CHAPTER 3 TEST PREP

The following test prep table summarizes essential concepts in this chapter. The references given in the right-hand column list Examples and Exercises that can be used to test your understanding of a concept.

3.1 Remainder Theorem and Factor Theorem

Synthetic division Synthetic division is a procedure that can be used to expedite the division of a polynomial by a binomial of the form $x - c$.	See Example 2, page 263, and then try Exercises 1 and 2, page 328.
Remainder Theorem If a polynomial $P(x)$ is divided by $x - c$, then the remainder equals $P(c)$.	See Example 3, page 265, and then try Exercises 3 and 5, pages 328 and 329.
Factor Theorem A polynomial $P(x)$ has a factor $(x - c)$ if and only if $P(c) = 0$.	See Example 4, page 266, and then try Exercises 11 and 12, page 329.

3.2 Polynomial Functions of Higher Degree

Leading Term Test The far-left and far-right behavior of the graph of a polynomial function P can be determined by examining its leading term, $a_n x^n$. • If $a_n > 0$ and n is even, then the graph of P goes up to the far left and up to the far right. • If $a_n > 0$ and n is odd, then the graph of P goes down to the far left and up to the far right. • If $a_n < 0$ and n is even, then the graph of P goes down to the far left and down to the far right. • If $a_n < 0$ and n is odd, then the graph of P goes up to the far left and down to the far right.	See Example 1, page 272, and then try Exercises 13 and 14, page 329.
Definition of Relative Minimum and Relative Maximum If there is an open interval I containing c on which • $f(c) \le f(x)$ for all x in I, then $f(c)$ is a relative minimum of f. • $f(c) \ge f(x)$ for all x in I, then $f(c)$ is a relative maximum of f.	See the Integrating Technology feature, page 273, and then try Exercises 15 and 16, page 329.
Intermediate Value Theorem If P is a polynomial function and $P(a) \neq P(b)$ for $a < b$, then P takes on every value between $P(a)$ and $P(b)$ in the interval $[a, b]$. The following statement is a special case of the Intermediate Value Theorem. If $P(a)$ and $P(b)$ have opposite signs, then you can conclude by the Intermediate Value Theorem that P has a zero between a and b.	See Example 4, page 277, and then try Exercises 17 and 18, page 329.
Even and Odd Powers of $(x - c)$ Theorem If c is a real number and the polynomial function P has $(x - c)$ as a factor exactly k times, then the graph of P will intersect but not cross the x-axis at $(c, 0)$, provided k is an even positive integer, and the graph of P will cross the x-axis at $(c, 0)$, provided k is an odd positive integer.	See Example 5, page 278, and then try Exercises 19 and 20, page 329.

(continued)

Procedure for Graphing Polynomial Functions
To graph a polynomial function P

1. Examine the leading coefficient of P to determine the far-left and far-right behavior of the graph.
2. Find the y-intercept by evaluating $P(0)$.
3. Find the x-intercept(s). If $(x - c)$, where c is a real number, is a factor of P, then $(c, 0)$ is an x-intercept of the graph. Use the Even and Odd Powers of $(x - c)$ Theorem to determine where the graph crosses the x-axis and where the graph intersects but does not cross the x-axis.
4. Find additional points on the graph.
5. Check for symmetry with respect to the y-axis and with respect to the origin.
6. Use all of the information obtained to sketch the graph. The graph should be a smooth, continuous curve that passes through the points determined in steps 2 through 4. The graph should have a maximum of $n - 1$ turning points.

See Example 6, page 279, and then try Exercises 22 and 25, page 329.

3.3 Zeros of Polynomial Functions

Rational Zero Theorem If $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients $(a_n \neq 0)$ and $\frac{p}{q}$ is a rational zero (in simplest form) of P, then p is a factor of the constant term a_0 and q is a factor of the leading coefficient a_n.

See Example 1, page 288, and then try Exercises 27 and 29, page 329.

