Chapter 5 Notes

Algebra 2

5.1 *n*th Roots and Rational Exponents

Targets:

1. I can explain the meaning of a rational exponent.
2. I can evaluate expressions with rational exponents.
3. I can solve equations using the *n*th roots.

Explore it! Writing Expressions in Different Forms

Work with a partner.

a. Use the definition of a rational exponent and the properties of exponents to write each expression as a base with a single rational exponent or as a radical raised to an exponent.

	Radical raised to an exponent	Base with a single rational exponent
i.	$(\sqrt{5})^3$	
ii.	$(\sqrt[4]{4})^2$	
iii.	$(\sqrt[3]{9})^2$	
iv.	$(\sqrt[5]{10})^4$	
v.	$(\sqrt{15})^3$	
vi.	$(\sqrt[3]{27})^4$	
vii.		$8^{2/3}$
viii.		$6^{5/2}$
ix.		$12^{3/4}$
x.		$10^{3/2}$
xi.		$16^{3/2}$
xii.		$20^{6/5}$

b. Use technology to evaluate each expression in part (a). Round your answer to two decimal places, if necessary.

c. Simplify $\sqrt[n]{a^n}$. What does this imply about the relationship between raising an expression to the *n*th power and taking the *n*th root? How can you use this result to solve the equation $x^4 = 6$?

Finding nth Roots

KEY IDEA

Real *n*th Roots of *a*

Let n be an integer greater than 1 and let a be a real number.

	***n* is an even integer.**		***n* is an odd integer.**
$a < 0$	No real nth roots	$a < 0$	One real nth root: $\sqrt[n]{a} = a^{1/n}$
$a = 0$	One real nth root: $\sqrt[n]{0} = 0$	$a = 0$	One real nth root: $\sqrt[n]{0} = 0$
$a > 0$	Two real nth roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$	$a > 0$	One real nth root: $\sqrt[n]{a} = a^{1/n}$

Example 1: Find the indicated real *n*th root(s) of *a*.

a. $n = 3, a = -216$

b. $n = 4, a = 81$

c. $n = 8, a = 256$

d. $n = 5, a = -243$

Rational Exponents

KEY IDEA

Rational Exponents

Let $a^{1/n}$ be an nth root of a, and let m be a positive integer.

$$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m \quad \text{or} \quad a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$$

$$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\sqrt[n]{a})^m}, a \neq 0 \quad \text{or} \quad a^{-m/n} = \frac{1}{(a^m)^{1/n}} = \frac{1}{\sqrt[n]{a^m}}, a \neq 0$$

Example 2: Evaluate.

a. $16^{3/2}$

b. $32^{-3/5}$

c. $64^{4/3}$

d. $16^{-5/4}$

Approximating Expressions with Rational Exponents

Example 3: Evaluate each expression using technology. Round your answer to two decimal places.

a. $9^{1/5}$

b. $12^{3/8}$

c. $\left(\sqrt[4]{7}\right)^3$

Solving Equations Using the nth roots

Example 4: Solving Equations Using the *n*th Roots

a. $4x^5 = 128$

b. $(x-3)^4 = 21$

c. $5x^3 = 320$

d. $(x+3)^4 = 24$

Example 5: Modeling Real Life

A hospital purchases an ultrasound machine for $50,000. The hospital expects the useful life of the machine to be 10 years, at which time its value will have depreciated to $8,000. The hospital uses the declining balances method for depreciation, so the annual depreciation rate *r* (in decimal form) is given by the formula $r = 1 - \left(\frac{S}{C}\right)^{1/n}$. In the formula, *n* is the useful life of the item (in years), *S* is the salvage value (in dollars), and *C* is the original cost (in dollars). What annual depreciation rate did the hospital use?

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5.2 Properties of Rational Exponents and Radicals

Targets:

1. I can simplify radical expressions wit rational exponents.
2. I can explain when radical expressions are in simplest form.
3. I can simplify variable expressions containing rational exponents and radicals.

Explore it! Reviewing Properties of Exponents

Work with a partner.

a. The Product Property of Square Roots state that the square root of a product equals the product of the square of the factors.

$\sqrt{64x^2} = \sqrt{64} \cdot \sqrt{x^2}$	Product Property of Square Roots
$= 8x$	Simplify.

Describe the end behavior of the graphs of $y = \sqrt{64x^2}$ and $y = 8x$. What do you notice? Use technology to check your graphs and explain the results.

b. You can extend the Product Property of Square Roots to other radicals such as cube roots.

$\sqrt[3]{64x^3} = \sqrt[3]{64} \cdot \sqrt[3]{x^3}$	Product Property of Cube Roots
$= 4x$	Simplify.

