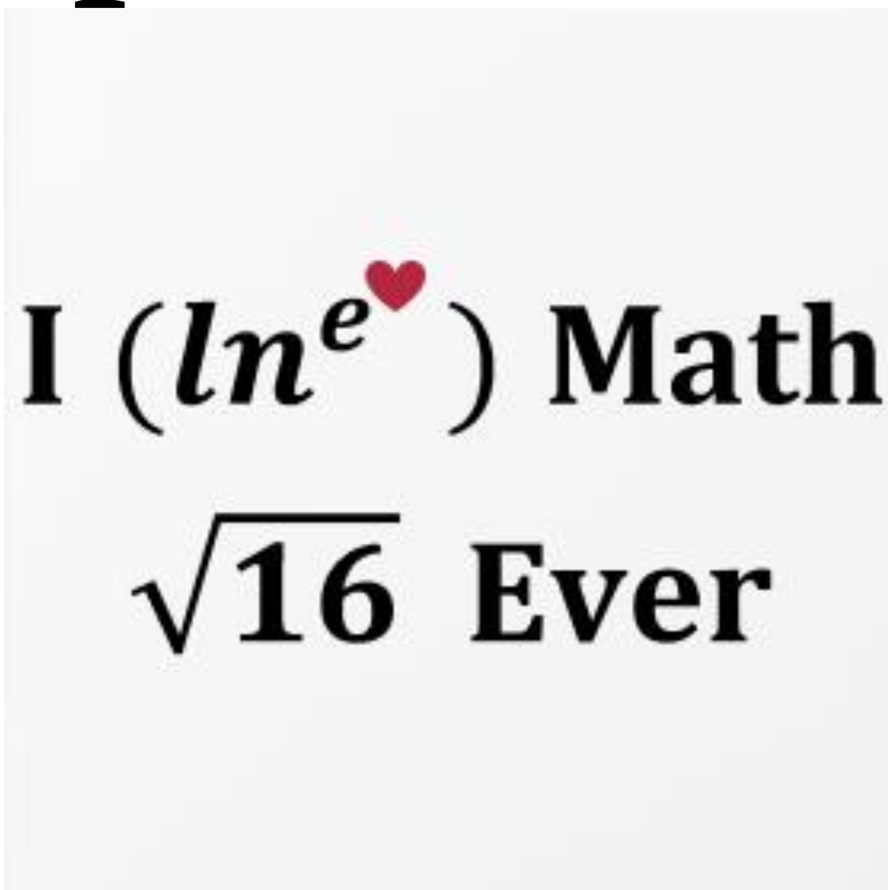
Chapter 6 Notes



Systems of Equations and Inequalities

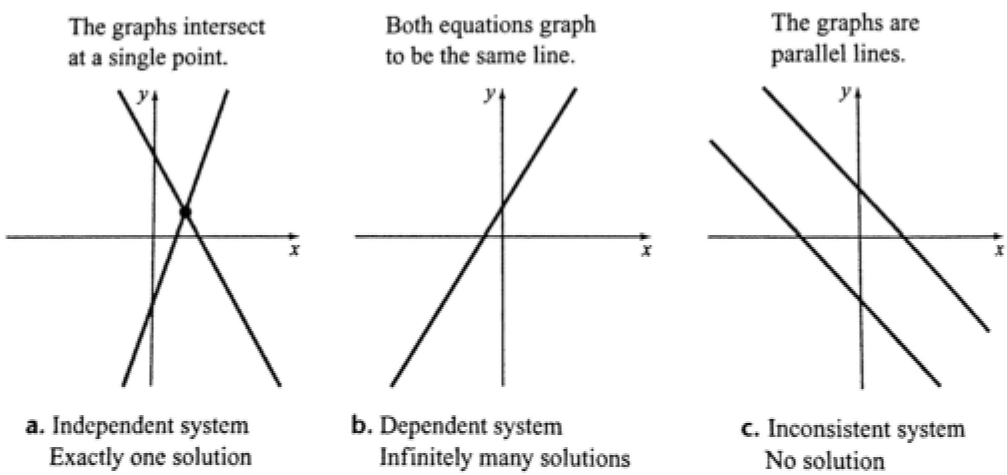
Section 6.1: Systems of Linear Equations in Two Variables Notes
Targets: I can solve a system of equations in two variables using the substitution or elimination method.

