

# 4 Chapter Review WITH CalcChat®



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## Chapter Learning Target

Understand polynomial functions.

## Chapter Success Criteria

- ◆ I can graph polynomial functions.
  - ◆ I can add, subtract, multiply, and divide, and factor polynomials.
  - I can solve polynomial equations.
  - I can model with and analyze graphs of polynomial functions.
- ◆ Surface  
■ Deep

**SELF-ASSESSMENT** 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

## 4.1 Graphing Polynomial Functions (pp. 153–160)



**Learning Target:** Graph and describe polynomial functions.

**Graph the polynomial function.**

1.  $h(x) = x^2 + 6x^5 - 5$
  2.  $p(x) = 2x^3 - 4x + 6$
  3.  $f(x) = 3x^4 - 5x^2 + 1$
  4.  $g(x) = -x^4 + x + 2$
5. The life expectancy (in years) at birth of a person in the United States  $t$  years after 2000 can be modeled by the polynomial function  $f(t) = -0.0013t^3 + 0.023t^2 + 0.09t + 76.8$ .
- a. Use technology to graph the function for  $0 \leq t \leq 16$ . Describe the behavior of the graph on this interval.
  - b. What was the average rate of change in life expectancy at birth from 2000 to 2016?
  - c. Do you think this model can be used for years after 2016? Explain your reasoning.

### Vocabulary

AZ  
VOCAB

polynomial  
polynomial function  
end behavior

## 4.2 Adding, Subtracting, and Multiplying Polynomials (pp. 161–168)



**Learning Target:** Add, subtract, and multiply polynomials.

**Find the sum or difference.**

6.  $(4x^3 - 12x^2 - 5) - (-8x^2 + 4x + 3)$
7.  $(x^4 + 3x^3 - x^2 + 6) + (2x^4 - 3x + 9)$
8.  $(3x^2 + 9x + 13) - (x^2 - 2x + 12)$

**Find the product.**

9.  $(2y^2 + 4y - 7)(y + 3)$
10.  $(2m + n)^3$
11.  $(s + 2)(s + 4)(s - 3)$

**Use Pascal's Triangle to expand the binomial.**

12.  $(m + 4)^4$
13.  $(3s + 2)^5$
14.  $(z + 1)^6$

### Vocabulary

AZ  
VOCAB

Pascal's Triangle



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**4.3** Dividing Polynomials (pp. 169–174)**Learning Target:** Divide polynomials by other polynomials and use the Remainder Theorem.

Divide using polynomial long division or synthetic division.

15.  $(x^3 + x^2 + 3x - 4) \div (x^2 + 2x + 1)$   
 16.  $(x^4 + 3x^3 - 4x^2 + 5x + 3) \div (x^2 + x + 4)$   
 17.  $(-2x^3 + 3x - 5) \div (x - 2)$   
 18.  $(x^4 - x^2 - 7) \div (x + 4)$

Use synthetic division to evaluate the function for the indicated value of  $x$ .

19.  $f(x) = 4x^3 + 2x^2 - 4$ ;  $x = 5$       20.  $f(x) = -x^4 - 3x^3 + 6x$ ;  $x = -3$

21. Find the missing values. Then write the dividend, divisor, and quotient represented by the synthetic division.

		1		4
		-6	15	
2	-5	13	-35	

22. The total numbers  $N$  (in thousands) of overnight stays at Zion National Park  $x$  years after 2000 can be modeled by

$$N = -0.032x^3 + 0.73x^2 + 2.2x + 248$$

where  $0 \leq x \leq 18$ . Use synthetic division to find the total number of overnight stays in 2018. Justify your answer.

**Vocabulary**AZ  
VOCAB

polynomial long  
division  
synthetic division

**4.4** Factoring Polynomials (pp. 175–182)**Learning Target:** Factor polynomials and use the Factor Theorem.

Factor the polynomial completely.