Descartes' Rule of Signs Let P be a polynomial function with real coefficients and with the terms arranged in order of decreasing powers of x.

- The number of positive real zeros of P is equal to the number of variations in sign of $P(x)$ or to that number decreased by an even integer.
- The number of negative real zeros of P is equal to the number of variations in sign of $P(-x)$ or to that number decreased by an even integer.

See Example 3, page 290, and then try Exercises 33 and 36, page 329.

Guidelines for Finding the Zeros of a Polynomial Function with Integer Coefficients

1. Determine the degree of the function. The number of distinct zeros of the polynomial function is at most n. Apply Descartes' Rule of Signs to find the possible number of positive zeros and the possible number of negative zeros.
2. Apply the Rational Zero Theorem to list rational numbers that are possible zeros. Use synthetic division to test numbers in your list. If you find an upper bound or lower bound, then eliminate from your list any number that is greater than the upper bound or less than the lower bound.
3. Work with the reduced polynomial.
   - If the reduced polynomial is of degree 2, find its zeros either by factoring or by applying the quadratic formula.
   - If the degree of the reduced polynomial is 3 or greater, repeat the preceding steps for this reduced polynomial.

See Example 4, page 292, and then try Exercises 37 and 39, page 329.

3.4 Fundamental Theorem of Algebra

Fundamental Theorem of Algebra If P is a polynomial function of degree $n \geq 1$ with complex coefficients, then P has at least one complex zero. The Fundamental Theorem of Algebra can be used to establish the following theorem. • **Linear Factor Theorem** If P is a polynomial function of degree $n \geq 1$ with leading coefficient $a_n \neq 0$, then P has exactly n linear factors and can be written as $$P(x) = a_n(x - c_1)(x - c_2) \cdots (x - c_n)$$ where $c_1, c_2, \ldots, c_n$ are complex numbers. The Linear Factor Theorem can be used to establish the following theorem. • **Number of Zeros of a Polynomial Function Theorem** If P is a polynomial function of degree $n \geq 1$, then P has exactly n complex zeros, provided each zero is counted according to its multiplicity.	See Example 1, page 300, and then try Exercises 43 and 44, page 329.
Conjugate Pair Theorem If $a + bi$ $(b \neq 0)$ is a complex zero of a polynomial function with real coefficients, then the conjugate $a - bi$ is also a complex zero of the polynomial function.	See Example 3, page 302, and then try Exercises 45 and 46, page 330.
Finding a Polynomial Function with Given Zeros If $c_1, c_2, \ldots, c_n$ are given as zeros, then the product $(x - c_1)(x - c_2) \cdots (x - c_n)$ yields a polynomial function that has the given zeros.	See Example 5, page 304, and then try Exercises 49 and 50, page 330.

3.5 Graphs of Rational Functions and Their Applications

Vertical Asymptotes • **Definition of a Vertical Asymptote** The line given by $x = a$ is a vertical asymptote of the graph of a function F, provided $F(x) \to \infty$ or $F(x) \to -\infty$ as x approaches a from either the left or the right. • **Theorem on Vertical Asymptotes** If the real number a is a zero of the denominator $Q(x)$, then the graph of $F(x) = P(x)/Q(x)$, where $P(x)$ and $Q(x)$ have no common factors, has the vertical asymptote $x = a$.	See Example 1, page 310, and then try Exercises 53 and 56, page 330.
Horizontal Asymptotes • **Definition of a Horizontal Asymptote** The line given by $y = b$ is a horizontal asymptote of the graph of a function F, provided $F(x) \to b$ as $x \to \infty$ or $x \to -\infty$. • **Theorem on Horizontal Asymptotes** See the Theorem on Horizontal Asymptotes on page 311. The method used to determine the horizontal asymptote of a rational function depends on the relationship between the degree of the numerator and the degree of the denominator of the rational function.	See Example 2, page 312, and then try Exercises 55 and 56, page 330.
Slant Asymptotes • **Definition of a Slant Asymptote** The line given by $y = mx + b$, $m \neq 0$, is a slant asymptote of the graph of a function F, provided $F(x) \to mx + b$ as $x \to \infty$ or $x \to -\infty$.	See Example 5, page 317, and then try Exercises 57 and 58, page 330.

