> **Note**
> WolframAlpha does not provide a convenient method for finding the logistic regression function for a given data set.

Logarithmic Regression
Enter the following text in the input field to find a logarithmic regression function for {(1, 3), (3, 4), (5, 4.5), (7, 5.2)}.

```
logarithmic fit {(1,3),(3,4),(5,4.5),(7,5.2)}
```

WolframAlpha displays the logarithmic regression function

$$y = 1.07211 \log(15.3438\, x)$$

Recall that WolframAlpha uses "log" to represent the natural logarithmic function. Thus this logarithmic regression function can be written as

$$y = 1.07211 \ln(15.3438\, x)$$

Caution: The TI-83/84 calculators and WolframAlpha use different algorithms to find exponential and logarithmic regression functions. Thus, for a given data set, the exponential and logarithmic regression functions produced using the TI-83/84 calculators and the exponential and logarithmic regression functions produced using WolframAlpha are not identical.

In the *Answers to Selected Exercises* appendix, the exponential and logarithmic regression functions produced using the TI calculators as well as the exponential and logarithmic regression functions produced using WolframAlpha are provided to accommodate users of each of these technologies.

CHAPTER 4 TEST PREP

The following test prep table summarizes essential concepts in this chapter. The references given in the right-hand column list Examples and Exercises that can be used to test your understanding of a concept.

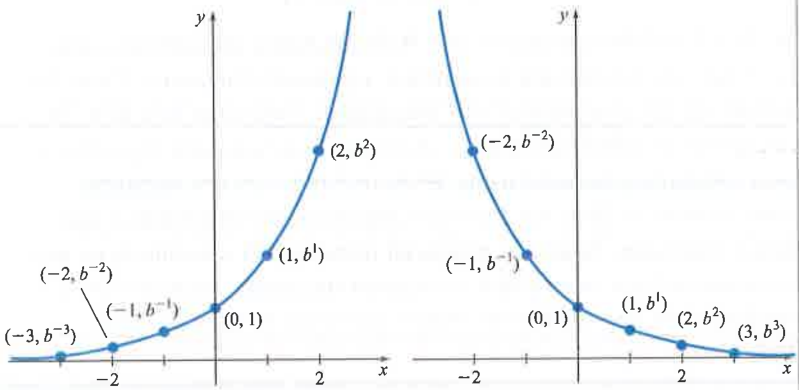
4.1 Inverse Functions

Graph the Inverse of a Function A function f has an inverse function if and only if it is a one-to-one function. The graph of f and the graph of its inverse f^{-1} are symmetric with respect to the line given by $y = x$.	See Example 1, page 338, and then try Exercises 1 and 2, page 423.
Composition of Inverse Functions Property If f is a one-to-one function, then f^{-1} is the inverse function of f if and only if $(f \circ f^{-1})(x) = f[f^{-1}(x)] = x$ for all x in the domain of f^{-1} and $(f^{-1} \circ f)(x) = f^{-1}[f(x)] = x$ for all x in the domain of f.	See Example 2, page 339, and then try Exercises 3 and 6, page 423.
Find the Inverse of a Function If a one-to-one function f is defined by an equation, then you can often use the following procedure to find the equation of f^{-1}. **1.** Substitute y for $f(x)$. **2.** Interchange x and y. **3.** Solve, if possible, for y in terms of x. **4.** Substitute $f^{-1}(x)$ for y.	See Examples 4 and 5, pages 341 and 342, and then try Exercises 9 and 11, page 423.

4.2 Exponential Functions and Their Applications

Properties of $f(x) = b^x$ For positive real numbers b, $b \neq 1$, the exponential function defined by $f(x) = b^x$ has the following properties.

- The function f is a one-to-one function. It has the set of real numbers as its domain and the set of positive real numbers as its range.
- The graph of f is a smooth, continuous curve with a y-intercept of $(0, 1)$, and the graph passes through $(1, b)$.
- If $b > 1$, f is an increasing function and its graph is asymptotic to the negative x-axis. See Figure a.
- If $0 < b < 1$, f is a decreasing function and its graph is asymptotic to the positive x-axis. See Figure b.



a. $f(x) = b^x, b > 1$ **b.** $f(x) = b^x, 0 < b < 1$

See Example 2, page 351, and then try Exercises 25 and 26, page 423.