Describe the behavior of the graphs of $y = \sqrt[3]{64x^3}$ and $y = 4x$. What do you notice? Use technology to check your graphs and explain the results.

c. How can change the function $y = 8x$ so that coincides with the graph of $y = \sqrt{64x^2}$ for all values of x? Explain your reasoning.

d. Determine the values of n for which $\sqrt[n]{x^n} = x$ and $\sqrt[n]{x^n} = |x|$.

Properties of Rational Exponents

KEY IDEA

Properties of Rational Exponents

Let a and b be real numbers and let m and n be rational numbers, such that the quantities in each property are real numbers.

Property Name	Definition	Example
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2+3/2)} = 5^2 = 25$
Power of a Power	$(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \cdot 2)} = 3^5 = 243$
Power of a Product	$(ab)^m = a^m b^m$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$
Negative Exponent	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{6}$
Zero Exponent	$a^0 = 1, a \neq 0$	$213^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)} = 4^2 = 16$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{64}\right)^{1/3} = \frac{27^{1/3}}{64^{1/3}} = \frac{3}{4}$

Example 1: Using Properties of Exponents

a. $7^{1/4} \cdot 7^{1/2}$

b. $\left(6^{1/2} \cdot 3^{1/3}\right)^2$

c. $(4^5 \cdot 3^5)^{-1/5}$

d. $\frac{5}{5^{1/3}}$

e. $\left(\frac{42^{1/3}}{6^{1/3}}\right)^2$

f. $13^{1/3} \cdot 13^{1/6}$

g. $\left(5^{1/5} \cdot 8^{1/4}\right)^5$

h. $\left(\frac{56^{1/4}}{8^{1/4}}\right)^3$

Simplifying Radical Expressions

KEY IDEA

Properties of Radicals

Let a and b be real numbers such that the indicated roots are real numbers, and let n be an integer greater than 1.

Property Name	Definition	Example
Product Property	$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt[3]{4} \cdot \sqrt[3]{2} = \sqrt[3]{8} = 2$
Quotient Property	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$	$\frac{\sqrt[4]{162}}{\sqrt[4]{2}} = \sqrt[4]{\frac{162}{2}} = \sqrt[4]{81} = 3$

Example 2: Using Properties of Radicals

a. $\sqrt[3]{12} \cdot \sqrt[3]{18}$

b. $\frac{\sqrt[4]{80}}{\sqrt[4]{5}}$

c. $\sqrt[3]{4} \cdot \sqrt[3]{128}$

d. $\frac{\sqrt[5]{192}}{\sqrt[5]{6}}$

Example 3: Writing Radicals in Simplest Form

a. $\sqrt[3]{135}$

b. $\frac{\sqrt[5]{7}}{\sqrt[5]{8}}$

c. $\frac{\sqrt[4]{19}}{\sqrt[4]{4}}$

Example 4: Writing a Radical Expression in Simplest Form

a. Write $\frac{1}{5+\sqrt{3}}$ in simplest form.

b. Write $\frac{1}{\sqrt{7}-2}$ in simplest form.

Example 5: Adding and Subtracting Like Radicals and Roots

Simplify each expression.

a. $\sqrt[4]{10} + 7\sqrt[4]{10}$

b. $2\left(8^{1/5}\right) + 10\left(8^{1/5}\right)$

c. $\sqrt[3]{54} - \sqrt[3]{2}$

d. $\sqrt[6]{17} + 9\sqrt[6]{17}$

e. $5\left(7^{1/3}\right) + 6\left(7^{1/3}\right)$

f. $\sqrt[4]{48} - \sqrt[4]{3}$

Example 6: Simplifying Variable Expressions

Simplify each expression.

a. $\sqrt[3]{64y^6}$

b. $\sqrt[4]{\frac{x^4}{y^8}}$

c. $\sqrt[3]{27y^{12}}$

d. $\sqrt[4]{\frac{x^4}{y^{16}}}$

Example 7: Writing Variable Expressions in Simplest Form

Write each expression in simplest form. Assume all variables are positive.

a. $\sqrt[4]{16a^7b^{11}c^4}$

b. $\frac{x}{\sqrt[3]{y^8}}$

c. $\frac{14xy^{1/3}}{2x^{3/4}z^{-6}}$

d. $\sqrt[3]{2a^{13}b^3c^8}$

e. $\frac{x^2}{\sqrt[3]{y^{13}}}$

f. $\frac{16xy^{1/3}}{8x^{2/3}z^{-4}}$

Example 8: Adding and Subtracting Variable Expressions

Perform each indicated operation. Assume all variables are positive.

a. $5\sqrt{y} + 6\sqrt{y}$

b. $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2}$

c. $6\sqrt[3]{y} + 2\sqrt[3]{y}$

d. $16\sqrt[4]{3z^6} - z\sqrt[4]{48z^2}$

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5.3 Graphing Radical Functions

Targets:

1. I can graph radical functions.
2. I can describe transformations of radical functions.
3. I can write functions that represent transformations of radical functions.

