Graphing Systems of Equations
§6.1

Systems of Equations – a set of equations with the same variables.

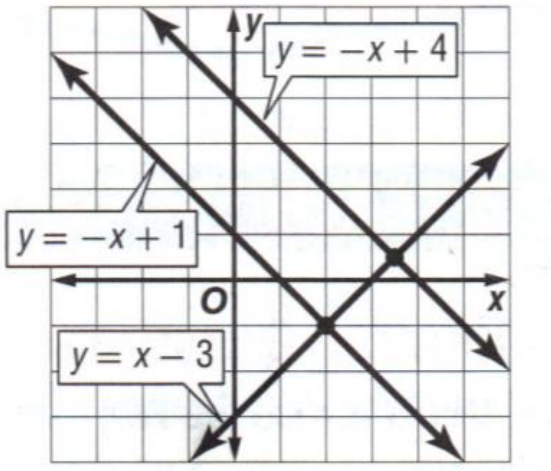
Consistent System – a system that has ***at least one*** solution.

Inconsistent System – a system that ***does not have*** a solution.

Independent System – a system that has ***exactly one*** solution.

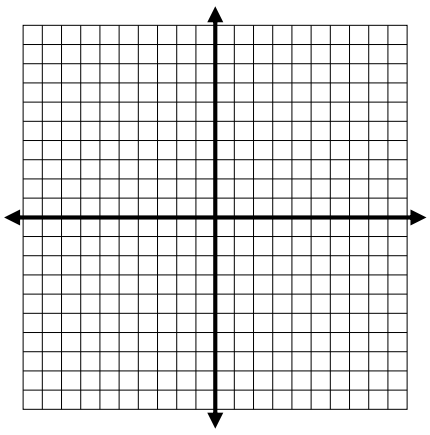
Dependent System – a system that has ***infinite amount*** of solutions.

Example 1
Use the graph to determine whether each system is consistent or inconsistent; independent or dependent.



Example 2
Graph each system of equations and state its solution. Also, state whether the system is consistent or inconsistent and dependent or independent.

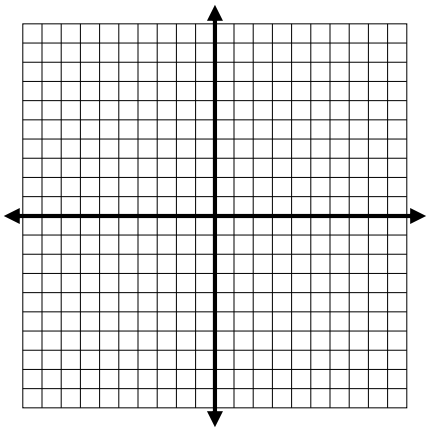
$x + y = 5$
$3x - 2y = 20$



Example 3

Graph each system of equations and state its solution. Also, state whether the system is consistent or inconsistent and dependent or independent.

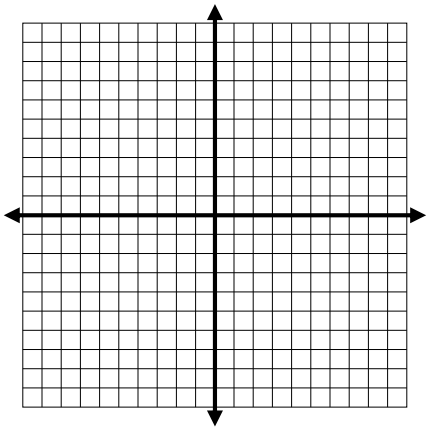
$y = -3x + 5$
$9x + 3y = 15$



Example 4

Graph each system of equations and state its solution. Also, state whether the system is consistent or inconsistent and dependent or independent.

