After you have found the solution, you can edit one of the constraints, or the objective function, to see what effect this has on the solution and tell whether the new solution still occurs at the same ordered pair. For instance, edit the objective function so that it becomes $3x + y$. Then you will find that the new solution is 18 and it occurs at $(x, y) = (6, 0)$.

Use the following text to produce a graph of the set of feasible solutions.

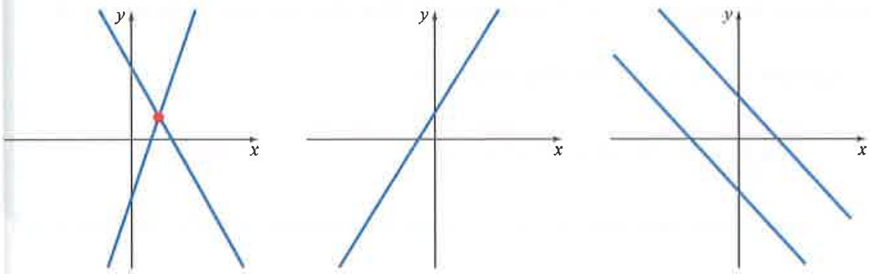
```
plot {2x + y ≤ 12 && 0.3x + y ≤ 6 && 8x + 7y ≤ 56 && x ≥ 0 && y ≥ 0}
```

CHAPTER 6 TEST PREP

The following test prep table summarizes essential concepts in this chapter. The references given in the right-hand column list Examples and Exercises that can be used to test your understanding of a concept.

6.1 Systems of Linear Equations in Two Variables

Systems of Linear Equations in Two Variables A solution of a system of two linear equations in two variables is an ordered pair that satisfies each equation of the system. Systems of equations are equivalent if the systems have exactly the same solutions. The substitution method and the elimination method are often used to solve these systems. • ***Substitution Method*** Solve one of the equations to find an expression for one variable in terms of the other variable. Substitute this expression into the other equation to produce an equation that involves only one variable. • ***Elimination Method*** Multiply one or both equations by appropriate nonzero constants so that the sum of the resulting equations is an equation in one variable. The elimination method uses the following operations to produce equivalent systems until the solution or solutions of the original system are apparent. 1. Interchange any two equations. 2. Replace an equation with a nonzero constant multiple of that equation. 3. Replace an equation with the sum of that equation and a nonzero constant multiple of another equation.	See Examples 1 and 4, pages 479 and 482, and then try Exercises 2 and 3, page 536.
Classification of Systems of Equations A system of equations is a consistent system if it has at least one solution. A system of equations with no solution is an inconsistent system. • A system of linear equations with exactly one solution is an independent system. A system of linear equations with an infinite number of solutions is a dependent system. The graphs of the two equations in a linear system of two variables can intersect at a single point, be the same line, or be parallel lines. See the graphs on page 534.	See Examples 2 and 3, pages 479 and 480, and then try Exercises 7 and 8, page 536.



a. Independent system
Exactly one solution

b. Dependent system
Infinitely many solutions

c. Inconsistent system
No solution

6.2 Systems of Linear Equations in Three Variables

Systems of Linear Equations in Three Variables An equation of the form $ax + by + cz = d$, with constants a, b, and c not all zero, is a linear equation in three variables. A solution of a linear system of equations in three variables is an ordered triple whose coordinates satisfy each of the equations in the system. The elimination method is often used to solve systems of linear equations in three variables by rewriting the system in an equivalent triangular form.	See Examples 1 and 2, pages 491 and 492, and then try Exercises 9 and 11, page 536.
Nonsquare Systems of Equations A system of linear equations with fewer equations than variables forms a nonsquare system of equations. These systems of equations have either no solution or an infinite number of solutions. The elimination method can often be used to solve these systems.	See Example 4, page 495, and then try Exercises 17 and 18, page 536.
Homogeneous Systems of Equations A system of linear equations in which the constant term is 0 for all equations is called a homogeneous system of equations. The ordered triple (0, 0, 0) is always a solution of a homogeneous system of linear equations in three variables. This solution is called the trivial solution. A homogeneous system of linear equations will have exactly one solution (the trivial solution) or an infinite number of solutions.	See Example 5, page 495, and then try Exercises 15 and 16, page 536.