Graphing Techniques The graphs of some functions can be constructed by translating, stretching, compressing, or reflecting another graph or by combining these techniques.

See Examples 3 and 4, pages 352 and 353, and then try Exercises 29 and 30, page 423.

Natural Exponential Function The number e is defined as the number that

$$\left(1 + \frac{1}{n}\right)^n$$

approaches as n increases without bound. The value of e accurate to 8 decimal places is 2.71828183. The function $f(x) = e^x$, where x is a real number, is called the natural exponential function. Many applications can be modeled by functions that involve e^{kx}, where k is a constant.

See Example 5, page 355, and then try Exercise 84, page 424.

4.3 Logarithmic Functions and Their Applications

Exponential and Logarithmic Form

The exponential form of $y = \log_b x$ is $b^y = x$.

The logarithmic form of $b^y = x$ is $y = \log_b x$.

See Examples 1 and 2, page 362, and then try Exercises 39 and 43, page 423.

Basic Logarithmic Properties

$\log_b b = 1 \qquad \log_b 1 = 0 \qquad \log_b (b^x) = x \qquad b^{\log_b x} = x$

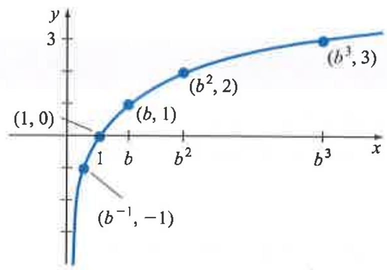
See Example 3, page 363, and then try Exercise 16, page 423.

Properties of $f(x) = \log_b x$ For positive real numbers b, $b \neq 1$, the logarithmic function $f(x) = \log_b x$ has the following properties.

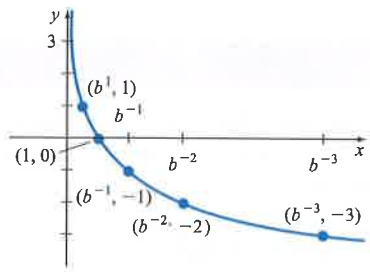
- The domain of f is the set of positive real numbers, and its range is the set of all real numbers.

See Example 4, page 364, and then try Exercises 31 and 32, page 423.

- The graph of f is a smooth, continuous curve with an x-intercept of $(1, 0)$, and the graph passes through $(b, 1)$.
- If $b > 1$, f is an increasing function and its graph is asymptotic to the negative y-axis. See Figure a.
- If $0 < b < 1$, f is a decreasing function and its graph is asymptotic to the positive y-axis. See Figure b.



a. $f(x) = \log_b x$, $b > 1$



b. $f(x) = \log_b x$, $0 < b < 1$

4.4 Properties of Logarithms and Logarithmic Scales

Properties of Logarithms

- *Product property* $\log_b(MN) = \log_b M + \log_b N$
- *Quotient property* $\log_b \dfrac{M}{N} = \log_b M - \log_b N$
- *Power property* $\log_b(M^p) = p \log_b M$
- *Logarithm-of-each-side property* $M = N$ implies $\log_b M = \log_b N$
- *One-to-one property* $\log_b M = \log_b N$ implies $M = N$

See Examples 1 and 2, pages 372 and 373, and then try Exercises 47 and 52, page 423.

Change-of-Base Formula If x, a, and b are positive real numbers with $a \neq 1$ and $b \neq 1$, then

$$\log_b x = \frac{\log_a x}{\log_a b}$$

See Example 3, page 374, and then try Exercises 55 and 58, pages 423 and 424.

Richter Scale Magnitude An earthquake with an intensity of I has a Richter scale magnitude of

$$M = \log\left(\frac{I}{I_0}\right)$$

where I_0 is the measure of the intensity of a zero-level earthquake.

See Examples 5 to 7, pages 375 and 376, and then try Exercises 75 and 77, page 424.

pH The pH of a solution with a hydronium-ion concentration of $[H^+]$ mole per liter is given by $pH = -\log[H^+]$.

See Examples 9 and 10, pages 378 and 379, and then try Exercises 79 and 80, page 424.