23.  $64x^3 - 8$       24.  $8y^3 + 125y^6$   
 25.  $2z^5 - 12z^3 + 10z$       26.  $2a^3 - 7a^2 - 8a + 28$   
 27. Show that  $x + 2$  is a factor of  $f(x) = x^4 + 2x^3 - 27x - 54$ .  
Then factor  $f(x)$  completely.  
 28. A rectangular prism has a volume of  $x^3 + 13x^2 + 34x - 48$  cubic inches.  
Give one possible set of dimensions for the prism in terms of  $x$ . Justify  
your answer.  
 29. The profit  $P$  (in millions of dollars) for a game company can be modeled  
by  $P = -x^3 + 3x^2 + 3$ , where  $x$  is the number (in millions) of copies of a  
new game produced. The company now produces 1 million copies of the  
new game and makes a profit of \$5 million, but it would like to increase  
production. What greater number of copies could the company produce and  
still make the same profit?

**Vocabulary**AZ  
VOCAB

factored completely  
factor by grouping  
quadratic form



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**4.5 Solving Polynomial Equations** (pp. 183–190)**Learning Target:** Solve polynomial equations and find zeros of polynomial functions.**Find all the real solutions of the equation.**

30.  $x^3 + 3x^2 - 10x - 24 = 0$

31.  $x^3 + 5x^2 - 2x - 24 = 0$

**Find the zeros of the function. Then sketch a graph of the function.**

32.  $f(x) = -3x^3 - 6x^2$

33.  $f(x) = -x^4 + 18x^2 - 81$

**Write a polynomial function  $f$  of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros.**

34.  $1, 2 - \sqrt{3}$

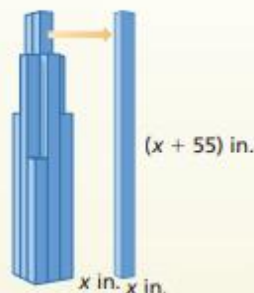
35.  $2, 3, \sqrt{5}$

36.  $-2, 5, 3 + \sqrt{6}$

37. You use 240 cubic inches of clay to make a sculpture shaped like a rectangular prism. The width is 4 inches less than the length, and the height is 2 inches more than three times the length. What are the dimensions of the sculpture? Justify your answer.

38. The Willis Tower in Chicago is composed of nine rectangular prisms with square bases and varying heights. A model of one of the two tallest prisms is shown.

- a. The volume of the prism in the model is 522 cubic inches. What are the dimensions?
- b. The height of the actual prism is about 1450 feet. Use the model to approximate the volume of the actual prism.

**Vocabulary**

repeated solution

**4.6 The Fundamental Theorem of Algebra** (pp. 191–198)**Learning Target:** Use the Fundamental Theorem of Algebra to find all complex roots of polynomial equations.**Find all the zeros of the polynomial function.**

