College Prep Algebra

Chapter P Notes

Section P.1: The Real Number System

Targets: I can answer questions in set notation accurately.

I can answer questions in interval notation accurately.

The Real Number System

Natural Numbers - $\{1, 2, 3, 4, 5, 6, \ldots\}$

Whole Numbers – All the natural numbers including 0; $\{0, 1, 2, 3, 4, 5, 6, \ldots\}$

Integers - $\{\ldots, -1, -2, -3, 0, 1, 2, 3 \ldots\}$

Rational Numbers = $\left\{\frac{p}{q}, \text{where } p \text{ and } q \text{ are intergers and } q \neq 0\right\}$

- Rational numbers can be written as a fraction or a decimal. If written as a decimal it will be either a terminating decimal such as 0.65 or a repeating decimal such as 0.218181818…

Irrational Numbers – numbers that *cannot* be expressed as terminating or repeating decimals.

Properties of Real Numbers

Let *a*, *b*, and *c* be real numbers.

	Addition Properties	**Multiplication Properties**
Closure	$a+b$ is a unique real number.	ab is a unique real number.
Commutative	$a+b=b+a$	$ab=ba$
Associative	$(a+b)+c=a+(b+c)$	$(ab)c=a(cb)$
Identity	There exists a unique real number 0 such that $a+0=0+a=a$.	There exists a unique real number 1 such that $a \bullet 1 = 1 \bullet a = a$.
Inverse	For each real number *a*, there is a unique real number $-a$ such that $a+(-a)=(-a)+a=0$.	For each *nonzero* real number *a*, there is a unique real number $\frac{1}{a}$ such that $a\left(\frac{1}{a}\right)=\left(\frac{1}{a}\right)a=1$.
Distributive	$a(b+c)=ab+ac$	

Properties of Equality

Let *a*, *b*, and *c* be real numbers.

Reflexive	$a=a$
Symmetric	If $a=b$, then $b=a$.
Transitive	If $a=b$ and $b=c$, the $a=c$.
Substitution	If $a=b$, then a may be replaced by b in any expression that involves a.

Set Notation

Element ($\in$) – every member of a set is called an element. Ex: If $C = \{1,5,7\}$, then the elements of C are 1, 5, and 7.

The notation $1 \in C$ is read "1 is an element of C."

Subset ($\subseteq$) – Set A is a subset of B if every $\in$ in A is also an $\in$ of B. The notation $A \subseteq B$ is read "A is a subset of B."

Ex: $A = \{1,2,3\}$ and $B = \{\text{natural numbers}\}$

Empty Set or **Null Set** ($\varnothing$) is a set that contains no elements. Ex: The set of people who have run a 2-minute mile is the empty set.

Finite Set – all $\in$ of the set can be listed. Ex: The set of natural numbers less than 6 is $\{1,2,3,4,5\}$.

Infinite Set – All the elements of the set cannot be listed. Ex: The set of all integers.

Set-builder Notation – The set of real numbers greater than 2 is written; $\{x \mid x > 2, x \in \text{real numbers}\}$ and is read "the set of x such that x is greater than 2 and *x* is an element of real numbers.

Shortened form: $\{x \mid x > 2\}$ for this we assume that *x* is a real number.

List the four smallest elements of each set.

1. $\{n^3 \mid n \in \text{natural numbers}\}$ ______________________
2. $\{y \mid y = x^2 - 1, x \in \text{integers}\}$ ______________________

Union and Intersection of Sets

Union ($\cup$) – Written $A \cup B$, is the set of all elements that belong to either A or B. In set-builder notation, this is written $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

Intersection ($\cap$) - Written $A \cap B$, is the set of all elements that are common to both A and B. In set-builder notation, this is written $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

Examples:

Find the intersection or union given $A = \{0,2,4,6,10,12\}$, $B = \{0,3,6,12,15\}$, $C = \{1,2,3,4,5,6,7\}$, and $D = \{18,20,22\}$.