4.5 Exponential and Logarithmic Equations

Equality of Exponents Theorem Equations that can be written in the form $b^x = b^y$ can generally be solved by using the Equality of Exponents Theorem, which states that if $b^x = b^y$, where b is a positive real number $(b \neq 1)$, then $x = y$.

See Example 1, page 383, and then try Exercises 17 and 18, page 423.

(continued)

Exponential Equations Many exponential equations can be solved by writing the equation in its logarithmic form or by taking the logarithm of each side of the equation.

See Examples 2 and 3, page 384, and then try Exercises 59 and 60, page 424.

Logarithmic Equations Many logarithmic equations can be solved by using the properties of logarithms and the definition of a logarithm.

See Examples 6 and 7, page 386, and then try Exercises 61 and 62, page 424.

4.6 Exponential Growth and Decay

Exponential Growth and Decay Functions The function $N(t) = N_0e^{kt}$ is an exponential growth function if k is a positive constant, and it is an exponential decay function if k is a negative constant.

See Examples 1 to 3, pages 393–395, and then try Exercises 89 and 90, page 424.

Compound Interest Formula A principal P invested at an annual interest rate r, expressed as a decimal and compounded n times per year for t years, produces the balance

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

See Example 4, page 396, and then try Exercises 81a and 82a, page 424.

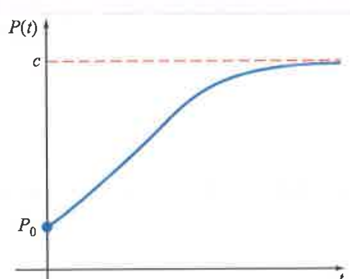
Continuous Compounding Interest Formula If an account with principal P and annual interest rate r is compounded continuously for t years, then the balance is $A = Pe^{rt}$.

See Example 5, page 398, and then try Exercises 81c, page 424.

Logistic Model In the logistic model, the magnitude of a population at time t is given by

$$P(t) = \frac{c}{1 + ae^{-bt}}$$

where $P_0 = P(0)$ is the population at time $t=0$, c is the carrying capacity of the population, and b is the growth rate constant. The constant a is given by the formula $a = \dfrac{c - P_0}{P_0}$.



$P(t) = \dfrac{c}{1+ae^{-bt}}, 0 < P_0 < c, t \geq 0$

See Example 7, page 400 and then try Exercise 93, page 425,

4.7 Modeling Data with Exponential and Logarithmic Functions

Modeling Process

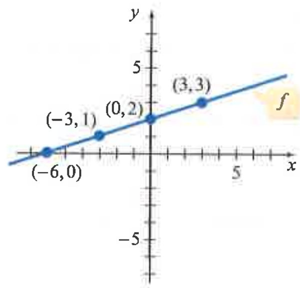
1. Construct a scatter plot of the data to determine which type of function will effectively model the data.
2. Use a graphing utility to find the modeling function and the correlation coefficient or the coefficient of determination for the model.
3. Examine the correlation coefficient or the coefficient of determination and view a graph that displays both the function and the scatter plot to determine how well the function fits the data.

See Examples 2 and 3, pages 409 and 411, and then try Exercises 91 and 92, pages 424 and 425.

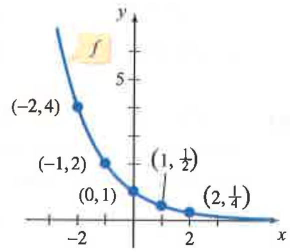
CHAPTER 4 REVIEW EXERCISES

In Exercises 1 and 2, draw the graph of the inverse of the given function.

1.



2.



In Exercises 3 to 6, use composition of functions to determine whether the given functions are inverse functions.

3. $F(x) = 2x - 5 \qquad G(x) = \dfrac{x+5}{2}$

4. $h(x) = \sqrt{x} \qquad k(x) = x^2, \quad x \geq 0$

5. $l(x) = \dfrac{x+3}{x} \qquad m(x) = \dfrac{3}{x-1}$

6. $p(x) = \dfrac{x-5}{2x} \qquad q(x) = \dfrac{2x}{x-5}$

In Exercises 7 to 10, find the inverse of the function. Sketch the graph of the function and its inverse on the same set of coordinate axes.