(continued)

- **Theorem on Slant Asymptotes** The rational function $F(x) = P(x)/Q(x)$, where $P(x)$ and $Q(x)$ have no common factors, has a slant asymptote if the degree of $P(x)$ is 1 greater than the degree of $Q(x)$. The equation of the asymptote can be determined by setting y equal to the quotient of $P(x)$ divided by $Q(x)$.

General Procedure for Graphing Rational Functions That Have No Common Factors
To graph a rational function F

1. Find the real zeros of the denominator. For each real zero a, the vertical line $x = a$ will be a vertical asymptote. Use the Theorem on Horizontal Asymptotes and the Theorem on Slant Asymptotes to determine whether F has a horizontal asymptote or a slant asymptote. Use dashed lines to graph all asymptotes.
2. Find the real zeros of the numerator. For each real zero c, plot $(c, 0)$. These are the x-intercepts. The y-intercept of the graph is the point $(0, F(0))$, provided $F(0)$ is a real number.
3. Use the tests for symmetry to determine whether the graph has symmetry with respect to the y-axis or with respect to the origin.
4. Find and plot additional points that lie in the intervals between and beyond the vertical asymptotes and the x-intercepts.
5. Determine the behavior of the graph near asymptotes.
6. Use all of the information obtained in steps 1 through 5 to sketch the graph.

See Examples 3 and 4, pages 313 and 315, and then try Exercises 60, 63, and 65, page 330.

General Procedure for Graphing Rational Functions That Have a Common Linear Factor
To graph a rational function F that has a numerator and a denominator with $(x - a)$ as a common factor

1. Reduce the rational function to simplest form. Then use the general procedure for graphing rational functions that have no common factors.
2. If the reduced rational function does not have $(x - a)$ as a factor of the denominator, then the graph produced in step 1 is the graph of F, provided you place an open circle on the graph at $x = a$. The height of the open circle can be determined by evaluating the reduced rational function at $x = a$.

   If $(x - a)$ is a factor of the denominator of the reduced rational function, then the graph produced in step 1 is the graph of F and it will have a vertical asymptote at $x = a$.

See Example 6, page 318, and then try Exercise 61, page 330.

CHAPTER 3 REVIEW EXERCISES

In Exercises 1 and 2, use synthetic division to divide the first polynomial by the second.

1. $4x^3 - 11x^2 + 5x - 2, \quad x - 3$

2. $x^4 + 9x^3 + 6x^2 - 65x - 63, \quad x + 7$

In Exercises 3 to 6, use the Remainder Theorem to find $P(c)$.

3. $P(x) = -2x^5 + 3x^3 - x^2 - 5x + 11, \quad c = 5$

4. $P(x) = 3x^4 - 2x^3 - 11x^2 + 15x - 2, \quad c = -3$

5. $P(x) = 6x^4 - 12x^2 + 8x + 1, \quad c = -2$

6. $P(x) = 5x^5 - 8x^4 + 2x^3 - 6x^2 - 9, \quad c = 3$

In Exercises 7 to 10, use synthetic division to show that *c* is a zero of the given polynomial function.

7. $P(x) = x^3 + 2x^2 - 26x + 33, \quad c = 3$

8. $P(x) = 2x^4 + 8x^3 - 8x^2 - 31x + 4, \quad c = -4$

9. $P(x) = x^5 - x^4 - 2x^2 + x + 1, \quad c = 1$

10. $P(x) = 2x^3 + 3x^2 - 8x + 3, \quad c = \frac{1}{2}$

In Exercises 11 and 12, use the Factor Theorem to determine whether the given binomial is a factor of *P*.

11. $P(x) = x^3 - 11x^2 + 39x - 45, \quad (x - 5)$

12. $P(x) = 2x^4 - 11x^3 + 11x^2 - 33x + 15, \quad (x + 2)$

In Exercises 13 and 14, determine the far-left and the far-right behavior of the graph of the function.