6.3 Nonlinear Systems of Equations

Solutions of Nonlinear Systems of Equations A nonlinear system of equations is a system in which one or more equations of the system are nonlinear. The substitution method and the elimination method are often used to find the solutions to these systems. A graph of the equations in the system can be used to visualize how many solutions to expect and the approximate coordinates of the solutions.	See Examples 1 and 2, pages 501 and 502, and then try Exercises 23 and 28, page 536.

6.4 Partial Fractions

Partial Fraction Decomposition A rational expression can be written as the sum of terms whose denominators are factors of the denominator of the rational expression. This sum is called a partial fraction decomposition of the rational expression. The procedure for finding a partial fraction decomposition of a rational expression depends on the factors in its denominator. The Partial Fraction Decomposition Theorem requires that the degree of the numerator of the rational expression be less than the degree of its denominator. If this is not the case, use long division to first write the rational expression as the sum of a polynomial and a rational expression whose numerator is of lesser degree than its denominator.	See Examples 1, 2, 3, 4, and 5, pages 510–513, and then try Exercises 31, 34, and 36, page 536.

6.5 Inequalities in Two Variables and Systems of Inequalities

Inequalities in Two Variables The solution set of an inequality in two variables is the set of all ordered pairs that satisfy the inequality. The graph of an inequality is the graph of its solution set.	See Examples 1 and 2, page 516, and then try Exercises 39 and 41, page 536.
Systems of Inequalities The solution set of a system of inequalities is the intersection of all solution sets of the individual inequalities.	See Examples 4 and 6, pages 517 and 519, and then try Exercises 51 and 59, pages 536–537.

6.6 Linear Programming

Optimization A linear programming problem involves a linear objective function that is to be maximized or minimized subject to a number of constraints, which are inequalities or equations that restrict the values of the variables. The Fundamental Linear Programming Theorem states that if an objective function has an optimal solution, then that solution will occur at a vertex of the set of feasible solutions.	See Examples 1, 2, and 3, pages 525–527, and then try Exercises 67 and 72, page 537.

CHAPTER 6 REVIEW EXERCISES

In Exercises 1 to 30, solve each system of equations.

1. $\begin{cases} 5x - 3y = -4 \\ 2x + 5y = 11 \end{cases}$

2. $\begin{cases} 6x + 3y = 4 \\ 5x - 2y = 8 \end{cases}$

3. $\begin{cases} 7x + 2y = 4 \\ y = \frac{2}{5}x + 3 \end{cases}$

4. $\begin{cases} 3x + y = -7 \\ y = -\frac{3}{5}x + 1 \end{cases}$

5. $\begin{cases} y = 2x - 5 \\ x = 4y - 1 \end{cases}$ $y - 2x = -5$, $x - 4y = -1$

6. $\begin{cases} y = 3x + 4 \\ x = 4y - 5 \end{cases}$

7. $\begin{cases} 6x + 9y = 15 \\ 10x + 15y = 25 \end{cases}$

8. $\begin{cases} 4x - 8y = 9 \\ 2x - 4y = 5 \end{cases}$

9. $\begin{cases} 2x - 3y + z = -9 \\ 2x + 5y - 2z = 18 \\ 4x - y + 3z = -4 \end{cases}$

10. $\begin{cases} x - 3y + 5z = 1 \\ 2x + 3y - 5z = 15 \\ 3x + 6y + 5z = 15 \end{cases}$

11. $\begin{cases} x + 3y - 5z = -12 \\ 3x - 2y + z = 7 \\ 5x + 4y - 9z = -17 \end{cases}$

12. $\begin{cases} 2x - y + 2z = 5 \\ x + 3y - 3z = 2 \\ 5x - 9y + 8z = 13 \end{cases}$

13. $\begin{cases} 3x + 4y - 6z = 10 \\ 2x + 2y - 3z = 6 \\ x - 6y + 9z = -4 \end{cases}$

14. $\begin{cases} x - 6y + 4z = 6 \\ 4x + 3y - 4z = 1 \\ 5x - 9y + 8z = 13 \end{cases}$

15. $\begin{cases} 2x + 3y - 2z = 0 \\ 3x - y - 4z = 0 \\ 5x + 13y - 4z = 0 \end{cases}$