7. $f(x) = 3x - 4$

8. $g(x) = -2x + 3$

9. $h(x) = -\dfrac{1}{2}x - 2$

10. $k(x) = \dfrac{1}{x}$

In Exercises 11 and 12, find the inverse of the given function.

11. $f(x) = \dfrac{2x}{x-1}$, where the domain of f is $\{x \mid x > 1\}$

12. $g(x) = x^2 + 2x$, where the domain of g is $\{x \mid x \geq -1\}$

In Exercises 13 to 24, solve each equation. Do not use a calculator.

13. $\log_5 25 = x$

14. $\log_3 81 = x$

15. $\ln e^3 = x$

16. $\ln e^{\pi} = x$

17. $3^{2x+7} = 27$

18. $5^{x-4} = 625$

19. $3^x = \dfrac{1}{243}$

20. $32(2^x) = 1024$

21. $\log x^2 = 2$

22. $\dfrac{2}{3}\log|x| = 2$

23. $10^{\log 2x} = 14$

24. $e^{\ln x^2} = 64$

In Exercises 25 to 36, sketch the graph of each function.

25. $f(x) = (2.5)^x$

26. $f(x) = \left(\dfrac{1}{4}\right)^x$

27. $f(x) = 3^{|x|}$

28. $f(x) = 4^{-|x|}$

29. $f(x) = 2^x - 3$

30. $f(x) = 2^{(x-3)}$

31. $f(x) = \log_5 x$

32. $f(x) = \log_{1/3} x$

33. $f(x) = \dfrac{1}{3}\log x$

34. $f(x) = 3\log x^{1/3}$

35. $f(x) = -\dfrac{1}{2}\ln x$

36. $f(x) = -\ln|x|$

In Exercises 37 and 38, use a graphing utility to graph each function.

37. $f(x) = \dfrac{4^x + 4^{-x}}{2}$

38. $f(x) = \dfrac{3^x - 3^{-x}}{2}$

In Exercises 39 to 42, change each logarithmic equation to its exponential form.

39. $\log 1000 = 3$

40. $\log_7 2401 = 4$

41. $\log_{\sqrt{2}} 4 = 4$

42. $\ln 1 = 0$

In Exercises 43 to 46, change each exponential equation to its logarithmic form.

43. $5^3 = 125$

44. $2^{10} = 1024$

45. $10^0 = 1$

46. $8^{1/2} = 2\sqrt{2}$

In Exercises 47 to 50, expand the given logarithmic expression.

47. $\log_b\left(\dfrac{x\sqrt{y}}{z^3}\right)$

48. $\log_5\left(\dfrac{25\sqrt{x}}{y^2}\right)$

49. $\ln xy^3$

50. $\ln \dfrac{\sqrt{xy}}{z^4}$

In Exercises 51 to 54, write each logarithmic expression as a single logarithm with a coefficient of 1.

51. $2\log x + \dfrac{1}{3}\log(x+1)$

52. $5\log x - 2\log(x+5)$

53. $\dfrac{1}{2}\ln 2xy - 3\ln z$

54. $\ln x - (\ln y - \ln z)$

In Exercises 55 to 58, use the change-of-base formula and a calculator to approximate each logarithm accurate to six significant digits.

55. $\log_2 551$

56. $\log_{12} 43$

57. $\log_4 0.85$

58. $\log_8 0.3$

In Exercises 59 to 74, solve each equation for *x*. Give exact answers. Do not use a calculator.

59. $4^x = 30$

60. $5^{x+1} = 41$

61. $\ln 3x - \ln(x - 1) = \ln 4$

62. $\ln 3x + \ln 2 = 1$

63. $e^{\ln(x+2)} = 6$

64. $10^{\log(2x+1)} = 31$

65. $\dfrac{4^x + 4^{-x}}{4^x - 4^{-x}} = 2$

66. $\dfrac{5^x + 5^{-x}}{2} = 8$

67. $\log(\log x) = 3$

68. $\ln(\ln x) = 2$

69. $\log \sqrt{x - 5} = 3$

70. $\log x + \log(x - 15) = 1$

71. $\log_4(\log_3 x) = 1$

72. $\log_7(\log_5 x^2) = 0$

73. $\log_5 x^3 = \log_5 16x$

74. $25 = 16^{\log_4 x}$

75. Earthquake Magnitude Determine, to the nearest 0.1, the Richter scale magnitude of an earthquake with an intensity of $I = 63{,}280{,}000I_0$.