Explore it! Graphing Radical Functions

Work with a partner.

a. In your own words, define a *radical* function. Give several examples.

b. Graph each function. How are the graphs alike? How are they different?

i. $f(x) = \sqrt{x}$ ii. $f(x) = \sqrt[3]{x}$ iii. $f(x) = \sqrt[4]{x}$ iv. $f(x) = \sqrt[5]{x}$

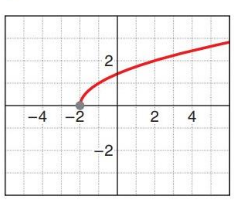
c. Match each function with its graph. Explain your reasoning. Then describe *g* as a transformation of its parent function *f*.

i. $g(x) = \sqrt{x+2}$ **ii.** $g(x) = \sqrt{x-2}$

iii. $g(x) = \sqrt[3]{x} + 2$ **iv.** $g(x) = \sqrt[4]{x} - 2$

v. $g(x) = \sqrt[3]{x+2} - 2$ **vi.** $g(x) = -\sqrt[5]{x+2}$

A.



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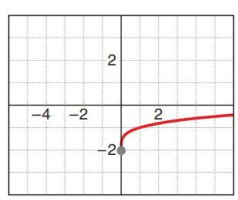

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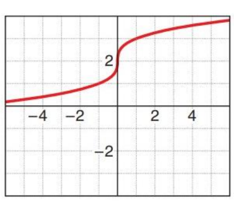
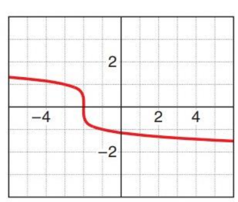
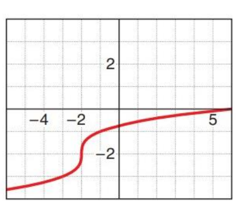


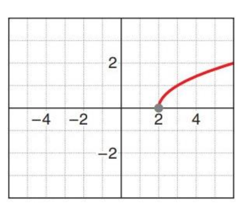
D.



E.



F.



d. Describe the transformation of $f(x) = \sqrt{x}$ represented by $g(x) = -\sqrt{x+1}$. Then graph each function.

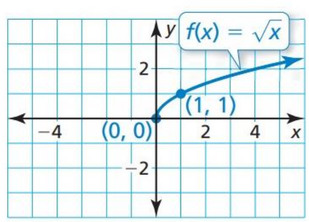
Graphing Radical Functions

A **radical function** contains a radical expression with the independent variable in the radicand. When the radical is a square root, the function is called a *square root function*. When the radical is a cube root, the function is called a *cube root function*.

KEY IDEA

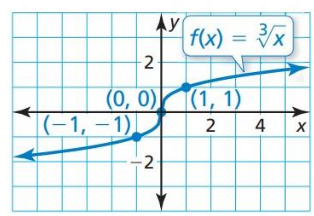
Parent Functions for Square Root and Cube Root Functions

The parent function for the family of square root functions is $f(x) = \sqrt{x}$.



Domain: $x \geq 0$, Range: $y \geq 0$

The parent function for the family of cube root functions is $f(x) = \sqrt[3]{x}$.

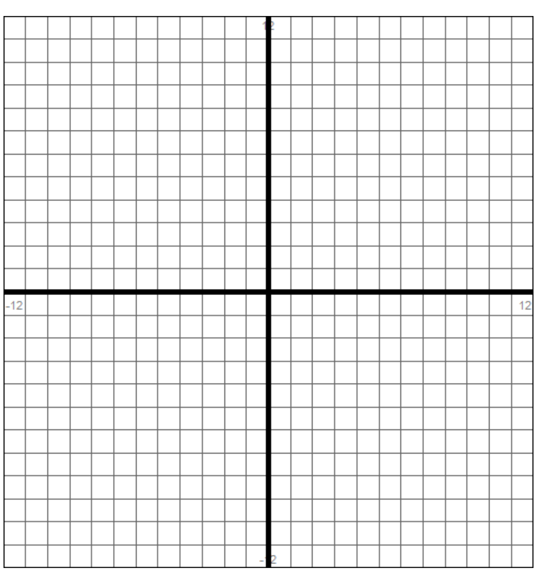


Domain and range: All real numbers

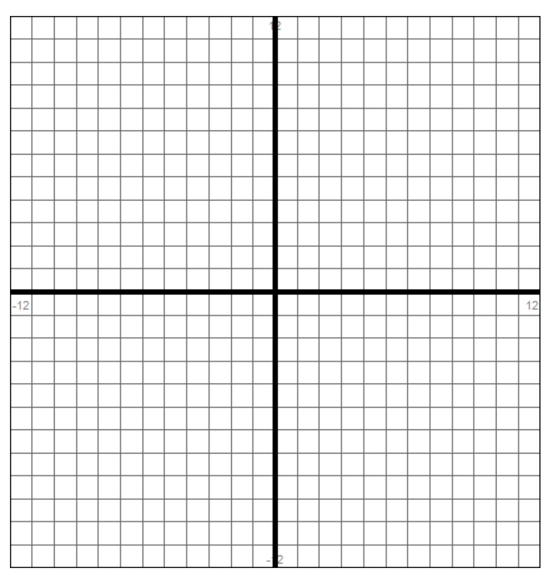
Example 1: Graphing Radical Functions

Graph each function. Find the domain and range of each function.

