a-8}$

30. $\dfrac{6x^2+13xy+6y^2}{4x^2-9y^2}\div\dfrac{3x^2-xy-2y^2}{2x^2+xy-3y^2}$ $\dfrac{2x+3y}{2x-3y}$

31. $\dfrac{p+5}{r}+\dfrac{2p-7}{r}$ $\dfrac{3p-2}{r}$

32. $\dfrac{2s+5t}{4t}+\dfrac{-2s+3t}{4t}$ 2

33. $\dfrac{5y-7}{y+4}-\dfrac{2y-3}{y+4}$ $\dfrac{3y-4}{y+4}$

34. $\dfrac{6x-5}{x-3}-\dfrac{3x-8}{x-3}$ $\dfrac{3(x+1)}{x-3}$

35. $\dfrac{x}{x-5}+\dfrac{7x}{x+3}$ $\dfrac{8x(x-4)}{(x-5)(x+3)}$

36. $\dfrac{2x}{3x+1}+\dfrac{5x}{x-7}$ $\dfrac{x(17x-9)}{(3x+1)(x-7)}$

37. $\dfrac{4z}{2z-3}+\dfrac{5z}{z-5}$ $\dfrac{7z(2z-5)}{(2z-3)(z-5)}$

38. $\dfrac{3y-1}{3y+1}-\dfrac{2y-5}{y-3}$ $\dfrac{-3y^2+3y+8}{(3y+1)(y-3)}$

39. $\dfrac{x}{x^2-9}-\dfrac{3x-1}{x^2+7x+12}$ $\dfrac{-2x^2+14x-3}{(x-3)(x+3)(x+4)}$

40. $\dfrac{m-n}{m^2-mn-6n^2}+\dfrac{3m-5n}{m^2+mn-2n^2}$ $\dfrac{4(m-2n)^2}{(m+2n)(m-3n)(m-n)}$

41. $\dfrac{1}{x}+\dfrac{2}{3x-1}\cdot\dfrac{3x^2+11x-4}{x-5}$ $\dfrac{(2x-1)(x+5)}{x(x-5)}$

42. $\dfrac{2}{y}-\dfrac{3}{y+1}\cdot\dfrac{y^2-1}{y+4}$ $\dfrac{-3y^2+5y+8}{y(y+4)}$

43. $\dfrac{q+1}{q-3}-\dfrac{2q}{q-3}\div\dfrac{q+5}{q-3}$ $\dfrac{-q^2+12q+5}{(q-3)(q+5)}$

44. $\dfrac{p}{p+5}+\dfrac{p}{p-4}\div\dfrac{p+2}{p^2-p-12}$ $\dfrac{p(p^2+9p+17)}{(p+5)(p+2)}$

45. $\dfrac{1}{x^2+7x+12}+\dfrac{1}{x^2-9}+\dfrac{1}{x^2-16}$ $\dfrac{3x^2-7x-13}{(x+3)(x+4)(x-3)(x-4)}$

46. $\dfrac{2}{a^2-3a+2}+\dfrac{3}{a^2-1}-\dfrac{5}{a^2+3a-10}$ $\dfrac{3(7a-5)}{(a-1)(a-2)(a+1)(a+5)}$

47. $\left(1+\dfrac{2}{x}\right)\left(3-\dfrac{1}{x}\right)$ $\dfrac{(x+2)(3x-1)}{x^2}$

48. $\left(4-\dfrac{1}{z}\right)\left(4+\dfrac{2}{z}\right)$ $\dfrac{2(4z-1)(2z+1)}{z^2}$

In Exercises 49 to 66, simplify each complex fraction.

49. $\dfrac{4+\dfrac{1}{x}}{1-\dfrac{1}{x}}$ $\dfrac{4x+1}{x-1}$

50. $\dfrac{3-\dfrac{2}{a}}{5+\dfrac{3}{a}}$ $\dfrac{3a-2}{5a+3}$

51. $\dfrac{\dfrac{x}{y}-2}{y-x}$ $\dfrac{x-2y}{y(y-x)}$

Test Bonus

52. $\dfrac{3+\dfrac{2}{x-3}}{4+\dfrac{1}{2+\dfrac{1}{x}}}$ $\dfrac{(3x-7)(2x+1)}{(x-3)(9x+4)}$