76. Earthquake Magnitude A seismogram has an amplitude of 18 millimeters, and the difference in time between the s-wave and the p-wave is 21 seconds. Find, to the nearest tenth, the Richter scale magnitude of the earthquake that produced the seismogram.

77. Comparison of Earthquakes An earthquake had a Richter scale magnitude of 7.2. Its aftershock had a Richter scale magnitude of 3.7. Compare the intensity of the earthquake with the intensity of the aftershock by finding, to the nearest unit, the ratio of the larger intensity to the smaller intensity.

78. Comparison of Earthquakes On March 28, 1964, an earthquake of magnitude 9.2 on the Richter scale struck Prince William Sound, Alaska. On May 2, 2008, an earthquake of magnitude 6.6 on the Richter scale struck the Aleutian Islands in Alaska. Compare the intensity of the larger earthquake to the intensity of the smaller earthquake by finding the ratio of the larger intensity to the smaller intensity.

79. Chemistry Find the pH of milk of magnesia that has a hydronium-ion concentration of 3.16×10^{-11} mole per liter. Round to the nearest tenth.

80. Chemistry Find the hydronium-ion concentration of lemon juice that has a pH of 2.3.

81. Compound Interest Find the balance when \$3750 is invested at an annual interest rate of 2.5% for 5 years if the interest is compounded

a. monthly **b.** daily **c.** continuously

82. Compound Interest Find the balance when \$48,000 is invested at an annual interest rate of 3.75% for 25 years if the interest is compounded

a. semiannually **b.** monthly **c.** daily

83. Depreciation The scrap value S of a product with an expected life span of n years is given by $S(n) = P(1 - r)^n$, where P is the original purchase price of the product and r is the annual rate of depreciation. A taxicab is purchased for \$12,400 and is expected to last 3 years. What is its scrap value if it depreciates at a rate of 29% per year?

84. Medicine A skin wound heals according to the function given by $N(t) = N_0 e^{-0.12t}$, where N is the number of square centimeters of unhealed skin t days after the injury and N_0 is the number of square centimeters covered by the original wound.

a. What percentage of the wound will be healed after 10 days?

b. How many days, to the nearest day, will it take for 50% of the wound to heal?

c. How long, to the nearest day, will it take for 90% of the wound to heal?

In Exercises 85 to 88, find the exponential growth or decay function $N(t) = N_0e^{kt}$ that satisfies the given conditions.

85. $N(0) = 1, N(2) = 5$

86. $N(0) = 2, N(3) = 11$

87. $N(1) = 4, N(5) = 5$

88. $N(-1) = 2, N(0) = 1$

89. Population Growth

a. Find the exponential growth function for a city whose population was 25,200 in 2007 and 26,800 in 2008. Use $t = 0$ to represent 2007.

b. Use the growth function to predict, to the nearest hundred, the population of the city in 2014.

90. Carbon Dating Determine, to the nearest 10 years, the age of a bone if it now contains 96% of its original amount of carbon-14. The half-life of carbon-14 is 5730 years.

91. Movie Theater Admissions The following table shows the average number of United States movie theater admissions, per week, for the selected years from 2002 to 2011.

Average Number of U.S. Movie Theater Admissions per Week

Year	Admissions per Week (millions)	Year	Admissions per Week (millions)
2002	30.8	2008	26.2
2004	28.5	2010	25.8
2006	26.8	2011	24.7

Source: The World Almanac and Book of Facts 2013.

a. Find an exponential regression function and a logarithmic regression function that model the average number of admissions per week, N, as a function of the year t. Use $t = 2$ to represent 2002, $t = 4$ to represent 2004, $t = 6$ to represent 2006, $t = 10$ to represent 2010, and $t = 11$ to represent 2011.

b. Examine the coefficients of determination to determine which function provides a better fit to the data.

c. Use the regression function you selected in part **b** to predict the average number of admissions per week in 2015 ($t = 15$). Round to the nearest tenth of a million.

92. **Internet Use** The following table shows the number of people, in the United States, that used the Internet for the years from 2001 to 2010.