13. $P(x) = -2x^3 - 5x^2 + 6x - 3$

14. $P(x) = -x^4 + 3x^3 - 2x^2 + x - 5$

In Exercises 15 and 16, use the maximum and minimum features of a graphing utility to estimate, to the nearest thousandth, the *x* and *y* coordinates of the points where *P* has a relative maximum or a relative minimum.

15. $P(x) = 2x^3 - x^2 - 3x + 1$

16. $P(x) = x^4 - 2x^2 + x + 1$

In Exercises 17 and 18, use the Intermediate Value Theorem to verify that *P* has a zero between *a* and *b*.

17. $P(x) = 3x^3 - 7x^2 - 3x + 7; a = 2, b = 3$

18. $P(x) = 3x^4 - 5x^3 - 6x^2 - 10x - 24; a = -2, b = -1$

In Exercises 19 and 20, determine the *x*-intercepts of the graph of *P*. For each *x*-intercept, use the Even and Odd Powers of $(x - c)$ Theorem to determine whether the graph of *P* crosses the *x*-axis or intersects but does not cross the *x*-axis.

19. $P(x) = (x + 3)(x - 5)^2$

20. $P(x) = (x - 4)^4(x + 1)$

In Exercises 21 to 26, graph the polynomial function.

21. $P(x) = x^3 - x$

22. $P(x) = -x^3 - x^2 + 8x + 12$

23. $P(x) = x^4 - 6$

24. $P(x) = x^5 - x$

25. $P(x) = x^4 - 10x^2 + 9$

26. $P(x) = x^5 - 5x^3$

In Exercises 27 to 32, use the Rational Zero Theorem to list all possible rational zeros for each polynomial function.

27. $P(x) = x^3 - 7x - 6$

28. $P(x) = 2x^3 + 3x^2 - 29x - 30$

29. $P(x) = 3x^3 - 20x^2 + 23x + 10$

30. $P(x) = 2x^5 + 3x^2 - x - 5$

31. $P(x) = x^3 + x^2 - x - 1$

32. $P(x) = 6x^5 + 3x - 2$

In Exercises 33 to 36, use Descartes' Rule of Signs to state the number of possible positive and negative real zeros of each polynomial function.

33. $P(x) = -2x^5 + 4x^3 + 5x^2 - 2x - 6$

34. $P(x) = 7x^3 + x^4 + 3x^2 - 8x + 15$

35. $P(x) = x^4 - x - 1$

36. $P(x) = x^5 - 4x^4 + 2x^3 - x^2 + x - 8$

In Exercises 37 to 42, find the zeros of the polynomial function.

37. $P(x) = 2x^3 - 7x^2 - 33x + 18$

38. $P(x) = 2x^4 - 5x^3 - 15x^2 + 40x - 42$

39. $P(x) = 6x^4 + 35x^3 + 72x^2 + 60x + 16$

40. $P(x) = 2x^4 + 7x^3 + 5x^2 + 7x + 3$

41. $P(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$

42. $P(x) = 2x^3 - 7x^2 + 22x + 13$

In Exercises 43 and 44, find all the zeros of *P* and write *P* as a product of its leading coefficient and its linear factors.

43. $P(x) = 2x^4 - 9x^3 + 22x^2 - 29x + 10$

44. $P(x) = x^4 - 6x^3 + 21x^2 - 46x + 30$

In Exercises 45 and 46, use the given zero to find the remaining zeros of each polynomial function.

45. $P(x) = x^4 - 4x^3 + 6x^2 - 4x - 15;\quad 1 - 2i$

46. $P(x) = x^4 - x^3 - 17x^2 + 55x - 50;\quad 2 + i$

In Exercises 47 to 50, find the requested polynomial function.