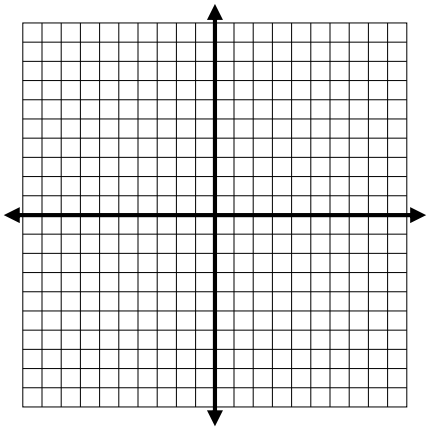
$8x - 4y = 12$
$12x - 6y = 12$



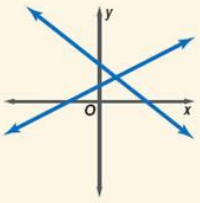
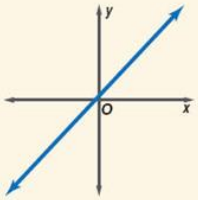
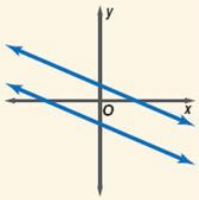
Example 5

Graph each system of equations and state its solution. Also, state whether the system is consistent or inconsistent and dependent or independent.

$y = 4$
$3x = -21$



CONCEPT SUMMARY

consistent and independent	consistent and dependent	inconsistent
		
intersecting lines; one solution	same line; infinitely many solutions	parallel lines; no solution

Pg 336, 8 – 42 even

Substitution
§6.2

Substitution – an algebraic method to find an exact solution of a system of equations.

Hints and Key Concepts
1. Write down equations by one another.
2. Solve one equation for one variable (coefficient of 1)
3. Substitute the resulting expression from step 1 into the other equation to replace the variable and then solve.
4. Substitute the value from step 2 into either original equation and solve for the other variable.
5. Write the answer as an ordered pair: (x, y).

Use substitution to solve the system of equations.

Example 1
$y = -4x + 12$
$2x + y = 2$

Example 2
$x - 2y = -3$
$3x + 5y = 24$

Example 3
$2x + 2y = 8$
$x + y = -2$

Example 4
$x + 2y = 6$
$3x - 4y = 28$

Example 5

$-3x + y = 0$

$4x + 2y = 30$

Example 6

A nature center charges $35.25 for a yearly membership and $6.25 for a single admission. Last week it sold a combined total of 50 yearly memberships and single admissions for $660.50. How many memberships and how many single admissions were sold?

Pg 345, 1-5,9-21 odds

Eliminations Using Addition and Subtraction
§6.3

Elimination – the use of addition or subtraction to eliminate one variable and solve a system of equation.

Hints:
1. Opposites: Add
2. Same: Subtract

Fill the blank with + or -.

$3x$ ☐ $-3x = 0$

$4x$ ☐ $x = 0$

Use elimination to solve the system of equations.

Example 1

$-3x + 4y = 12$
$3x - 6y = 18$

Example 2

$-2x + 4y = -20$
$5x - 4y = 32$

Example 3

$4x + 2y = 28$
$4x - 3y = 18$

Example 4

$3x - 5y = -34$
$-5x - 5y = -10$

Example 5

$-3x + 5y = -11$

$3x + 7y = -1$

Example 6

$3x - 7y = 6$

$8x - 7y = -24$

Example 7

Four times one number minus three times another number is 12. Two times the first number added to three times the second number is 6. Find the numbers.

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Elimination Using Multiplication
§6.4

Elimination – the use of addition or subtraction to eliminate one variable and solve a system of equation.

Hints:
1. Opposites: Add
2. Same: Subtract

Fill the blank with + or -.

$7x$ ☐ $7x = 0$

$5x$ ☐ $5x = 0$

Use elimination to solve the system of equations.