Number of U.S. Internet Users

Year	Number of Users, U (in millions)	Year	Number of Users, U (in millions)
2001	140	2006	206
2002	169	2007	227
2003	179	2008	226
2004	190	2009	240
2005	201	2010	245

Source: Time Almanac 2012.

a. Find an exponential regression function and a logarithmic regression function for the data. Use $t = 1$ to represent 2001, $t = 2$ to represent 2002, . . . , and $t = 10$ to represent 2010.

b. Use the exponential function to predict the number of U.S. Internet users in 2014. Round to the nearest million.

c. Use the logarithmic function to predict the number of U.S. Internet users in 2014. Round to the nearest million.

93. **Logistic Growth** The population of coyotes in a national park satisfies the logistic model with $P_0 = 210$ in 2001, $c = 1400$, and $P(3) = 360$ (the population in 2004).

a. Determine the logistic model.

b. Use the model to predict, to the nearest 10, the coyote population in 2014.

94. **Logistic Growth** Consider the logistic function

$$P(t) = \frac{128}{1 + 5e^{-0.27t}}$$

a. Find P_0.

b. What does $P(t)$ approach as $t \to \infty$?

CHAPTER 4 TEST

1. Find the inverse of $f(x) = 2x - 3$. Graph f and f^{-1} on the same coordinate axes.

2. Find the inverse of $f(x) = \dfrac{x}{4x - 8}$, where the domain of f is $\{x \mid x > 2\}$. State the domain and the range of f^{-1}.

3. **a.** Write $\log_b(5x - 3) = c$ in exponential form.

b. Write $3^{x/2} = y$ in logarithmic form.

4. Expand $\log_b \dfrac{z^2}{y^3\sqrt{x}}$.

5. Write $\log(2x + 3) - 3\log(x - 2)$ as a single logarithm with a coefficient of 1.

6. Use the change-of-base formula and a calculator to approximate $\log_4 12$. Round your result to the nearest ten-thousandth.

7. Graph: $f(x) = 3^{-x/2}$

8. Graph: $f(x) = \ln(x + 1)$

9. Solve $5^x = 22$. Round your solution to the nearest ten-thousandth.

10. Find the *exact* solution of $4^{5-x} = 7^x$.

11. Solve: $\log(x + 99) - \log(3x - 2) = 2$

12. Solve: $\ln(2 - x) + \ln(5 - x) = \ln(37 - x)$

13. **Compound Interest** Find the balance on \$2800 invested at an annual interest rate of 2.25% for 10 years provided the interest is compounded

a. monthly **b.** daily **c.** continuously

14. **Compound Interest** Find the time required for money invested at an annual interest rate of 3.75% to double in value if the investment is compounded daily. Round to the nearest hundredth of a year.

15. **Earthquake Magnitude**

a. What, to the nearest tenth, will an earthquake measure on the Richter scale if it has an intensity of $I = 42{,}304{,}000I_0$?

b. Compare the intensity of an earthquake that measures 6.3 on the Richter scale with the intensity of an earthquake that measures 4.5 on the Richter scale by finding the ratio of the larger intensity to the smaller intensity. Round to the nearest natural number.

16. **Exponential Decay** From 2000 to 2010, the population of Detroit, Michigan, declined exponentially. The population of Detroit was 951,270 in 2000 and 713,777 in 2010. (*Source: The World Almanac and Book of Facts 2012.*)

a. Find the exponential decay function that models the population of Detroit during this 10-year period.

b. Use the exponential decay function to predict Detroit's population in 2015. Use $t = 0$ to represent 2000, $t = 10$ to represent 2010, etc. Round the 2015 population prediction to the nearest thousand.

17. Determine, to the nearest 10 years, the age of a bone if it now contains 92% of its original amount of carbon-14. The half-life of carbon-14 is 5730 years.

18. **Value of a Diamond** A diamond merchant has determined the values of several white diamonds that have different weights, measured in carats, but are similar in quality.

Value of a Diamond

Weight (in carats)	Value (in dollars)	Weight (in carats)	Value (in dollars)
0.5	3900	2.5	18,200
1.0	5100	3.0	27,400
1.5	8700	3.5	41,100
2.0	12,300	4.0	65,400

a. Find an exponential regression function that models the value, y, of the diamonds as a function of their weight, x.

b. Use the exponential regression function to estimate the value of a 1.75-carat diamond of similar quality. Round to the nearest \$100.

c. Use the exponential regression function to estimate the value of a 4.25-carat diamond of similar quality. Round to the nearest \$100.