16. $\begin{cases} 3x - 5y + z = 0 \\ x + 4y - 3z = 0 \\ 2x + y - 2z = 0 \end{cases}$

17. $\begin{cases} 2x - y + 3z = 6 \\ x + 2y + 4z = 10 \end{cases}$

18. $\begin{cases} 2x - 3y + z = 1 \\ 4x + 2y + 3z = 21 \end{cases}$

19. $\begin{cases} y = x^2 - 2x - 3 \\ y = 2x - 7 \end{cases}$

20. $\begin{cases} y = 2x^2 + x \\ y = 2x + 1 \end{cases}$

21. $\begin{cases} (x - 1)^2 - y = 4 \\ (x + 2)^2 - y = 9 \end{cases}$

22. $\begin{cases} y = 4x^2 - 2x - 3 \\ y = 2x^2 + 3x - 6 \end{cases}$

23. $\begin{cases} (x + 1)^2 + (y - 2)^2 = 4 \\ 2x + y = 4 \end{cases}$

24. $\begin{cases} (x - 1)^2 + (y + 1)^2 = 5 \\ y = 2x - 3 \end{cases}$

25. $\begin{cases} (x - 2)^2 + (y - 2)^2 = 1 \\ (x - 1)^2 + (y + 2)^2 = 16 \end{cases}$

26. $\begin{cases} (x - 1)^2 - (y + 2)^2 = 4 \\ (x - 2)^2 + (y + 2)^2 = 9 \end{cases}$

27. $\begin{cases} x^2 - 3xy + y^2 = -1 \\ 3x^2 - 5xy - 2y^2 = 0 \end{cases}$

28. $\begin{cases} 2x^2 + 2xy - y^2 = -1 \\ 6x^2 + xy - y^2 = 0 \end{cases}$

29. $\begin{cases} 2x^2 - 5xy + 2y^2 = 56 \\ 14x^2 - 3xy - 2y^2 = 56 \end{cases}$

30. $\begin{cases} 2x^2 + 7xy + 6y^2 = 1 \\ 6x^2 + 7xy + 2y^2 = 1 \end{cases}$

In Exercises 31 to 36, find the partial fraction decomposition.

31. $\dfrac{5x + 1}{x^2 + x - 6}$

32. $\dfrac{3x - 1}{(x - 5)^2}$

33. $\dfrac{2x - 2}{(x^2 + 1)(x + 2)}$

34. $\dfrac{5x^2 - 10x + 9}{(x - 2)^2(x + 1)}$

35. $\dfrac{11x^2 - x - 2}{x^3 - x}$

36. $\dfrac{x^4 + x^3 + 4x^2 + x + 3}{(x^2 + 1)^2}$

37. $\dfrac{x^3 - 2x^2 + 5x - 3}{x^2 + 3x}$

38. $\dfrac{x^4 - 2x^2 + 5x - 1}{x^3 - 2x^2}$

In Exercises 39 to 50, graph the solution set of each inequality.

39. $4x - 5y < 20$

40. $2x + 7y \geq -14$

41. $y \geq 2x^2 - x - 1$

42. $y < x^2 - 5x - 6$

43. $(x - 2)^2 + (y - 1)^2 > 4$

44. $(x + 3)^2 + (y + 1)^2 \leq 9$

45. $\dfrac{(x - 3)^2}{16} - \dfrac{(y + 2)^2}{25} \leq 1$

46. $\dfrac{(x + 1)^2}{9} - \dfrac{(y - 3)^2}{4} < -1$

47. $(2x - y + 1)(x - 2y - 2) > 0$

48. $(2x - 3y - 6)(x + 2y - 4) < 0$

49. $x^2y^2 < 1$

50. $xy \geq 0$

In Exercises 51 to 62, graph the solution set of each system of inequalities.