A **system of equations** is two or more equations considered together. The following system of equations is a **linear system of equations** in two variables.

$$\begin{cases} 2x+3y=4 \\ 3x-2y=-7 \end{cases}$$

A **solution of a system of equations** in two variables is an ordered pair that is a solution of both equations.

- A system of equations is a **consistent system** if it has <u>at least one solution</u>.
- A system of equations with <u>no solution</u> is an **inconsistent system**.
- A system of equations with <u>exactly one solution</u> is an **independent system**.
- A system of equations with an <u>infinite number of solutions</u> is a **dependent system**.



<u>Substitution Method for Solving a System of Equations</u>

1. $\begin{cases} 3x-5y=7 \\ -2x+y=0 \end{cases}$

2. $\begin{cases} x+3y=6 \\ 2x+6y=-18 \end{cases}$

3. $\begin{cases} 5x+2y=-4 \\ y=-3x \end{cases}$

Elimination Method for Solving a System of Equations

4. $\begin{cases} 3x-4y=10 \\ 2x+5y=-1 \end{cases}$

5. $\begin{cases} x-2y=2 \\ 3x-6y=6 \end{cases}$

6. $\begin{cases} 8x+5y=9 \\ 3x-2y=-16 \end{cases}$

7. Suppose that the number x of bushels of apples a farmer is willing to sell is given by $x=100p-25$, where p is the price, in dollars, per bushel of apples. The number x of bushels of apples a grocer is willing to purchase is given by $x=-150p+655$, where p is the price per bushel of apples. Find the equilibrium price.

Section 6.2: System of Linear Equations in Three Variables
Targets: I can solve a system of linear equations in three variables accurately.

An equation of the form $Ax + By + Cz = D$, with *A*, *B*, and *C* not all zero, is a linear equation in three variables. A solution of an equation in three variables is an **ordered triple** (x, y, z).

A system of equations in more than two variables can be solved by using the **substitution method** or the **elimination method**.

Solve the system of equations.

1. $\begin{cases} x - 2y + z = 7 \\ 2x + y - z = 0 \\ 3x + 2y - 2z = -2 \end{cases}$

2. $\begin{cases} 2x - y - z = -1 \\ -x + 3y - z = -3 \\ -5x + 5y + z = -1 \end{cases}$

3. $\begin{cases} x-2y-z=-5 \\ 3x+y+z=9 \\ 2x-y-z=1 \end{cases}$

4. $\begin{cases} x+2y+3z=4 \\ 2x-y-z=3 \\ 3x+y+2z=5 \end{cases}$

Homogeneous System of Equations

A system of equations in which the constant term is zero for all equations is called a **homogeneous system of equations**. Two examples of homogeneous systems of equations are:

$$\begin{cases} 3x+4y=0 \\ 2x+3y=0 \end{cases} \qquad \begin{cases} 2x+3y+5z=0 \\ 3x+2y+z=0 \\ x-4y+5z=0 \end{cases}$$

The ordered pair $(0,0)$ is always a solution of a homogeneous system of equations in two variables, and the ordered triple $(0,0,0)$ is always a solution of a homogeneous system of equations in three variables. This solution is called the **trivial solution**.

Sometimes a homogeneous has other solutions other than the trivial solution, which is why we still have to solve the system using the substitution or elimination method.

5. Solve. $\begin{cases} x+2y-3z=0 \\ 2x-y+z=0 \\ 3x+y-2z=0 \end{cases}$

Applications of Systems of Equations

Curve Fitting

One application of system of equations is curve fitting. Given a set of points in the plane, we can find an equation whose graph passes through, or fits, all of the points.

1. Find an equation of the form $y = ax^2 + bx + c$ whose graph passes through the points located at $(1,4)$, $(-1,6)$, and $(2,9)$.

Section 6.3: Nonlinear System of Equations
Targets: I can solve a nonlinear system of equations using the substitution or elimination method.

A **nonlinear system of equations** is one in which one or more equations of the system are not linear equations. In this section, we will consider only solutions whose coordinates are real numbers. Therefore, if a system of equations does not have any solution in which both coordinates are real numbers, we will simple state that the system has no solution.