19. **Women's Javelin Throw** The following table shows the progression of the world record distances for the women's javelin throw from 1999 to 2012. (*Note:* No new world record distances were set during the years from 2009 to 2012.)

World Record Progression in the Women's Javelin Throw

Year	Distance d (m)
1999	67.09
2000	68.22
2000	69.48
2001	71.54
2005	71.70
2008	72.28

Source: www.nemethjavelins.hu/world-record-progression-women.

a. Find a logarithmic model for the data. Use $t = 1$ to represent the year 1999, $t = 2$ for 2000, $t = 3$ for 2001, $t = 7$ for 2005, and $t = 10$ for 2008.

b. Assume that a new world record distance will be established in 2015. Use the model from **a** to predict the women's world record javelin throw distance for 2015, represented by $t = 17$. Round to the nearest hundredth of a meter.

20. **Population Growth** The population of raccoons in a state park satisfies a logistic growth model with $P_0 = 160$ in 2011 and $P(1) = 190$ in 2012. A park ranger has estimated the carrying capacity of the park to be 1100 raccoons.

a. Determine the logistic growth model for the raccoon population where t is the number of years after 2011.

b. Use the logistic model from **a** to predict the raccoon population in 2018.

CUMULATIVE REVIEW EXERCISES

1. Solve $|x-4| \leq 2$. Write the solution set using interval notation.

2. Solve $\frac{x}{2x-6} \geq 1$. Write the solution set using set-builder notation.

3. Find, to the nearest tenth, the distance between the points $(5, 2)$ and $(11, 7)$.

4. **Height of a Ball** The height, in feet, of a ball released with an initial upward velocity of 44 feet per second at an initial height of 8 feet is given by $h(t) = -16t^2 + 44t + 8$, where t is the time in seconds after the ball is released. Find the maximum height the ball will reach.

5. Given $f(x) = 2x + 1$ and $g(x) = x^2 - 5$, find $(g \circ f)(x)$.

6. Find the inverse of $f(x) = 3x - 5$.

7. **Safe Load** The load that a horizontal beam can safely support varies jointly as the width and the square of the depth of the beam. It has been determined that a beam with a width of 4 inches and a depth of 8 inches can safely support a load of 1500 pounds. How many pounds can a beam of the same material and the same length safely support if it has a width of 6 inches and a depth of 10 inches? Round to the nearest hundred pounds.

8. Use Descartes' Rule of Signs to determine the number of possible positive and the number of possible negative real zeros of $P(x) = x^4 - 3x^3 + x^2 - x - 6$.

9. Find the zeros of $P(x) = x^4 - 5x^3 + x^2 + 15x - 12$.

10. Find a polynomial function of lowest degree that has 2, $1 - i$, and $1 + i$ as zeros.

11. Find the equations of the vertical and horizontal asymptotes of the graph of $r(x) = \frac{3x-5}{x-4}$.

12. Determine the domain and the range of the rational function $R(x) = \frac{4}{x^2+1}$.

13. State whether $f(x) = 0.4^x$ is an increasing function or a decreasing function.

14. Write $\log_4 x = y$ in exponential form.

15. Write $5^3 = 125$ in logarithmic form.

16. Find, to the nearest tenth, the Richter scale magnitude of an earthquake with an intensity of $I = 11{,}650{,}600I_0$.

17. Solve $2e^x = 15$. Round to the nearest ten-thousandth.

18. Find the age of a bone if it now has 94% of the carbon-14 it had at time $t = 0$. The half-life of carbon-14 is 5730 years. Round to the nearest 10 years.

19. Solve $\frac{e^x - e^{-x}}{2} = 12$ for x. Round to the nearest ten-thousandth.

20. **Population Growth** The wolf population in a national park satisfies a logistic growth model with $P_0 = 160$ in 2008 and $P(3) = 205$ (the population in 2011). It has been determined that the maximum population the park can support is 450 wolves.

a. Determine the logistic growth model for the data.

b. Use the logistic growth model to predict, to the nearest 10, the wolf population in 2018.