3. $A \cup C$ ________________________

4. $B \cap D$ ________________________

5. $A \cap (B \cup C)$ ____________________

6. $B \cup (A \cap C)$ ____________________

Interval Notation

< or > we will now us (or) instead of an open circle. $\leq$ **or** $\geq$ **we will not us** $[or]$ **instead of a closed circle.**

- (a,b) represents all real numbers between *a* and *b*. This is an **open interval**. In set-builder notation, we write $\{x|a<x<b\}$.
- $[a,b]$ represents all real number between *a* and *b*, including *a* and *b*. This is a **closed interval**. In set-builder notation, we write $\{x|a\leq x\leq b\}$.
- $(a,b]$ represents all real numbers between *a* and *b*, not including *a* but including *b*. This is a **half-open interval**. In set-builder notation, we write $\{x|a<x\leq b\}$.
- $[a,b)$ represents all real numbers between *a* and *b*, including *a* but not including *b*. This a **half-open interval**. In set-builder notation, we write $\{x|a\leq x<b\}$.

$(-\infty,a)$ represents all real numbers less than *a*.

(b,∞) represents all real numbers greater than *b*.

$(-\infty,a]$ represents all real numbers less than or equal to *a*.

$[b,\infty)$ represents all real numbers greater than or equal to *b*.

Graph Intervals

Graph the following. Write 7 and 8 using interval notation . Write 9 and 10 using set-builder notation.

7. $\{x|x\leq -1\}\cup\{x|x\geq 2\}$ ______________________________ 8. $\{x|x\geq -1\}\cap\{x|x<5\}$ ______________________________

9. $(-\infty,0)\cup[1,3]$ ______________________________ 10. $[-1,3]\cap(1,5)$ ______________________________

Section P.2 Notes: Integer and Rational Number Exponents

Targets: I can the properties of exponents to simplify and evaluate problems accurately.

I can evaluate and simplify problems with radicals and rational exponents accurately.

Exponents

Definition of b^0: For any nonzero real number b, $b^0 = 1$.

Definition of b^{-n}: If $b \neq 0$ and n is a natural number, then $b^{-n} = \frac{1}{b^n}$ **and** $\frac{1}{b^{-n}} = b^n$.

Properties of Exponents

If m, n, and p are integers and a and b are real numbers, then

Product: $b^m \bullet b^n = b^{m+n}$

Quotient: $\frac{b^m}{b^n} = b^{m-n},\ b \neq 0$

Power:

$$\left(b^m\right)^n = b^{mn}$$

$$\left(a^m b^n\right)^p = a^{mp} b^{mp}$$

$$\left(\frac{a^m}{b^n}\right)^p = \frac{a^{mp}}{b^{np}},\ b \neq 0$$

Evaluate.

1. $(-2^4)(-3)^2$

2. $\frac{(-4)^{-3}}{(-2)^{-5}}$

3. $-\pi^0$

Simplify.

4. $\left(5x^2y\right)\left(-4x^3y^5\right)$

5. $\left(3x^2yz^{-4}\right)^3$

6. $\frac{-12x^5y}{18x^2y^6}$

7. $\left(\frac{4p^2q}{6pq^4}\right)^{-2}$

Scientific Notation

Write the number in scientific notation.

8. 7,430,000

9. 0.00000078

Change the number from scientific notation to decimal notation.

10. 3.5×10^{5}

11. 2.51×10^{-8}

Simplifying Scientific notation

12. $\left(9.5 \times 10^{4}\right)\left(5.7 \times 10^{12}\right)$

13. $\dfrac{3.8 \times 10^{8}}{3.0 \times 10^{8}}$

Day 2: Rational Exponents and Radicals

$25^{\frac{1}{2}} = \sqrt{25}$

$4^{\frac{3}{2}} =$ ______________________ $= \sqrt{4^{3}}$

$64^{\frac{1}{3}} = \sqrt[3]{64}$

$5^{\frac{2}{5}} =$ ______________________ $= \sqrt[5]{5^{2}}$

$81^{\frac{1}{4}} =$ ______________

$\left(\sqrt[3]{5}\right)^{2} =$ ______________ $= 5^{\frac{2}{3}}$

Definition of $\sqrt[n]{b^{n}}$:

If n is an even natural number and b is a real number, then $\sqrt[n]{b^{n}} = |b|$

If n is an odd natural number and b is a real number, then $\sqrt[n]{b^{n}} = b$

Simplify.