47. Find a third-degree polynomial function with integer coefficients and zeros of 4, -3, and $\frac{1}{2}$.

48. Find a fourth-degree polynomial function with zeros of 2, -3, i, and $-i$.

49. Find a fourth-degree polynomial function with real coefficients that has zeros of 1, 2, and $5i$.

50. Find a fourth-degree polynomial function with real coefficients that has -2 as a zero of multiplicity 2 and has $1 + 3i$ as a zero.

In Exercises 51 and 52, determine the domain of the rational function.

51. $F(x) = \dfrac{2x - 3}{x^2 - 25}$

52. $F(x) = \dfrac{6x^2 - 5x - 4}{15x^2 - 24x + 15}$

In Exercises 53 and 54, determine the vertical asymptotes for the graph of each rational function.

53. $f(x) = \dfrac{2x^2 + 3x - 5}{6x^2 - x - 35}$

54. $f(x) = \dfrac{3x - 17}{x^3 - 16x}$

In Exercises 55 and 56, determine the horizontal asymptote for the graph of each rational function.

55. $f(x) = \dfrac{-2x^3 - 5x^2 + 4x + 1}{3x^4 - 2x^3 - 7x^2 + x - 3}$

56. $f(x) = \dfrac{5x^2 - 2x + 3}{\frac{1}{3}x^2 + x + 4}$

In Exercises 57 and 58, determine the slant asymptote for the graph of each rational function.

57. $f(x) = \dfrac{x^3 - 5x^2 + x + 12}{x^2 - 2x - 5}$

58. $f(x) = \dfrac{-2x^4 - 3x^3 - 2x^2 + 5x + 3}{x^3 - 2x^2 - 5}$

In Exercises 59 to 66, graph each rational function.

59. $f(x) = \dfrac{3x - 2}{x}$

60. $f(x) = \dfrac{x + 4}{x - 2}$

61. $f(x) = \dfrac{12x - 24}{x^2 - 4}$

62. $f(x) = \dfrac{4x^2}{x^2 + 1}$

63. $f(x) = \dfrac{2x^3 - 4x + 6}{x^2 - 4}$

64. $f(x) = \dfrac{x}{x^3 - 1}$

65. $f(x) = \dfrac{3x^2 - 6}{x^2 - 9}$

66. $f(x) = \dfrac{-x^3 + 6}{x^2}$

67. **Average Cost of Skateboards** The cost, in dollars, of producing x skateboards is given by

$$C(x) = 5.75x + 34{,}200$$

The average cost per skateboard is given by

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{5.75x + 34{,}200}{x}$$

a. Find the average cost per skateboard, to the nearest cent, of producing 5000 and 50,000 skateboards.

b. What is the equation of the horizontal asymptote of the graph of $\overline{C}$? Explain the significance of the horizontal asymptote as it relates to this application.

68. **Food Temperature** The temperature F, in degrees Fahrenheit, of a dessert placed in a freezer for t hours is given by the rational function

$$F(t) = \frac{60}{t^2 + 2t + 1},\ t \geq 0$$

a. Find the temperature of the dessert after it has been in the freezer for 1 hour.

b. What temperature will the dessert approach as $t \to \infty$?

69. **Tuition and Fees** The following table shows the average cost for tuition and fees, paid per school year, by a United States college student attending a private four-year college.

Average Tuition/Fees at U.S. Private Four-Year Colleges

School Year	Tuition/Fees (in dollars)	School Year	Tuition/Fees (in dollars)
2003–04	17,763	2007–08	21,427
2004–05	18,604	2008–09	22,036
2005–06	19,292	2009–10	21,908
2006–07	20,517	2010–11	22,771

Source: The World Almanac and Book of Facts 2013, p. 419.

a. Find a quartic regression function that models the data. Use $x = 3$ to represent 2003–04, $x = 4$ to represent 2004–05, . . . , and $x = 10$ to represent 2010–11.

b. Use the quartic regression function to estimate the average cost of tuition and fees for the 2013–14 ($x = 13$) school year. Round to the nearest hundred dollars.