Example 1

$2x + y = 23$
$3x + 2y = 37$

Example 2

$4x + 3y = 8$
$3x - 5y = -23$

Example 3

$4x + 2y = 8$
$3x + 3y = 9$

Example 4

$2x + 5y = -11$
$5x - 3y = 19$

Example 5

$6x + 3y = -3$

$4x + 5y = 19$

Pg 357,1,3,7-17 odds

Applying Systems of Linear Equations
§6.5

Methods for solving system of equations
1. Graphing (never use when given choice)
2. Substitution
3. Elimination

Example 1
$2x + 3y = 23$
$4x + 2y = 34$

Example 2
$-2x + y = -4$
$7x + 3y = 27$

Pg 365,1-4,6-11

Organizing Data Using Matrices
§6.6

Matrix – a rectangular arrangement of numbers in rows and columns enclosed in brackets.

columns

$$C = \begin{bmatrix} 2 & 3 & 5 & 6 \\ 4 & 8 & 7 & 2 \\ 1 & 0 & 5 & 9 \end{bmatrix} \text{ rows}$$

Each value is called an **Element**
Dimensions is the number of rows and columns

$$C_{3 \times 4}$$

State the dimensions of each matrix. Then identify the position of the circled element in each matrix.

Example 1

$$\begin{vmatrix} -8 & \textcircled{4} \\ 5 & 9 \end{vmatrix}$$

Example 2

$$\begin{vmatrix} 2 & 0 & -5 & 8 \\ 3 & \textcircled{8} & 10 & 9 \\ 6 & -7 & 4 & 1 \end{vmatrix}$$

Example 3

At a particular horseback riding competition, blue ribbons go to the highest score in an event. Use a matrix to organize the scores for each participant for each event. Which participant won the most blue ribbons? What are the dimensions of the matrix?

Participant	Jumping	Cutting	Reining
Luke	7	8	6
John	10	10	8
Mandy	12	9	11
Lisa	11	7	10

Find each sum or difference for $A = \begin{vmatrix} 2 & -3 & 0 \\ -4 & 7 & 6 \end{vmatrix}$, $B = \begin{vmatrix} 1 & 5 & -4 \end{vmatrix}$, and $C = \begin{vmatrix} 8 & 9 & -3 \\ 2 & 0 & 10 \end{vmatrix}$.

Example 4
$A + B$

Example 5
$A - C$

Example 6
$A + C$

Example 7
$C - A$

Scalar Multiplication (Matrix)

$$k\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} ka & kb & kc \\ kd & ke & kf \end{bmatrix}$$

Find the following for $A = \begin{vmatrix} 2 & -3 & 0 \\ -4 & 7 & 6 \end{vmatrix}$ and $B = \begin{vmatrix} 1 & 5 & -4 \end{vmatrix}$.

Example 8
$3A$

Example 9
$4B$

Pg 372,1-25

Using Matrices to Solve Systems of Equations
§6.7

Row Operations
1. Any two rows can be multiplied.
2. Any row can be multiplied by a non-zero multiple.
3. Any row can be replaced with the sum of that row and a multiple of another row.
4. Any row can be divided by a non-zero multiple.
5. Any row can be added to another row.

$2x + 7y = 4$
$-3x - 5y = 8$

$$\left[\begin{array}{cc|c} 2 & 7 & 4 \\ -3 & -5 & 8 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & - \\ 0 & 1 & - \end{array}\right]$$

$2x - 3y + 5z = 10$
$x + 2y - 4z = 15$
$3x - 8y + 9z = -7$

$$\left[\begin{array}{ccc|c} 2 & -3 & 5 & 10 \\ 1 & 2 & -4 & 15 \\ 3 & -8 & 9 & -7 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & - \\ 0 & 1 & 0 & - \\ 0 & 0 & 1 & - \end{array}\right]$$

Write an augmented matrix for the system of equations.
Example 1
$7x - 4y = 18$
$-2x + 5y = 11$

Use an augmented matrix to solve the system of equations
Example 2
$8x - 16y = 32$
$10x + 4y = 64$

Graphing Calculator

1. 2nd x^{-1} (MATRX)
2. $[A]$ Edit
3. Enter the proper dimensions.(2 x 3)
4. Enter Coefficients.
5. 2nd x^{-1} (MATRX), MATH
6. B: rref(2nd x^{-1} (MATRX), $[A]$, enter, enter

Use an augmented matrix to solve the system of equations.