39.  $h(x) = x^4 - 4x^3 - x^2 - 16x - 20$

40.  $f(x) = x^5 + 5x^4 + 8x^3 + 8x^2 + 7x + 3$

**Write a polynomial function  $f$  of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros.**

41.  $3, 1 + 2i$

42.  $-1, 2, 4i$

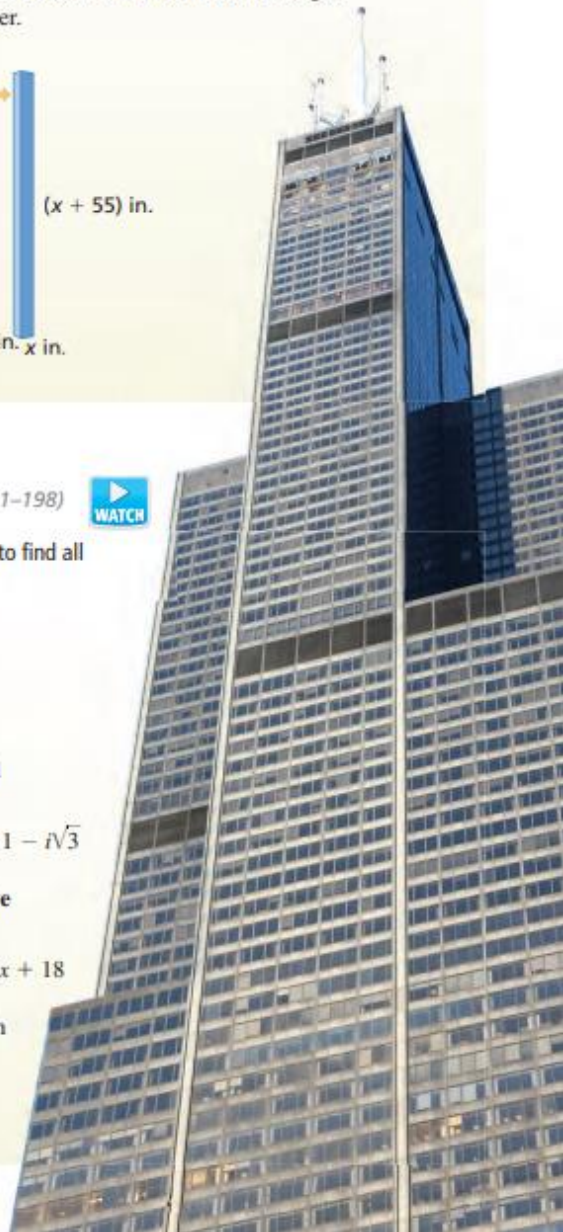
43.  $-5, -4, 1 - i\sqrt{3}$

**Determine the possible numbers of positive real zeros, negative real zeros, and imaginary zeros for the function.**

44.  $f(x) = x^4 - 10x + 8$

45.  $f(x) = -6x^4 - x^3 + 3x^2 + 2x + 18$

46. The average monthly low temperature (in degrees Celsius) in Fargo, North Dakota, can be modeled by  $f(t) = 0.0226t^4 - 0.645t^3 + 5.15t^2 - 8.0t - 14$ , where  $t = 1$  represents January. For what month(s) is the average low temperature about 8 degrees Celsius?





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**4.7** Transformations of Polynomial Functions (pp. 199–204)**Learning Target:** Describe and graph transformations of polynomial functions.Describe the transformation of  $f$  represented by  $g$ . Then graph each function.

47.  $f(x) = x^3$ ,  $g(x) = (-x)^3 + 2$

48.  $f(x) = x^4$ ,  $g(x) = -(x + 9)^4$

Write a rule for  $g$ .49. Let the graph of  $g$  be a horizontal stretch by a factor of 4, followed by a translation 3 units right and 5 units down of the graph of  $f(x) = x^5 + 3x$ .50. Let the graph of  $g$  be a translation 5 units up, followed by a reflection in the  $y$ -axis of the graph of  $f(x) = x^4 - 2x^3 - 12$ .Match the function with the correct transformation of the graph of  $f$ .

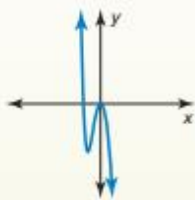
51.  $y = f(2x)$

52.  $y = -2f(x)$

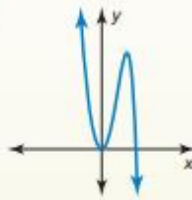
53.  $y = f(-2x)$

54.  $y = 2f(-x)$

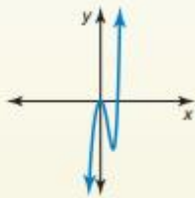
A.



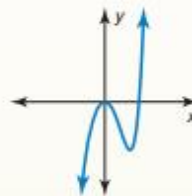
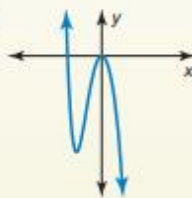
B.



C.



D.

**4.8** Analyzing Graphs of Polynomial Functions (pp. 205–212)**Learning Target:** Analyze graphs of polynomial functions.Graph the function. Identify the  $x$ -intercepts and the points where the local maximums and local minimums occur. Determine the intervals for which the function is increasing or decreasing.

55.  $f(x) = -2x^3 - 3x^2 - 1$

56.  $f(x) = x^4 + 3x^3 - x^2 - 8x + 2$

Determine whether the function is *even*, *odd*, or *neither*.