Solve a Nonlinear System by the Substitution Method

1. $\begin{cases} y = x^2 - x - 1 \\ 3x - y = 4 \end{cases}$

2. $\begin{cases} 5x + y = 3 \\ y = x^2 - 3x - 5 \end{cases}$

Solving a Nonlinear System by the Elimination Method

3. $\begin{cases} 2x^2 + 3y^2 = 21 \\ x^2 + 2y^2 = 12 \end{cases}$

4. $\begin{cases} 4x^2 + 9y^2 = 36 \\ x^2 - y^2 = 25 \end{cases}$

5. $\begin{cases} 4x^2 + 3y^2 = 48 \\ 3x^2 + 2y^2 = 35 \end{cases}$

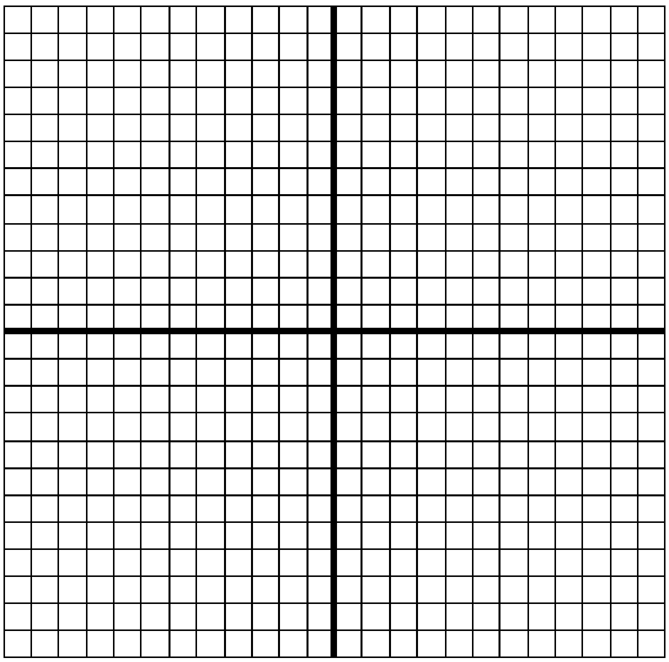
6. $\begin{cases} 9x^2 + 4y^2 = 144 \\ x^2 + y^2 = 9 \end{cases}$

7. A television screen has a diagonal of 63 inches. The ration of the width of the screen, x, to the height of the screen, y, is 16 to 9. Find the width and the height of the screen. Round to the nearest tenth of an inch.

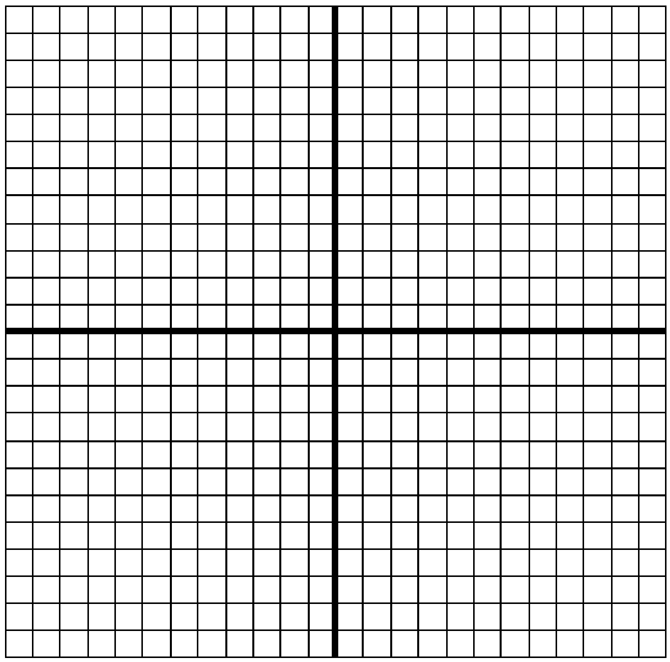
Section 6.5: Inequalities in Two Variables and System of Inequalities
Targets: I can graph a system of inequalities accurately.