1. $64^{\frac{2}{3}}$

2. $32^{-\frac{3}{5}}$

3. $\left(\dfrac{16}{81}\right)^{-\frac{3}{4}}$

4. $\left(2x^{\frac{1}{3}}y^{\frac{3}{5}}\right)^{2}\left(9x^{3}y^{\frac{3}{2}}\right)^{\frac{1}{2}}$

5. $\dfrac{\left(a^{\frac{3}{4}}b^{\frac{1}{2}}\right)^2}{\left(a^{\frac{2}{3}}b^{\frac{3}{4}}\right)^3}$

6. $\sqrt{48x^7y^2}$

7. $\sqrt[3]{162x^4y^6}$

8. $\sqrt[4]{32x^3y^4}$

Day 3: Combining Radical Expressions: To combine **like radicals** they must have the same radicand and the same index.

Example: $3\sqrt[3]{5xy^2} - 4\sqrt[3]{5xy^2} =$ ______________________

Simplify.

1. $2\sqrt{2x^3} + 4x\sqrt{8x}$

2. $5x\sqrt[3]{16x^4} - \sqrt[3]{128x^7}$

3. $2b\sqrt[3]{16b^2} + \sqrt[3]{128b^5}$

Multiplying Radical Expressions

4. $\left(5\sqrt{6} - 7\right)\left(3\sqrt{6} + 4\right)$

5. $\left(3 - \sqrt{x-7}\right)^2, x \geq 7$

<u>Rationalizing the Denominator:</u> Recall you are not allowed to have a radical in the denominator.

Simplify.

6. $\dfrac{5}{\sqrt[3]{a}}$

7. $\sqrt{\dfrac{3}{32y}}, y > 0$

8. $\sqrt{\dfrac{5x}{10y}}$

9. $\dfrac{3+2\sqrt{5}}{1-4\sqrt{5}}$

10. $\dfrac{2+4\sqrt{x}}{3-5\sqrt{x}}, x > 0$

Section P.3: Polynomials

Targets: I can simplify polynomials with different operations accurately.

	Examples:	Terms:	Degree:	Standard Form
Monomial	-8 z $7y$ $-12a^2bc^3$			
Binomial	$2xy-y^2$ $3x^4-7$			
Trinomial	$2x^2-3xy+7y^2$ $3x^2+6x-1$			
Polynomial	$5x^4-6x^3-8+5x^2-7x$			

Simplify.

1. $\left(3x^3-2x^2-6\right)+\left(4x^2-6x-7\right)$

2. $\left(6x^3-3x^2+5\right)\left(5x+4\right)$

3. $\left(7x-2\right)\left(5x+4\right)$

<u>Special Product Formulas</u>

(Sum)(Difference)	$(x+y)(x-y)=x^2-y^2$
(Binomial)^2	$(x+y)^2=x^2+2xy+y^2$ $(x-y)^2=x^2-2xy+y^2$

Simplify.

4. $(7x+10)(7x-10)$

5. $(3x+4y)^2$

Section P.4: Factoring

Targets: I can factor binomials and trinomials accurately.

I can factor by grouping or using special factors (difference of squares, difference or sum of perfect cubes, etc.).

I can factor a trinomial that is quadratic in form.

<u>Greatest Common Factor (GCF):</u> Factor out the GCF

1. $-6x^2y^2+3xy^2$
2. $12x^3y^4-24x^2y^5+18xy^6$
3. $(6x-5)(4x+3)-(4x+3)(3x-7)$

Factorization Theorem: The trinomial ax^2+bx+c, with integer coefficients a, b, and c, can be factored as the product of two binomials with integer coefficients if and only if b^2-4ac is a perfect square.

Factoring Trinomials: Factor

4. $x^2+7x-18$
5. $x^2+7xy+10y^2$
6. $6x^2-11x+4$
7. $4x^2-17x-21$

Special Factors

Difference of 2 Perfect Squares	$a^2 - b^2 = (a+b)(a-b)$
Perfect Square Trinomials	$a^2 + 2ab + b^2 = (a+b)^2$ $a^2 - 2ab + b^2 = (a-b)^2$
Sum or Difference of 2 Perfect Cubes	$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Factor.