Example 3

$2x + 6y = 102$

$2x + 7y = 114$

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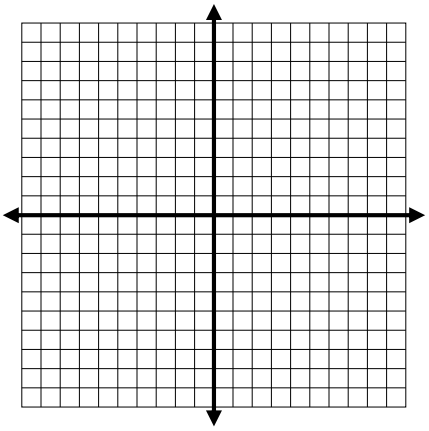
System of Inequalities
§6.8

System of Inequalities must be done by graphing

Example 1

$x \geq 5$

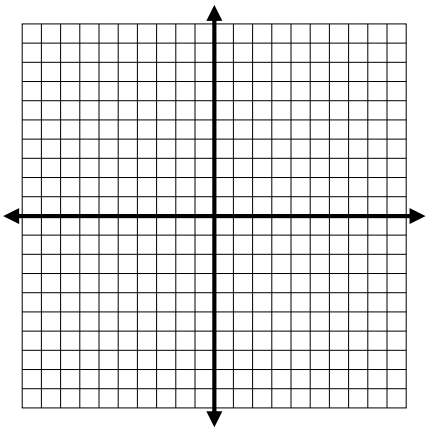
$x + y \leq 3$



Example 2

$5y > -4x - 4$

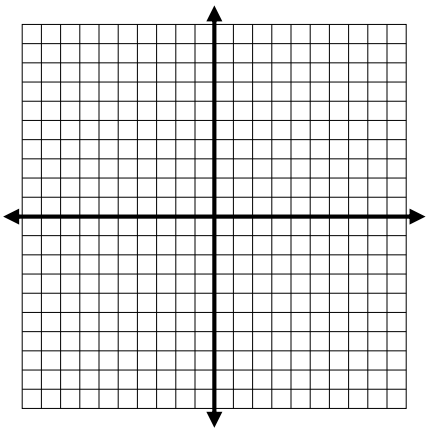
$4x + 5y > 10$



Example 3

$y \leq 2x - 3$

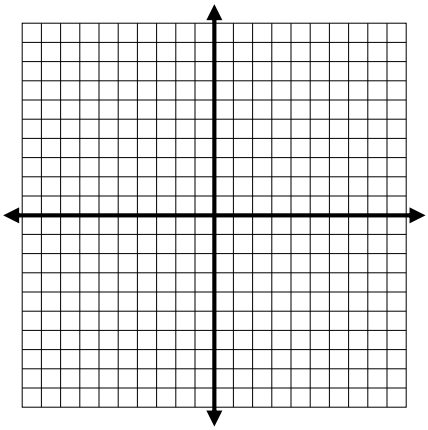
$x + 2y \leq 4$



Example 4

$3x - 2y \leq 8$

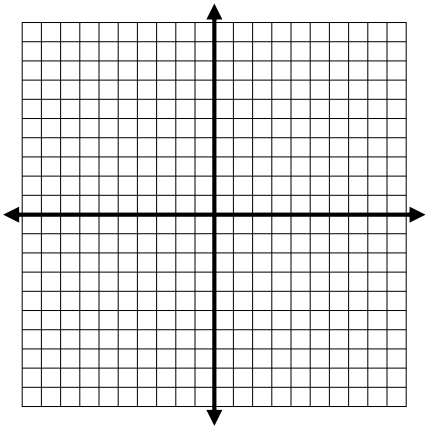
$-2x - 2y \leq -6$



Example 5

$3x + y \geq 1$

$3x + y \leq -2$



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