Graphing an Linear Inequality

1. $3x + 4y > 12$

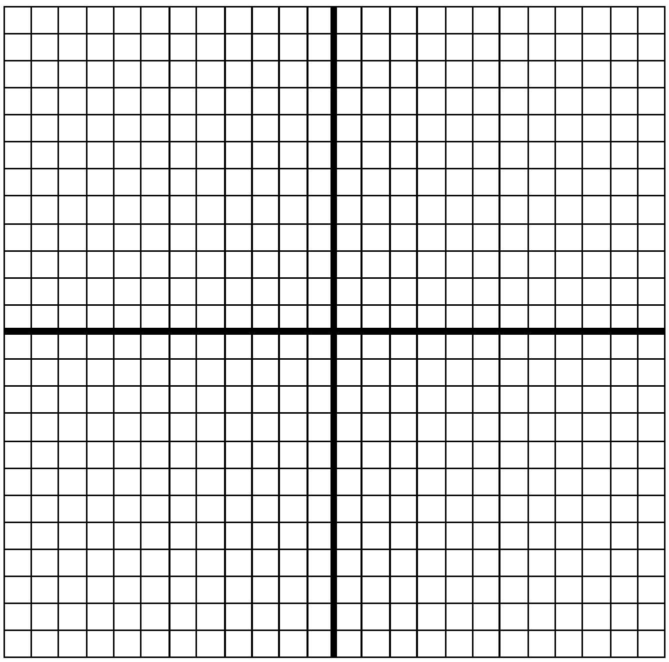


2. $2x - 3y \le 6$

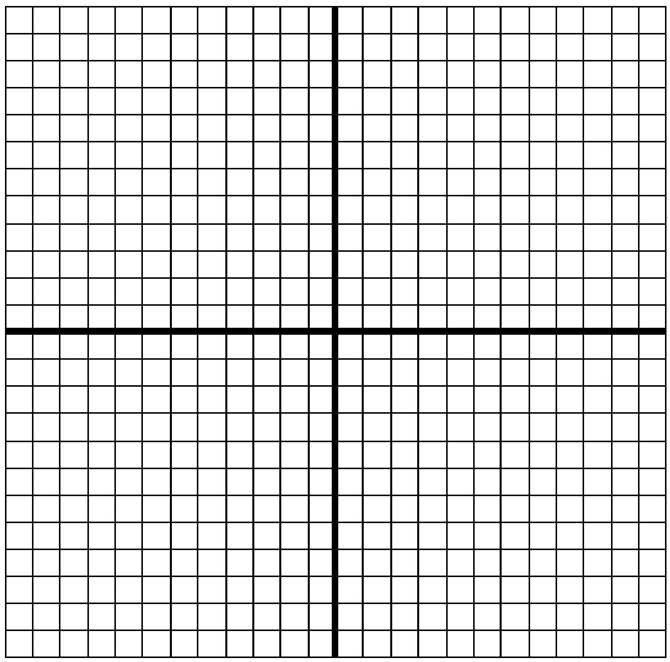


Graphing an Nonlinear Inequality

3. $y \le x^2 + 2x - 3$



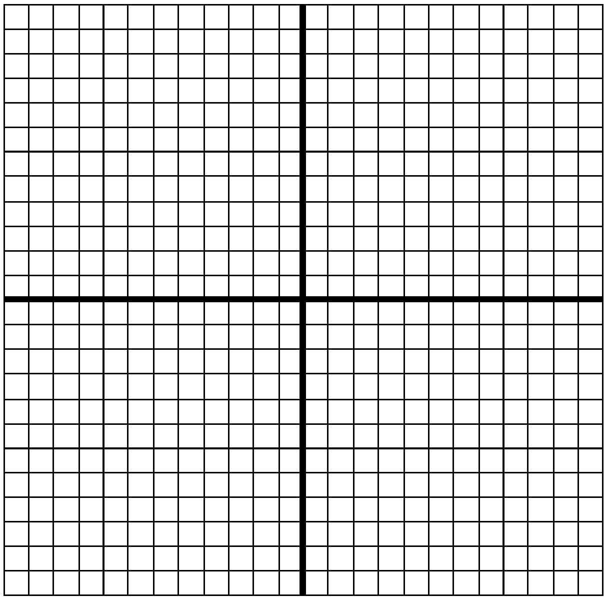
4. $y > x^2 - 4x - 5$



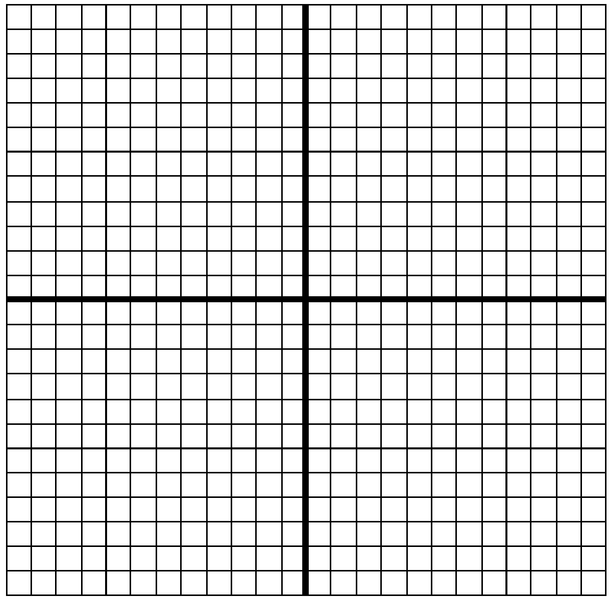
Graphing an Absolute Value Inequality: $y = a|x-h|+k$