a. $f(x) = \sqrt{\frac{1}{4}x}$



b. $f(x) = -3\sqrt[3]{x}$



Graphing Transformations of Radical Functions

KEY IDEAS

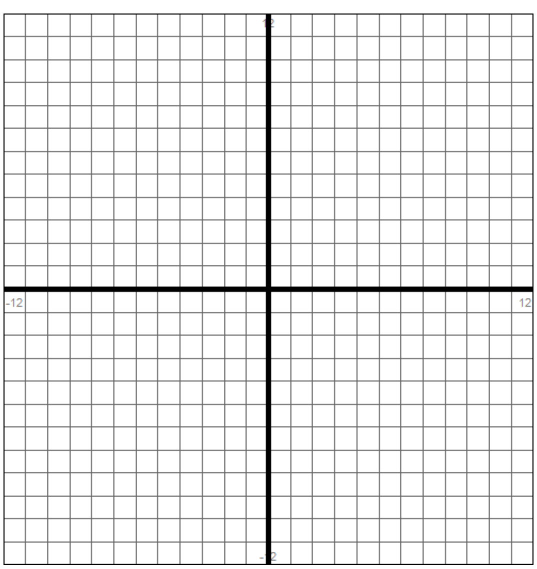
Transformation	f(x) Notation	Examples	
Horizontal Translation Graph shifts left or right.	$f(x-h)$	$g(x)=\sqrt{x-2}$ $g(x)=\sqrt{x+3}$	2 units right 3 units left
Vertical Translation Graph shifts up or down.	$f(x)+k$	$g(x)=\sqrt{x}+7$ $g(x)=\sqrt{x}-1$	7 units up 1 unit down
Reflection Graph flips over a line.	$f(-x)$ $-f(x)$	$g(x)=\sqrt{-x}$ $g(x)=-\sqrt{x}$	in the y-axis in the x-axis
Horizontal Stretch or Shrink Graph stretches away from or shrinks toward y-axis by a factor of $\frac{1}{a}$.	$f(ax)$	$g(x)=\sqrt{3x}$ $g(x)=\sqrt{\frac{1}{2}x}$	shrink by a factor of $\frac{1}{3}$ stretch by a factor of 2
Vertical Stretch or Shrink Graph stretches away from or shrinks toward x-axis by a factor of a.	$a \cdot f(x)$	$g(x)=4\sqrt{x}$ $g(x)=\frac{1}{5}\sqrt{x}$	stretch by a factor of 4 shrink by a factor of $\frac{1}{5}$

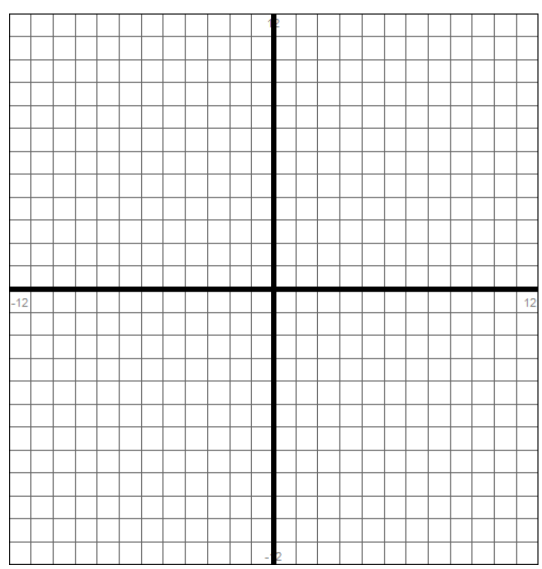
Example 2: Transforming Radical Functions

Describe the transformation of *f* represented by *g*. Then graph each function.

a. $f(x)=\sqrt{x}$, $g(x)=\sqrt{x-3}+4$

b. $f(x)=\sqrt[3]{x}$, $g(x)=-8\sqrt[3]{x}$





Example 3: Modeling Real Life

The function $E(d) = 0.25\sqrt{d}$ approximates the number of seconds in takes a dropped object to fall *d* feet on Earth. The function $M(d) = 1.6 \cdot E(d)$ approximates the number of seconds it takes a dropped object to fall *d* feet on Mars. How long does it take a dropped object to fall 64 feet on Mars?