1. $49x^2 - 144$
2. $8x^3 + y^3$
3. $16m^2 - 40mn + 25n^2$
4. $9x^2 - 121$
5. $12x^2 + 36x + 27$
6. $a^3 - 64$
7. $x^2 - 64$
8. $x^4 + 8x$

Factor a Polynomial that is Quadratic in Form

A trinomial can be expressed as quadratic trinomial by making suitable variable substitutions. A trinomial is **quadratic in form** if it can be written as, $au^2 + bu + c$.

Factor.

1. $6x^2y^2 - xy - 12$
2. $x^4 + 5x^2 - 36$
3. $2x^4 - 15x^2 - 27$

Factoring by Grouping

Factor.

4. $p^2 + p - q - q^2$
5. $2ax + 4bx - 3ay - 6by$
6. $a^2 + 10ab + 25b^2 - c^2$

Section P.5: Rational Expressions

Target: I can simplify rational expressions, complex fractions, and fractions.

A **rational expression** is a fraction in which the numerator and denominator are polynomials.

Examples: $\frac{3}{x+1}$ and $\frac{x^2-4x-21}{x^2-9}$

The **domain of a rational expression** is the set of all real numbers that can be used as replacements from the variables, except values that make the denominator zero.

Properties of Rational Expressions

For all rational expressions $\frac{P}{Q}$ and $\frac{R}{S}$, where $Q \neq 0$ and $S \neq 0$,

Equality: $\frac{P}{Q} = \frac{R}{S}$ if and only if $PS = QR$

Equivalent expressions: $\frac{P}{Q} = \frac{PR}{QR}, R \neq 0$

Sign: $-\frac{P}{Q} = \frac{-P}{Q} = \frac{P}{-Q}$

Simplify

1. $\frac{7+20x-3x^2}{2x^2-11x-21}$

2. $\frac{6x^3-15x^2}{12x^2-30x}$

3. $\frac{3x^2+10x-8}{8-14x+3x^2}$

4. $\frac{4-x^2}{x^2+2x-8} \cdot \frac{x^2-11x+28}{x^2-5x-14}$

5. $\frac{x^2+6x+9}{x^3+27} \div \frac{x^2+7x+12}{x^3-3x^2+9x}$

6. $\frac{2x+1}{x-3} + \frac{x+2}{x+5}$

7. $\frac{39x+36}{x^2-3x-10} - \frac{23x-16}{x^2-7x+10}$

Simplify

8. $\frac{x+3}{x-2}-\frac{x+4}{x-1}\div\frac{x^2+5x+4}{x^2+4x-5}$

9. $\frac{x+4}{x-5}+\frac{x-3}{x-4}\cdot\frac{x^2-5x+4}{x^2-9}$

10. $$\frac{\frac{2}{x-2}+\frac{1}{x}}{\frac{3x}{x-5}-\frac{2}{x-5}}$$

11. $$4-\frac{2x}{2-\frac{x-2}{x}}$$

Simplify a fraction.

12. $$\frac{c^{-1}}{a^{-1}+b^{-1}}$$

13. $$\frac{x^{-1}}{y^{-1}}+\frac{y^{-1}}{x^{-1}}$$

Section P.6: Complex Numbers

Targets: I can write complex numbers in standard form.
I can add, subtract, multiply, and divide complex numbers accurately.
I can evaluate the power of i.

Definition of an Imaginary Number

If a is a positive real number, then $\sqrt{-a} = i\sqrt{a}$. The number $i\sqrt{a}$ is called an **imaginary number**.

Definition of a Complex Number

A **complex number** is a number of the form $a+bi$, where a and b are real numbers and $i=\sqrt{-1}$. The number a is the **real part** of $a+bi$, and b is the **imaginary part**. Examples: ____

Powers of i: $i^1 = i,\quad i^2 = -1,\quad i^3 = -i,\quad i^4 = 1$

Write the complex number in standard form.

1. $7+\sqrt{-45}$

2. $4-\sqrt{-72}$

Simplify.

3. $(7-2i)+(-2+4i)$

4. $(-9+4i)-(2-6i)$

5. $3i(2-5i)$

6. $(3-4i)(2-5i)$

7. $\dfrac{3-6i}{2i}$

8. $\dfrac{3+2i}{5-i}$

Powers of i
If n is a positive integer, then $i^n = i^r$, which is the remainder of the division of n by 4.

Evaluate

9. i^{153}

10. i^{214}

11. i^{19}

12. i^{12}