53. $\dfrac{5-\dfrac{1}{x+2}}{1+\dfrac{3}{1+\dfrac{3}{x}}}$ $\dfrac{(5x+9)(x+3)}{(x+2)(4x+3)}$

54. $\dfrac{\dfrac{1}{(x+h)^2}-1}{h}$ $\dfrac{1-x^2-2xh-h^2}{h(x+h)^2}$

55. $\dfrac{1+\dfrac{1}{b-2}}{1-\dfrac{1}{b+3}}$ $\dfrac{(b+3)(b-1)}{(b-2)(b+2)}$

56. $r-\dfrac{r}{r+\dfrac{1}{3}}$ $\dfrac{r(3r-2)}{3r+1}$

57. $\dfrac{1-\dfrac{1}{x^2}}{1+\dfrac{1}{x}}$ $\dfrac{x-1}{x}$

58. $\dfrac{1}{\dfrac{1}{a}+\dfrac{1}{b}}$ $\dfrac{ab}{b+a}$

59. $2-\dfrac{m}{1-\dfrac{1-m}{-m}}$ $2-m^2$

60. $\dfrac{\dfrac{x+h+1}{x+h}-\dfrac{x}{x+1}}{h}$ $\dfrac{2x+h+1}{h(x+h)(x+1)}$

61. $\dfrac{\dfrac{1}{x}-\dfrac{x-4}{x+1}}{\dfrac{x}{x+1}}$ $\dfrac{-x^2+5x+1}{x^2}$

62. $\dfrac{\dfrac{2}{y}-\dfrac{3y-2}{y-1}}{\dfrac{y}{y-1}}$ $\dfrac{-3y^2+4y-2}{y^2}$

63. $\dfrac{\dfrac{1}{x+3}-\dfrac{2}{x-1}}{\dfrac{x}{x-1}+\dfrac{3}{x+3}}$ $\dfrac{-x-7}{x^2+6x-3}$

64. $\dfrac{\dfrac{x+2}{x^2-1}+\dfrac{1}{x+1}}{\dfrac{x}{2x^2-x-1}+\dfrac{1}{x-1}}$ $\dfrac{(2x+1)^2}{(3x+1)(x+1)}$

65. $\dfrac{\dfrac{x^2+3x-10}{x^2+x-6}}{\dfrac{x^2-x-30}{2x^2-15x+18}}$ $\dfrac{2x-3}{x+3}$

66. $\dfrac{\dfrac{2y^2+11y+15}{y^2-4y-21}}{\dfrac{6y^2+11y-10}{3y^2-23y+14}}$ 1

In Exercises 67 to 70, simplify each algebraic fraction. Write all answers with positive exponents.

67. $\dfrac{a^{-1}+b^{-1}}{a-b}$ $\dfrac{a+b}{ab(a-b)}$

68. $\dfrac{e^{-2}-f^{-1}}{ef}$ $\dfrac{f-e^2}{e^3f^2}$

69. $\dfrac{a^{-1}b-ab^{-1}}{a^2+b^2}$ $\dfrac{(b-a)(b+a)}{ab(a^2+b^2)}$

70. $(a+b^{-2})^{-1}$ $\dfrac{b^2}{ab^2+1}$

71. **Average Speed** According to Example 8, the average speed for a round-trip in which the average speed on the way to your destination is v_1 and the average speed on your return is v_2 is given by the complex fraction

$$\frac{2}{\dfrac{1}{v_1}+\dfrac{1}{v_2}}$$

a. Find the average speed for a round-trip by helicopter with $v_1 = 180$ mph and $v_2 = 110$ mph. ≈ 136.55 mph

b. Simplify the complex fraction. $\dfrac{2v_1v_2}{v_1+v_2}$

Enrichment Exercises

72. **Relativity Theory** Using Einstein's Theory of Relativity, the "sum" of the two speeds v_1 and v_2 is given by the complex fraction

$$\frac{v_1+v_2}{1+\dfrac{v_1v_2}{c^2}}$$

where c is the speed of light.