Example 4: Writing a Transformed Radical Function

a. Let the graph of g be a vertical shrink by a factor of $\frac{1}{6}$, followed by a translation 3 units left of the graph of $f(x) = \sqrt[3]{x}$. Write a rule for g.

b. Let the graph of g be translated 2 units right and 3 units down, followed by a vertical stretch by a factor of 2 of the graph $f(x) = \sqrt{x} + 1$. Write a rule for g.

Pg 250,1-49 odds

5.4 Solving Radical Equations and Inequalities

Targets:

1. I can identify radical equations and inequalities
2. I can solve radical equations and inequalities.
3. I can identify extraneous solutions of radical equations.
4. I can solve real-life problems involving radical equations.

Explore it! Solving Radical Equations

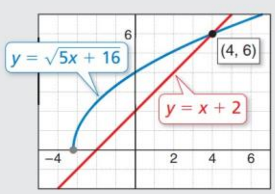
Work with a partner.

a. Two students solve the equation $x + 2 = \sqrt{5x + 16}$ as shown. Justify each solution step in the first student's solution. Then describe each student's method. Are the methods valid? Explain.

Student 1

$x + 2 = \sqrt{5x + 16}$	Write the equation.
$(x + 2)^2 = (\sqrt{5x + 16})^2$	______
$x^2 + 4x + 4 = 5x + 16$	______
$x^2 - x - 12 = 0$	______
$(x - 4)(x + 3) = 0$	______
$x - 4 = 0$ or $x + 3 = 0$	______
$x = 4$ or $x = -3$	______

Student 2



The graphs intersect at the point (4, 6). So, the only solution is $x = 4$.

b. Which student is correct? Explain why the other student's solution is incorrect and how the student arrived at an incorrect answer.

c. Explain how you might solve the equation $(9n)^{3/2} - 7 = 20$

Solving Equations

A **radical equation** contains radicals that have variables in the radicands. An example of a radical equations is: $2\sqrt{x+1} = 4$.

KEY IDEA

Solving Radical Equations

Step 1 Isolate the radical on one side of the equation, if necessary.

Step 2 Raise each side of the equation to the same exponent to eliminate the radical and obtain a linear, quadratic, or other polynomial equation.

Step 3 Solve the resulting equation using techniques you learned in previous chapters. Check your solution.

Example 1: Solving Radical Equations

Solve each equation.

a. $2\sqrt{x+1} = 4$

b. $\sqrt[3]{2x-9} - 1 = 2$

c. $2\sqrt{x+2} = 8$

d. $\sqrt[3]{2x-5} - 2 = 3$

Example 2: Modeling Real Life

The mean sustained wind velocity (in meters per second) of a hurricane is modeled by $v(p) = 6.3\sqrt{1013 - p}$, where p is the air pressure (in millibars) at the center of the hurricane. Estimate the air pressure at the center of the hurricane when the means sustained wind velocity is 54.5 meters per second?

Example 3: Solving an Equation with an Extraneous Solutions

Solve.

a. $x + 1 = \sqrt{7x + 15}$

b. $4 + \sqrt{2x + 7} = x$

Example 4: Solving an Equation with Two Radicals

Solve.

a. $\sqrt{x+2}+1=\sqrt{3-x}$

b. $\sqrt{x+6}+1=\sqrt{7-x}$

Example 5: Solving an Equation with a Rational Exponent

Solve.

a. $(2x)^{3/4}+2=10$

b. $(x+20)^{2/3}=4$

Example 6: Solving an Equation with a Rational Exponent

Solve.

a. $(x+30)^{1/2}=x$

b. $(x+12)^{1/2}=x$

Example 7: Solving a Radical Inequality

Solve.

a. $3\sqrt{x-1}\leq 12$

b. $3\sqrt{x}-4\geq 8$

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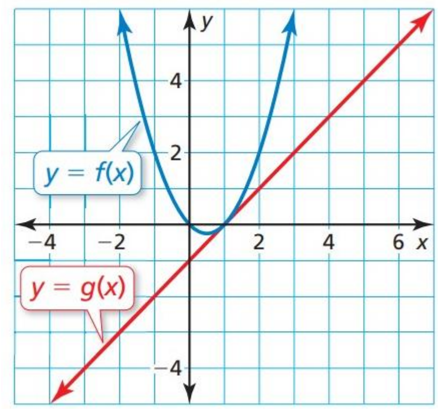
5.5 Performing Function Operations

Targets:

1. I can explain what it means to perform an arithmetic operation on two functions.
2. I can find arithmetic combinations of two functions.
3. I can state the domain of an arithmetic combination of two functions.
4. I can evaluate an arithmetic combination of two functions for a given input.

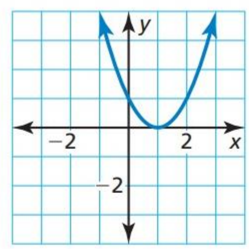
Explore it! Graphing Arithmetic Combinations of Two Functions

Work with a Partner. Consider the graphs of *f* and *g*.



a. Describe what it means to add two functions. Then describe what it means to subtract one function from another function.

b. Match each function with its graph. Explain your reasoning.
   i. $m(x) = f(x) + g(x)$
   ii. $n(x) = f(x) - g(x)$
   iii. $p(x) = f(x) \cdot g(x)$
   iv. $q(x) = f(x) \div g(x)$

A.