51. $\begin{cases} 2x - 5y < 9 \\ 3x + 4y \geq 2 \end{cases}$

52. $\begin{cases} 3x + y > 7 \\ 2x + 5y < 9 \end{cases}$

53. $\begin{cases} 2x + 3y > 6 \\ 2x - y > -2 \\ x \leq 4 \end{cases}$

54. $\begin{cases} 2x + 5y > 10 \\ x - y > -2 \\ x \leq 4 \end{cases}$

55. $\begin{cases} 2x + 3y \leq 18 \\ x + y \leq 7 \\ x \geq 0, y \geq 0 \end{cases}$

56. $\begin{cases} 3x + 5y \geq 25 \\ 2x + 3y \geq 16 \\ x \geq 0, y \geq 0 \end{cases}$

57. $\begin{cases} 3x + y \geq 6 \\ x + 4y \geq 14 \\ 2x + 3y \geq 16 \\ x \geq 0, y \geq 0 \end{cases}$

58. $\begin{cases} 3x + 2y \geq 14 \\ x + y \geq 6 \\ 11x + 4y \leq 48 \\ x \geq 0, y \geq 0 \end{cases}$

59. $\begin{cases} y < x^2 - x - 2 \\ y \geq 2x - 4 \end{cases}$

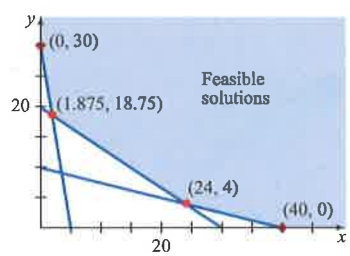
60. $\begin{cases} y > 2x^2 + x - 1 \\ y > x + 3 \end{cases}$

61. $\begin{cases} x^2 + y^2 - 2x + 4y > 4 \\ y < 2x^2 - 1 \end{cases}$

62. $\begin{cases} x^2 - y^2 - 4x - 2y < -4 \\ x^2 + y^2 - 4x + 4y > 8 \end{cases}$

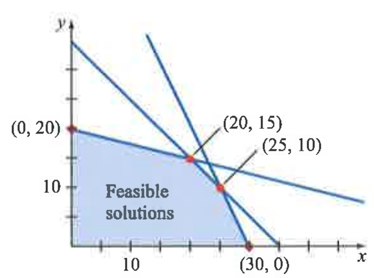
63. Find the minimum value of the given objective function for the given set of feasible solutions. Also state where the objective function takes on its minimum value.

Objective function: $C = \frac{1}{2}x + y$



64. Find the maximum value of the given objective function for the given set of feasible solutions. Also state where the objective function takes on its maximum value.

Objective function: $C = 3.5x + 5y + 4$



In Exercises 65 to 70, solve the linear programming problem. In each problem, assume $x \geq 0$ and $y \geq 0$.

65. Objective function: $P = 2x + 2y$

Constraints: $\begin{cases} x + 2y \leq 14 \\ 5x + 2y \leq 30 \end{cases}$

Maximize the objective function.

66. Objective function: $P = 4x + 5y$

Constraints: $\begin{cases} 2x + 3y \leq 24 \\ 4x + 3y \leq 36 \end{cases}$

Maximize the objective function.

67. Objective function: $P = 4x + y$

Constraints: $\begin{cases} 5x + 2y \geq 16 \\ x + 2y \geq 8 \\ x \leq 20, y \leq 20 \end{cases}$

Minimize the objective function.

68. Objective function: $P = 2x + 7y$

Constraints: $\begin{cases} 4x + 3y \geq 24 \\ 4x + 7y \geq 40 \\ x \leq 10, y \leq 10 \end{cases}$

Minimize the objective function.

69. Objective function: $P = 6x + 3y$

Constraints: $\begin{cases} 5x + 2y \geq 20 \\ x + y \geq 7 \\ x + 2y \geq 10 \\ x \leq 15, y \leq 15 \end{cases}$

Minimize the objective function.

70. Objective function: $P = x + y$

Constraints: $\begin{cases} x + 2y \leq 1000 \\ 3x + y \leq 900 \\ 2x + y \leq 1000 \end{cases}$

Maximize the objective function.

71. **Maximize Profit** An engine reconditioning company works on 4- and 6-cylinder engines. Each 4-cylinder engine requires 1 hour for cleaning, 5 hours for overhauling, and 3 hours for testing. Each 6-cylinder engine requires 1 hour for cleaning, 10 hours for overhauling, and 2 hours for testing. The cleaning station is available for at most 9 hours. The overhauling equipment is available for at most 80 hours, and the testing equipment is available for at most 24 hours. For each reconditioned 4-cylinder engine, the company makes a profit of \$150. A reconditioned 6-cylinder engine yields a profit of \$250. The company can sell all the reconditioned engines it produces. How many of each type should be produced to maximize profit? What is the maximum profit?