57.  $f(x) = 2x^3 + 3x$

58.  $g(x) = 3x^2 - 7$

59.  $h(x) = x^6 + 3x^5$

60. Compare the domains and ranges of odd-degree polynomial functions with the domains and ranges of even-degree polynomial functions.

61. Sketch a graph of a polynomial function  $f$  having the given characteristics.

- The graph of  $f$  has  $x$ -intercepts of  $x = -1$ ,  $x = 3$ , and  $x = 5$ .
- $f$  has a local maximum value when  $x = 0$  and when  $x = 4$ .
- $f$  has a local minimum value when  $x = 3$ .

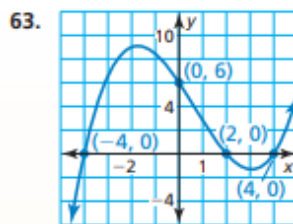
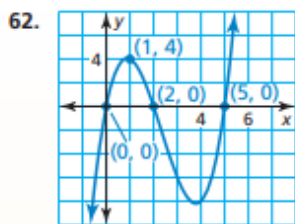
**Vocabulary**A-Z  
VOCABlocal maximum  
local minimum  
even function  
odd function

**4.9** Modeling with Polynomial Functions (pp. 213–218)



**Learning Target:** Write polynomial functions.

Write a cubic function whose graph is shown.



**Vocabulary** AZ VOCAB

finite differences

Use finite differences to determine the degree of the polynomial function that fits the data. Then use technology to find the polynomial function.

64. 

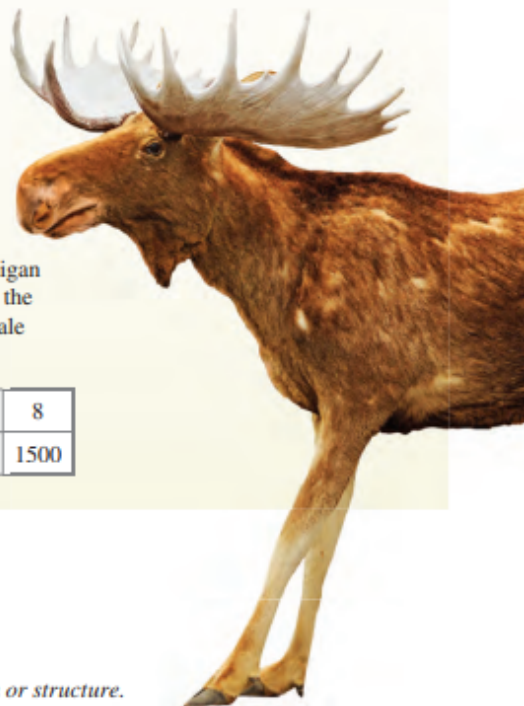
$x$	1	2	3	4	5	6	7
$f(x)$	-11	-24	-27	-8	45	144	301

65. 

$x$	-4	-2	0	2	4	6	8
$f(x)$	-60	10	4	-6	4	10	-60

66. The table shows the numbers  $y$  of moose on Isle Royale in Michigan in the year  $t$ , where  $t = 0$  corresponds to 2010. Find a model for the data. Use the model to predict the number of moose on Isle Royale in 2019.

$t$	0	1	2	3	4	5	6	7	8
$y$	510	515	750	975	1050	1250	1300	1600	1500

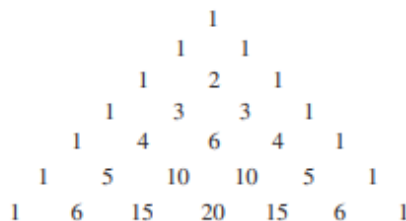


## Mathematical Practices

### Look for and Make Use of Structure

Mathematically proficient students look closely to discern a pattern or structure.

1. Describe any patterns that you see in the diagonals of Pascal's Triangle.



2. In Exercise 65(c) on page 181, explain how you used the structure of the polynomial  $a^5b^2 - a^2b^4 + 2a^4b - 2ab^3 + a^3 - b^2$  to factor it completely.
3. Explain how you used a pattern to write the polynomial function in Exercise 18 on page 218.