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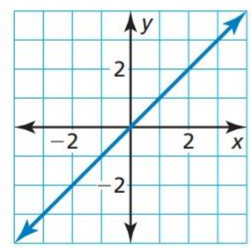
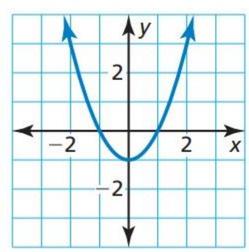
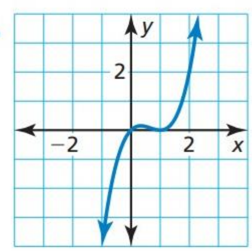


C.


D.


c. What is the domain of each function in part (b)? How do you know?

d. Check your answer in part (b) by writing function rules for *f* and *g*, performing each arithmetic combination and graphing the results.

Operations of Functions

KEY IDEA

Operations on Functions

Let f and g be any two functions. A new function can be defined by performing any of the four basic operations on f and g.

Operation	Definition	Example: $f(x) = 5x$, $g(x) = x + 2$
Addition	$(f + g)(x) = f(x) + g(x)$	$(f + g)(x) = 5x + (x + 2) = 6x + 2$
Subtraction	$(f - g)(x) = f(x) - g(x)$	$(f - g)(x) = 5x - (x + 2) = 4x - 2$
Multiplication	$(fg)(x) = f(x) \cdot g(x)$	$(fg)(x) = 5x(x + 2) = 5x^2 + 10x$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	$\left(\frac{f}{g}\right)(x) = \frac{5x}{x + 2}$

The domains of the sum, difference, product, and quotient functions consist of the x-values that are in the domains of both f and g. Additionally, the domain of the quotient does not include x-values for which $g(x) = 0$.

Example 1: Adding Two Functions

a. Let $f(x) = 3\sqrt{x}$ and $g(x) = 3 - 10\sqrt{x}$. Find $(f + g)(x)$ and state the domain. Then evaluate $(f + g)(4)$.

b. Let $f(x) = 5\sqrt{x}$ and $g(x) = -8\sqrt{x}$. Find $(f + g)(x)$ and state the domain. Then evaluate $(f + g)(16)$.

Example 2: Subtracting Two Functions

a. Let $f(x) = 3x^3 - 2x^2 + 5$ and $g(x) = x^3 - 3x^2 + 4x - 2$. Find $(f - g)(x)$ and state the domain. Then evaluate $(f - g)(-2)$.

b. Let $f(x) = 2x^3 + 4x^2 - 8x + 4$ and $g(x) = 3x^3 - 5x^2 + 6x - 9$. Find $(f - g)(x)$ and state the domain. Then evaluate $(f - g)(-1)$.

Example 3: Multiplying Two Functions

a. Let $f(x) = x^2$ and $g(x) = \sqrt{x}$. Find $(fg)(x)$ and state the domain. Then evaluate $(fg)(9)$.

b. Let $f(x) = x^2 - 4$ and $g(x) = 2x^3 - 3x^2 + x - 5$. Find $(fg)(x)$ and state the domain. Then evaluate $(fg)(-1)$.

Example 4: Dividing Two Functions

a. Let $f(x) = 6x$ and $g(x) = x^{3/4}$. Find $\left(\frac{f}{g}\right)(x)$ and state the domain. Then evaluate $\left(\frac{f}{g}\right)(16)$.

b. Let $f(x) = x^2 - 16$ and $g(x) = x^2 + x - 12$. Find $\left(\frac{f}{g}\right)(x)$ and state the domain. Then evaluate $\left(\frac{f}{g}\right)(2)$.

c. Let $f(x) = x + 6$ and $g(x) = 2x$. Find $\left(\frac{f}{g}\right)(x)$ and state the domain. Then evaluate $\left(\frac{f}{g}\right)(-3)$.

Example 6: Modeling Real Life

For a white rhino, heart rate (in beats per minute) and life span (in minutes) are related to body mass *m* (in kilograms) by the following functions.

Heart rate: $r(m) = 241m^{-0.25}$

Life span: $s(m) = (6 \times 10^6)m^{0.2}$

Find $(rs)(m)$ and explain what is represents.

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5.6 Composition of Functions

Targets:

1. I can evaluate a composition of functions.
2. I can find a composition of functions.
3. I can state the domain of a composition of functions.