72. **Maximize Profit** A manufacturer makes two types of golf clubs: a starter model and a professional model. The starter model requires 4 hours in the assembly room and 1 hour in the finishing room. The professional model requires 6 hours in the assembly room and 1 hour in the finishing room. The total number of hours available in the assembly room is 108. There are 24 hours available

in the finishing room. The profit for each starter model is \$35, and the profit for each professional model is \$55. Assuming all the sets produced can be sold, find how many of each set should be manufactured to maximize profit.

In Exercises 73 to 79, solve each exercise by solving a system of equations.

73. Find an equation of the form $y = ax^2 + bx + c$ whose graph passes through the points $(1, 0)$, $(-1, 5)$, and $(2, 3)$.

74. Find an equation of the circle that passes through the points $(4, 2)$, $(0, 1)$, and $(3, -1)$.

75. Find an equation of the plane that passes through the points $(2, 1, 2)$, $(3, 1, 0)$, and $(-2, -3, -2)$. Use the equation $z = ax + by + c$.

76. **Chemistry** How many liters of a 20% acid solution should be mixed with 10 liters of a 10% acid solution so that the result is a 16% acid solution?

77. **Uniform Motion** Flying with the wind, a small plane traveled 855 miles in 5 hours. Flying against the wind, the same plane traveled 575 miles in the same time. Find the rate of the wind and the rate of the plane in calm air.

78. **Commerce** A collection of 10 coins has a value of \$1.25. The collection consists of only nickels, dimes, and quarters. How many of each coin are in the collection? (*Hint:* There is more than one solution.)

79. Consider the ordered triple (a, b, c). Find all real number values for a, b, and c so that the product of any two numbers equals the remaining number.

CHAPTER 6 TEST

In Exercises 1 to 8, solve each system of equations.

1. $\begin{cases} 3x + 2y = -5 \\ 2x - 5y = -16 \end{cases}$

2. $\begin{cases} x - \frac{1}{2}y = 3 \\ 2x - y = 6 \end{cases}$

3. $\begin{cases} 2x - 3y + z = 9 \\ x - 5y + 3z = 5 \\ 3x - y - 2z = 8 \end{cases}$

4. $\begin{cases} 3x - 2y + z = 2 \\ x + 2y - 2z = 1 \\ 4x - z = 3 \end{cases}$

5. $\begin{cases} 2x - 3y + z = -1 \\ x + 5y - 2z = 5 \end{cases}$

6. $\begin{cases} 4x + 2y + z = 0 \\ x - 3y - 2z = 0 \\ 3x + 5y + 3z = 0 \end{cases}$

7. $\begin{cases} 8x = 3y + 10 \\ y = x^2 - 2x - 5 \end{cases}$

8. $\begin{cases} x^2 + y^2 = 16 \\ y = 2x^2 - 4 \end{cases}$

In Exercises 9 and 10, graph each inequality.

9. $x^2 + 4y^2 \geq 16$

10. $x + y^2 < 0$

In Exercises 11 to 14, graph each system of inequalities.

11. $\begin{cases} 2x - 5y \leq 16 \\ x + 3y \geq -3 \end{cases}$

12. $\begin{cases} x^2 + y^2 > 9 \\ x^2 + y^2 < 4 \end{cases}$

13. $\begin{cases} x + y \geq 8 \\ 2x + y \geq 11 \\ x \geq 0, y \geq 0 \end{cases}$

14. $\begin{cases} 2x + 3y \leq 12 \\ x + y \leq 5 \\ 3x + 2y \leq 11 \\ x \geq 0, y \geq 0 \end{cases}$

In Exercises 15 to 17, find the partial fraction decomposition.