a. Evaluate this complex fraction with $v_1 = 1.2\times10^8$ mph, $v_2 = 2.4\times10^8$ mph, and $c = 6.7\times10^8$ mph. $\approx 3.4\times10^8$ mph

b. Simplify the complex fraction. $\dfrac{c^2(v_1+v_2)}{c^2+v_1v_2}$

73. An expression such as the one at the right is called a **continued fraction.** The pattern of the fraction *continues* on forever. A **convergent** of the continued fraction is an approximation that is found by stopping the process at some step. The first four convergents are shown at the right.

$$3+\cfrac{1}{6+\cfrac{3^2}{6+\cfrac{5^2}{6+\cfrac{7^2}{6+\cfrac{9^2}{6+\cfrac{11^2}{6+\dots}}}}}}$$

(i) 3 (ii) $3+\dfrac{1}{6}$

(iii) $3+\cfrac{1}{6+\cfrac{3^2}{6}}$

(iv) $3+\cfrac{1}{6+\cfrac{3^2}{6+\cfrac{5^2}{6}}}$

a. Write the expression for the fifth convergent.

b. What is the value of the sixth convergent? Round to the nearest ten-thousandth.

Note: It can be shown that as more and more convergents are evaluated, the values of the convergents become closer to π.

a. $3+\cfrac{1}{6+\cfrac{3^2}{6+\cfrac{5^2}{6+\cfrac{7^2}{6}}}}$

b. 3.1397

SECTION P.6

Complex Numbers

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A3.

In Exercises PS1 to PS5, simplify the expression.

PS1. $(2 - 3x)(4 - 5x)$ [P.3] $15x^2 - 22x + 8$

PS2. $(2 - 5x)^2$ [P.3] $25x^2 - 20x + 4$

PS3. $\sqrt{96}$ [P.2] $4\sqrt{6}$

PS4. $(2 + 3\sqrt{5})(3 - 4\sqrt{5})$ [P.2] $-54 + \sqrt{5}$

PS5. $\dfrac{5 + \sqrt{2}}{3 - \sqrt{2}}$ [P.2] $\dfrac{17 + 8\sqrt{2}}{7}$

PS6. Which of the following polynomials, if any, does not factor over the integers? [P.4] ii

i. $81 - x^2$ **ii.** $9 + z^2$

Math Matters

It may seem strange to just invent new numbers, but that is how mathematics evolves. For instance, negative numbers were not an accepted part of mathematics until well into the thirteenth century. In fact, these numbers often were referred to as "fictitious numbers."

In the seventeenth century, René Descartes called square roots of negative numbers "imaginary numbers," an unfortunate choice of words, and started using the letter *i* to denote these numbers. These numbers were subjected to the same skepticism as negative numbers.

It is important to understand that these numbers are not *imaginary* in the dictionary sense of the word. This misleading word is similar to the situation of negative numbers being called *fictitious*.

If you think of a number line, then the numbers to the right of zero are positive numbers and the numbers to the left of zero are negative numbers. One way to think of an imaginary number is to visualize it as *up* or *down* from zero.

Introduction to Complex Numbers

Recall that $\sqrt{9} = 3$ because $3^2 = 9$. Now consider the expression $\sqrt{-9}$. To find $\sqrt{-9}$, we need to find a number c such that $c^2 = -9$. However, the square of any real number c (except zero) is a *positive* number. Consequently, we must expand our concept of number to include numbers whose squares are negative numbers.

Around the seventeenth century, a new number, called an *imaginary number*, was defined so that a negative number would have a square root. The letter i was chosen to represent the number whose square is -1.

Definition of *i*

The **imaginary unit,** designated by the letter i, is the number such that $i^2 = -1$.

The principal square root of a negative number is defined in terms of i.

Definition of an Imaginary Number

If a is a positive real number, then $\sqrt{-a} = i\sqrt{a}$. The number $i\sqrt{a}$ is called an **imaginary number.**

EXAMPLE

$\sqrt{-36} = i\sqrt{36} = 6i$ $\sqrt{-18} = i\sqrt{18} = 3i\sqrt{2}$

$\sqrt{-23} = i\sqrt{23}$ $\sqrt{-1} = i\sqrt{1} = i$

Math Matters

The imaginary unit *i* is important in the field of electrical engineering. However, because the letter *i* is used by engineers as the symbol for electric current, these engineers use *j* for the complex unit.

It is customary to write i in front of a radical sign, as we did for $i\sqrt{23}$, to avoid confusing $\sqrt{a}i$ with $\sqrt{ai}$.