Step 1: Find the vertex. (opposite of h, same as k)
Step 2: Graph the vertex.
Step 3: Count the slope in going both left and right.
Step 4: Shade graph in the correct region.

5. $y < |x| + 1$

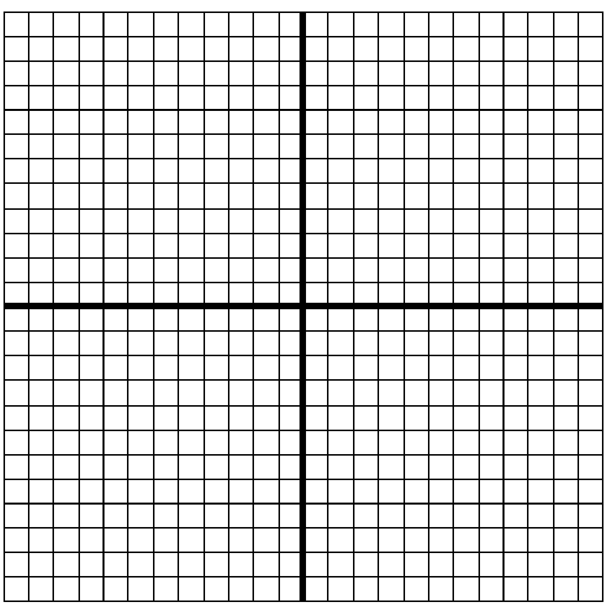


6. $y \geq |x-1| + 3$

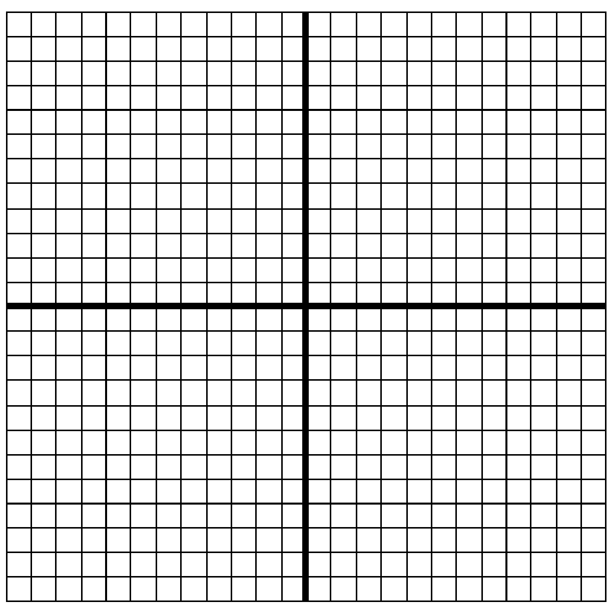


Graphing a System of Linear Inequalities

7. $\begin{cases} 3x - 2y > 6 \\ 2x - 5y \leq 10 \end{cases}$



8. $\begin{cases} y > -\dfrac{1}{2}x + 2 \\ 2x - 3y \leq 12 \end{cases}$



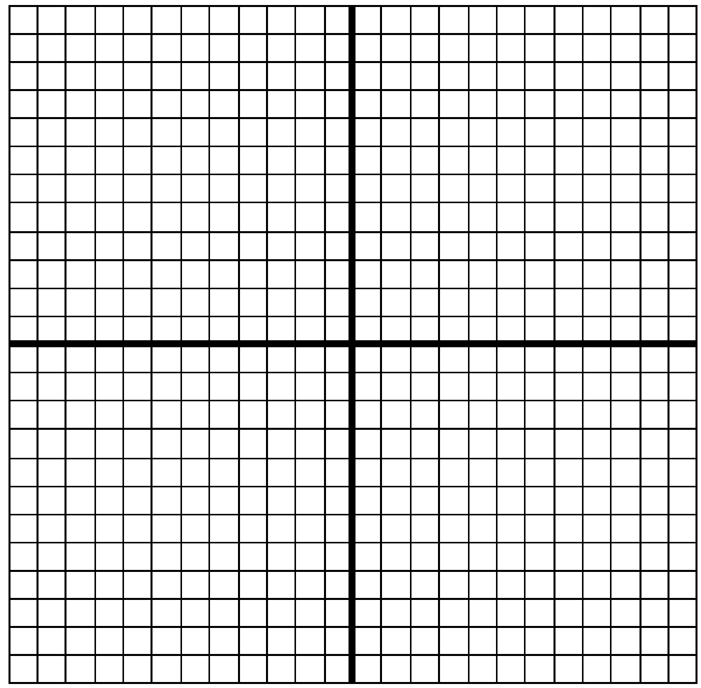
Section 6.6 Linear Programming

Target: I can solve the optimization problems accurately.

Fundamental Linear Programming Theorem
If an objective function has an optimal solution, then that solution will be at a vertex of the set of feasible solutions.

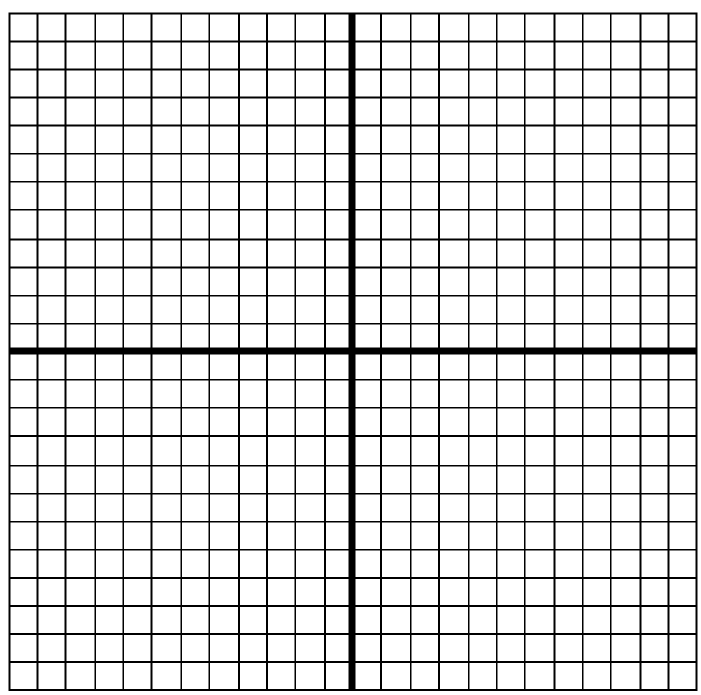
1. Minimize the objective function $C = 4x + 7y$ with the constraints:

$$\begin{cases} 3x + y \geq 6 \\ x + y \geq 4 \\ x + 3y \geq 6 \\ x \geq 0, y \geq 0 \end{cases}$$



2. Maximize the objective function $P = 3x + 5y$ with the constraints:

$$\begin{cases} x + y \leq 5 \\ 2x + y \leq 6 \\ x \geq 0, y \geq 0 \end{cases}$$



Section 7.1: Augmented Matrices