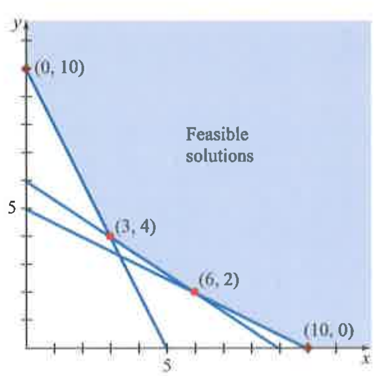
15. $\dfrac{3x - 5}{x^2 - 3x - 4}$

16. $\dfrac{2x + 1}{x(x^2 + 1)}$

17. $\dfrac{x^3 + x - 2}{x^2 + x - 12}$

18. Find the minimum value of the given objective function for the given set of feasible solutions. Also state where the objective function takes on its minimum value.

Objective function: $C = 5x + 4y$



19. Field Dimensions A soccer stadium has a grass field in the shape of a rectangle with semicircles at the two ends. See the following figure. The perimeter of the entire grass field is approximately 554.16 meters, and the distance x is 20 meters longer than the distance y. Use a system of equations to find the x and y dimensions of the field. Round to the nearest meter.

20. Parking Rates A parking garage charges its customers a certain amount for the first hour and another amount for each additional half-hour or part of the half-hour. One day Nicole parked her car for 3 hours and 50 minutes. The parking fee was \$14.50. The next day she parked her car for 4 hours and 45 minutes. The parking fee was \$18.00. Determine the fee the parking garage charges for the first hour and the fee the garage charges for each additional half-hour or a portion of the half-hour.

21. Maximize Profit A farmer has 160 acres available on which to plant oats and barley. It costs \$15 per acre for oat seed and \$13 per acre for barley seed. The labor cost is \$15 per acre for oats and \$20 per acre for barley. The farmer has \$2200 available to purchase seed and has set aside \$2600 for labor. The profit per acre for oats is \$120, and the profit per acre for barley is \$150. How many acres of oats and how many acres of barley should the farmer plant to maximize profit?

22. Curve Fitting Find an equation of the circle that passes through the points $(3, 5)$, $(-3, -3)$, and $(4, 4)$. (*Hint:* Use $x^2 + y^2 + ax + by + c = 0$.)

CUMULATIVE REVIEW EXERCISES

1. Find the slope of the line that passes through the points $\left(-\frac{1}{2}, 2\right)$ and $\left(4, -\frac{1}{3}\right)$.

2. Find the range of $f(x) = -x^2 + 2x - 4$.

3. Evaluate $3x^4 - 4x^3 + 2x^2 - x + 1$ for $x = -2$.

4. Write $\log_6(x - 5) + 3\log_6(2x)$ as a single logarithm with a coefficient of 1.

5. Find the equation in standard form of the parabola that has the vertex $(4, 2)$, has an axis of symmetry parallel to the y-axis, and passes through the point $(-1, 1)$.

6. Solve $\frac{1}{F} = \frac{1}{d_0} + \frac{1}{d_1}$ for d_0.

7. Find the equation of the line that passes through $P_1(-4, 2)$ and $P_2(2, -1)$.

8. Let $f(x) = \frac{x^2 - 1}{x^4}$. Is f an even function, an odd function, or neither?

9. Solve: $\log x - \log(2x - 3) = 2$

10. Find the equation in standard form of the hyperbola with vertices $(2, 2)$ and $(10, 2)$ and an eccentricity of 3.

11. Given $g(x) = \frac{x - 2}{x}$, find $g\left(-\frac{1}{2}\right)$.

12. Given $f(x) = x^2 - 1$ and $g(x) = x^2 - 4x - 2$, find $(f \cdot g)(-2)$.

13. Evaluate: $\log_{0.25} 0.015625$

14. Find the quadratic regression model for the data

$$\{(1, 1), (2, 3), (3, 10), (4, 17), (5, 26)\}$$

15. Find the polynomial of lowest degree that has zeros of -2, $3i$, and $-3i$.

16. Find the inverse function of $Q(r) = \frac{2}{1 - r}$.

17. Find the slant asymptote of the graph of

$$H(x) = \frac{2x^3 - x^2 - 2}{x^2 - x - 1}$$

18. Given that $f(x) = 2^x$ and $g(x) = 3^{2x}$, find $g[f(1)]$.

19. Sketch the graph of $F(x) = \frac{2^x - 2^{-x}}{3}$.

20. **Compound Continuously** How long will it take \$2000 to double if it is invested at an annual interest rate of 6.5% compounded continuously? Round to the nearest year.