70. **Physiology** One of Poiseuille's laws states that the resistance R encountered by blood flowing through a blood vessel is given by

$$R(r) = C\frac{L}{r^4}$$

where C is a positive constant determined by the viscosity of the blood, L is the length of the blood vessel, and r is its radius.

a. Explain the meaning of $R(r) \to \infty$ as $r \to 0$.

b. Explain the meaning of $R(r) \to 0$ as $r \to \infty$.

CHAPTER 3 TEST

1. Use synthetic division to divide

$$(3x^3 + 5x^2 + 4x - 1) \div (x + 2)$$

2. Use the Remainder Theorem to find $P(-2)$ if

$$P(x) = -3x^3 + 7x^2 + 2x - 5$$

3. Use the Factor Theorem to show that $x - 1$ is a factor of

$$x^4 - 4x^3 + 7x^2 - 6x + 2$$

4. Determine the far-left and far-right behavior of the graph of

$$P(x) = x^5 - 3x^4 + 4x^3 - x^2 - 2x + 1$$

5. Find the real zeros of $P(x) = 3x^3 + 7x^2 - 6x$.

6. Use the Intermediate Value Theorem to verify that

$$P(x) = 2x^3 - 3x^2 - x + 1$$

has a zero between 1 and 2.

7. Find the zeros of

$$P(x) = (x^2 - 4)^2(2x - 3)(x + 1)^3$$

and state the multiplicity of each.

8. Use the Rational Zero Theorem to list the possible rational zeros of

$$P(x) = 6x^3 - 3x^2 + 2x - 3$$

9. Use Descartes' Rule of Signs to state the number of possible positive and negative real zeros of

$$P(x) = x^4 - 5x^3 + 12x^2 - 23x - 15$$

10. Find the zeros of $P(x) = 2x^4 - 15x^3 + 41x^2 - 47x + 15$.

11. Given that $2 + 3i$ is a zero of

$$P(x) = 6x^4 - 5x^3 + 12x^2 + 207x + 130$$

find the remaining zeros.

12. Find all the zeros of

$$P(x) = x^5 - 6x^4 + 14x^3 - 14x^2 + 5x$$

13. Find a polynomial function of smallest degree that has integer coefficients and zeros $3 - 2i$, 2, and $-\frac{1}{2}$.

14. Find the vertical asymptotes and the horizontal asymptotes of the graph of

$$f(x) = \frac{3x^2 - 2x + 1}{x^2 - 5x + 6}$$

15. Graph: $P(x) = x^3 - 6x^2 + 9x + 1$

16. Graph: $f(x) = \dfrac{x^2 - 1}{x^2 - 2x - 3}$

17. Graph: $f(x) = \dfrac{2x^2 + 2x + 1}{x + 1}$

18. **Weight of a Great White Shark** The following table shows the estimated weights of great white sharks of various lengths.

Estimated Weight of Great White Sharks

Length (in feet, x)	Weight (in pounds)	Length (in feet, x)	Weight (in pounds)
7	256	11	1031
8	386	12	1348
9	555	13	1726
10	768	14	2169

Source: National Oceanic and Atmospheric Administration.

a. Find the cubic regression function that models the data.

b. What is the coefficient of determination for this cubic regression?

c. What does the coefficient of determination indicate concerning how well the cubic regression function models the data?

d. Use the cubic regression function to estimate the weight of a great white shark that has a length of 12.5 feet and the weight of a great white shark that has a length of 15 feet. Round to the nearest pound.