Target: I can use augmented matrices to solve a system of linear equations.

Steps for using a graphing calculator to solve the system of equations:

1. **2nd Matrix, Edit, Enter**
2. Input the Rows and Columns of the augmented matrix
3. Input in each value of the augmented matrix, then hit **2nd Quit** when done.
4. **2nd Matrix, Math, rref(, Enter**
5. **2nd Matrix,** select the matrix that you used to enter the augmented matrix, then hit **Enter**

   a. Solutions

      i. $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -9 \end{bmatrix}$ **Solution:** $(3, 2, -9)$

      ii. $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **Last row is all ZEROS (i.e. 0 = 0):** Infinitely Many Solutions

      iii. $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ **Last row is all Zeros except the last spot (0 = 1):** No Solution

Solve the system of equations.

1. $\begin{cases} x - 2y + 3z = 5 \\ 3x - 3y + z = 9 \\ 5x + y - 3z = 3 \end{cases}$

2. $\begin{cases} x - 3y + z = 5 \\ 3x - 7y + 2z = 12 \\ 2x - 4y + z = 3 \end{cases}$

3. $\begin{cases} 4x + y = 2 \\ 8x + 2y = 4 \end{cases}$

8.6-8.7 Permutations and Combinations Notes

Target: I can use combinations and permutations to calculate the probability of winning the lottery.