Explore it! Finding Composition of Functions

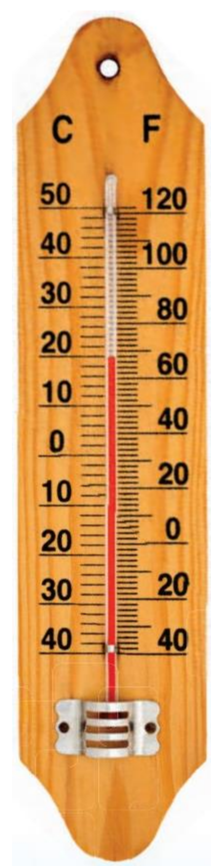
Work with a partner. The formula below represents the temperature *F* (in degrees Fahrenheit) when the temperature is *C* degrees Celsius, and the temperature *C* when the temperature is *K* (Kelvin).

$$F = \frac{9}{5}C + 32 \qquad\qquad C = K - 273$$

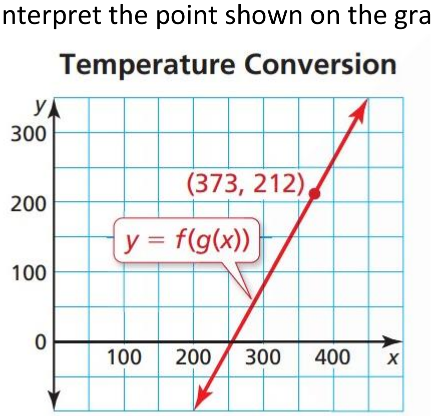
a. Write an expression for *F* in terms of *K*.

b. Given that $f(x) = \frac{9}{5}x + 32$ and $g(x) = x - 273$ write an equation for $f(g(x))$. What does $f(g(x))$ represent in the situation?

c. Water freezes at about 273 Kelvin. Find $f(g(273))$. Does your answer make sense? Explain your reasoning.



d. Interpret the point shown on the graph.



Evaluating Compositions of Functions

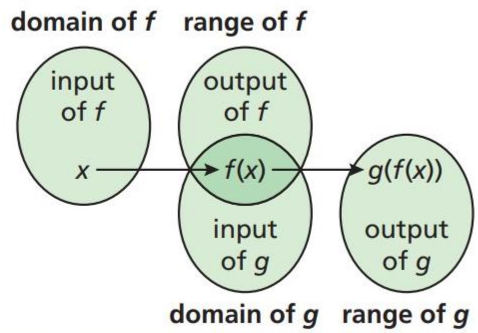
KEY IDEA

Composition of Functions

The **composition** of a function g with a function f is

$$h(x) = g(f(x)).$$

The domain of h is the set of all x-values such that x is in the domain of f and $f(x)$ is in the domain of g.



Example 1: Evaluating Compositions of Functions

Let $f(x) = \sqrt{2x+1}$ and $g(x) = x^2 - 4$. Find the indicated value.

a. $f(g(4))$ b. $f(g(2))$ c. $g(g(-2))$ d. $g(f(1))$

Example 2: Finding Compositions of Functions

a. Let $f(x) = 5x^{-1}$ and $g(x) = 3x - 3$. Perform the indicated composition and stat the domain.

i. $f(g(x))$ ii. $g(f(x))$ iii. $f(f(x))$

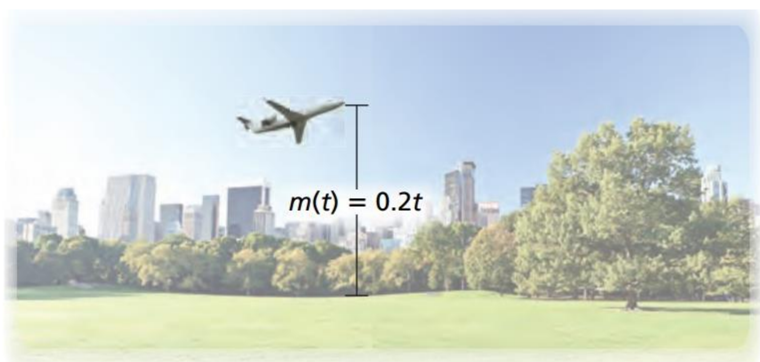
b. Let $f(x) = 2x^2 + 3x - 4$ and $g(x) = x + 3$. Perform the indicated composition and stat the domain.

i. $f(g(x))$

ii. $g(f(x))$

Example 3: Modeling Real Life

The function $C(m) = 15 - 10.5m$ approximates the temperature (in degrees Celsius) at an altitude of m miles. The diagram shows the altitude of (in miles) of an airplane t minutes after taking off, where $0 \leq t \leq 30$. Find $C(m(t))$. Evaluate $C(m(30))$ and explain what it represents.



Pg 271,1-27 odds

5.7 Inverse of a Function

Targets:

1. I can explain what inverse functions are.
2. I can find inverses of linear and nonlinear functions.
3. I can determine whether a pair of functions are inverses.