19. **Typing Speed** The rational function

$$w(t) = \frac{70t + 120}{t + 40}, \, t \geq 0$$

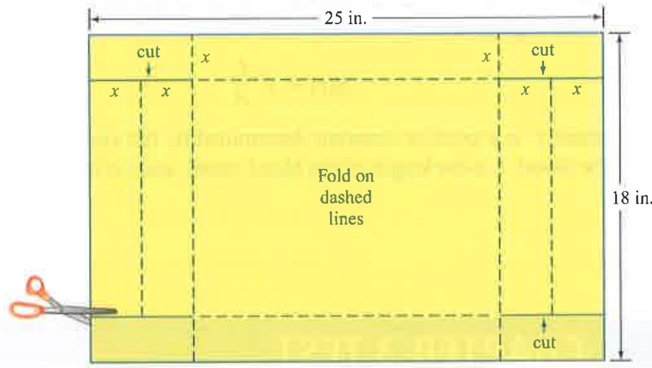
models Rene's typing speed, in words per minute, after t hours of typing lessons.

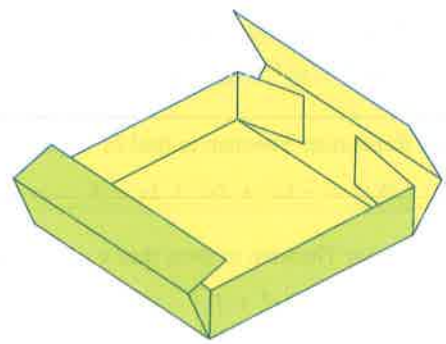
a. Find $w(1)$, $w(10)$, and $w(20)$. Round to the nearest word per minute.

b. How many hours of typing lessons will be needed before Rene can expect to type at 60 words per minute?

c. What will Rene's typing speed approach as $t \to \infty$?

20. **Maximizing Volume** You are to construct an open box from a rectangular sheet of cardboard that measures 18 inches by 25 inches. To assemble the box, you make the four cuts shown in the figure below and then fold on the dashed lines. What value of x (to the nearest 0.01 inch) will produce a box with maximum volume? What is the maximum volume (to the nearest 0.1 cubic inch)?





CUMULATIVE REVIEW EXERCISES

1. Write $\frac{3 + 4i}{1 - 2i}$ in $a + bi$ form.

2. Use the quadratic formula to solve $x^2 - x - 1 = 0$.

3. Solve: $\sqrt{2x + 5} - \sqrt{x - 1} = 2$

4. Solve: $|x - 3| \leq 11$

5. Find the distance between the points $(2, 5)$ and $(7, -11)$.

6. Explain how to use the graph of $y = x^2$ to produce the graph of $y = (x - 2)^2 + 4$.

7. Find the difference quotient for the function $P(x) = x^2 - 2x - 3$.

8. Given $f(x) = 2x^2 + 5x - 3$ and $g(x) = 4x - 7$, find $(f \circ g)(x)$.

9. Given $f(x) = x^3 - 2x + 7$ and $g(x) = x^2 - 3x - 4$, find $(f - g)(x)$.

10. Use synthetic division to divide $(4x^4 - 2x^2 - 4x - 5)$ by $(x + 2)$.

11. Use the Remainder Theorem to find $P(3)$ for the polynomial function $P(x) = 2x^4 - 3x^2 + 4x - 6$.

12. Determine the far-right behavior of the graph the of polynomial function $P(x) = -3x^4 - x^2 + 7x - 6$.

13. Determine the relative maximum of the polynomial function $P(x) = -3x^3 - x^2 + 4x - 1$. Round to the nearest ten-thousandth.

14. Use the Rational Zero Theorem to list all possible rational zeros of $P(x) = 3x^4 - 4x^3 - 11x^2 + 16x - 4$.

15. Use Descartes' Rule of Signs to state the number of possible positive and negative real zeros of
$$P(x) = x^3 + x^2 + 2x + 4$$

16. Find all zeros of $P(x) = x^3 + x + 10$.

17. Find a polynomial function of smallest degree that has real coefficients and -2 and $3 + i$ as zeros.

18. Write $P(x) = x^3 - 2x^2 + 9x - 18$ as a product of linear factors.

19. Determine the vertical and horizontal asymptotes of the graph of $F(x) = \frac{4x^2}{x^2 + x - 6}$.

20. Find the equation of the slant asymptote for the graph of $F(x) = \frac{x^3 + 4x^2 + 1}{x^2 + 4}$.