Explore it! Finding Composition of Functions

Work with a partner.

a. Consider each pair of functions, *f* and *g*, below. For each pair, create an input-output table of values for each function. Use the outputs of *f* as the inputs of *g*. What do you notice about the relationship between the equations *f* and *g*?

i. $f(x) = 4x + 3$, $g(x) = \frac{x - 3}{4}$

ii. $f(x) = x^3 + 1$, $g(x) = \sqrt[3]{x - 1}$

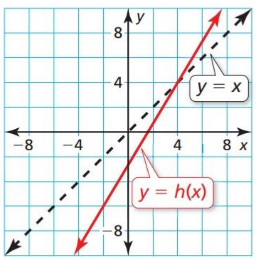
iii. $f(x) = \sqrt{x - 3}$, $g(x) = x^2 + 3, x \geq 0$

b. What do you notice about the graphs of each pair of functions in part (a)?

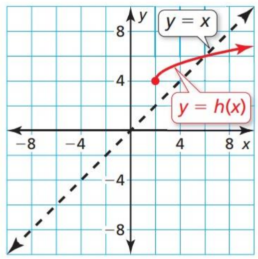
c. For each pair of functions in part (a), find $f(g(x))$ and $g(f(x))$. What do you notice?

d. The functions *h* and *j* are inverses of each other. Use the graph of *h* to find the given value. Explain how you found your answer.

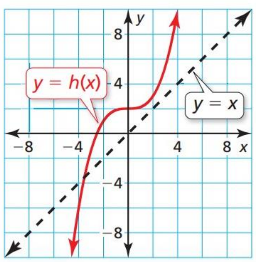
i. $j(-6)$



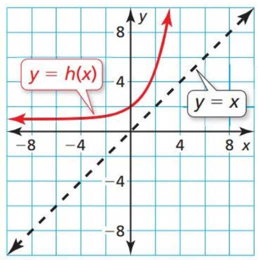
ii. $j(4)$



iii. $j(-6)$



iv. $j(2)$



Exploring Inverse Functions

Example 1: Writing a Formula for the input of a Function

a. Let $f(x) = 2x + 3$. Solve $y = f(x)$ for x. Then find the input when the output is -7.

b. Let $f(x) = 3x - 5$. Solve $y = f(x)$ for x. Then find the input when the output is -11.

An inverse function can be denoted by f^{-1}, read as "*f* inverse." Because an inverse function switches the input and output values of the original function, the domain and range are also switched.

Original function: $f(x) = 2x + 3$

x	−2	−1	0	1	2
y	−1	1	3	5	7

Inverse function: $f^{-1}(x) = \dfrac{x - 3}{2}$

x	−1	1	3	5	7
y	−2	−1	0	1	2

The graph of f^{-1} is a *reflection* of the graph of *f*. The *line of reflection* is $y = x$. This is true for all inverses.

Steps for Finding the Inverse of a Function
To find the equation of the inverse f^{-1} of the one-to-one function *f*, follow these steps: 1. Substitute *y* for $f(x)$. 2. Solve for *x*. 3. Interchange *x* and *y*. 4. Substitute $f^{-1}(x)$ for *y*.

Example 2: Find the Inverse of a Linear Function

a. Find the inverse of $f(x) = 3x - 1$.

b. Find $f^{-1}(x)$ of $f(x) = \frac{x-8}{2}$.

Example 3: Finding the Inverse of a Quadratic Function

a. Find the inverse of $f(x) = x^2, x \geq 0$. Then graph the function and its inverse.

b. Find the inverse of $f(x) = x^2 - 2, x \geq 0$. Then graph the function and its inverse.

Example 4: Finding the Inverse of a Cubic Function

a. Consider the function $f(x) = 2x^3 + 1$. Determine whether the inverse of f is a function. Then find the inverse.

b. Consider the function $f(x) = 3x^3 - 2$. Determine whether the inverse of f is a function. Then find the inverse.

Example 5: Finding the Inverse of a Radical Function

a. Consider the function $f(x) = 2\sqrt{x - 3}$. Determine whether the inverse of f is a function. Then find the inverse.

b. Consider the function $f(x) = 3\sqrt{x + 4}$. Determine whether the inverse of f is a function. Then find the inverse.

Composition of Inverse Functions Property
Let f and g be inverse functions if and only if $(f \circ g)(x) = f[g(x)] = x$ for all x in the domain of g and $(g \circ f)(x) = g[f(x)] = x$ for all x in the domain of f.

Example 6: Determining Whether Functions Are Inverses

Determine whether the functions are inverses.

a. $f(x) = 3x - 1,\ g(x) = \frac{x+1}{3}$

b. $f(x) = 8x^3,\ g(x) = \sqrt[3]{2x}$

Example 7: Modeling Real Life

The speed of sound (in meters per second) through air is approximated by $f(x) = 20\sqrt{x + 273}$ where *x* is the temperature in degrees Celsius. Find and interpret $f^{-1}(340)$.

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