Basic Identities
§3.1(Day 1)

Reciprocal Identities

$\sin x = \dfrac{1}{\csc x}$ $\csc x = \dfrac{1}{\sin x}$

$\cos x = \dfrac{1}{\sec x}$ $\sec x = \dfrac{1}{\cos x}$

$\tan x = \dfrac{1}{\cot x}$ $\cot x = \dfrac{1}{\tan x}$

In addition, $\tan x = \dfrac{\sin x}{\cos x}$ and $\cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

Example 1

Write each in terms of sine and/or cosine, and then simplify.

a. $\dfrac{\tan x}{\sec x}$

b. $\sin x + \cot x \cos x$

Writing one function in terms of another.

Example 2

Express tangent in terms of sine.

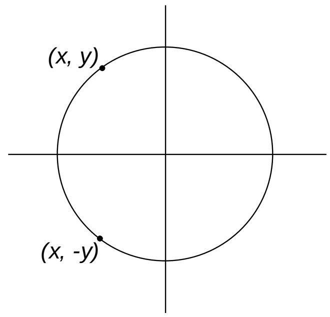
Using identities to find function values.

Example 3

$\tan \alpha = -\frac{2}{3}$ and α is in Quadrant IV. Find all the remaining trig functions.

Pg 173, 1-21 odd

Basic Identities
§3.1 (Day 2)



$\cos(-s) = \cos(s)$, therefore EVEN
$\sin(-s) = -\sin(s)$, therefore ODD

Odd and Even Identities

Odd: $\sin(-x) = -\sin(x)$ $\quad$ $\csc(-x) = -\csc(x)$
$\tan(-x) = -\tan(x)$ $\quad$ $\cot(-x) = -\cot(x)$

Even: $\cos(-x) = \cos(x)$ $\quad$ $\sec(-x) = \sec(x)$

Example 1

Simplify each expression.

a. sin(-x) cot(-x)

b. $\dfrac{1}{1 + \cos(-x)} + \dfrac{1}{1 - \cos x}$

Example 2

Determine whether each function is odd, even, or neither.

a. $f(x) = \dfrac{\cos(2x)}{x}$

b. $g(x) = \sin x + \cos x$

Proving that an equation is not an identity.

Example 3

Show that $\sin(2t) = 2\sin(t)$ is not an identity.

Pg 173, 23-29, 33-39, 45-49 odds

Verifying Identities
§3.2

Example 1

Verify that $1 + \sec x \sin x \tan x = \sec^2 x$

Multiplying Binomials

Example 2

a. $(1 - \tan x)(1 + \tan x)$

b. $(2\sin\ + 1)^2$

Factoring Trig Functions

Example 3

a. $\sec^2 x - \tan^2 x$

b. $\sin^2 \beta + \sin\beta - 2$

c. $2\sin^2 \alpha - 5\sin\alpha - 3$

Example 4

Prove that each is an identity

a. $\dfrac{\cos \alpha}{1 - \sin \alpha} = \dfrac{1 + \sin \alpha}{\cos \alpha}$

b. $\dfrac{\csc x - \sin x}{\sin x} = \cot^2 x$

Pg 181, 1-45 odd

Sum and Difference Identities for Cosine
§3.3

Sum and Difference Identities for Cosine

Sum: $\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$

Difference: $\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$

Used for non-famous angles

Example 1

Find the exact values.

a. cos 75

b. $\cos\left(\frac{\pi}{12}\right)$

Example 2

Use an appropriate identity to simplify.

a. cos(49º)cos (4º) + sin(49º)sin(4º)

b. cos(2)cos(-3) – sin(-2)sin(3)

Pg 190, 1-45 odd

Sum and Difference Identities for Sine and Tangent
§3.4

Sum and Difference Identities for Sine and Tangent

$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

$\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$$

Example 1
Find the exact values.

a. sin(195º) b. $\tan\left(\frac{\pi}{12}\right)$ c. $\tan\left(\frac{7\pi}{12}\right)$

Example 2
Use an appropriate identity to simplify.

a. sin(7º)cos(2º) + cos(7º)sin(2º) b. sin(-t)cos(2t) – cos(-t)sin(-2t)

Pg 196, 1-21, 25, 33, 35, 41, 43 odds

Double-Angle and Half-Angle Identities
§3.5

Double Angle Identities

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Example 1

Find each using double angle identities.

a. sin(120)   b. cos(120)   c. tan(120)

Example 2

Use identities to simplify.

a. 2sin15cos15   b. $2\cos^2(22.5) - 1$ c. $\dfrac{2\tan(22.5)}{1 - \tan^2(22.5)}$

Half-Angle Identities

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}} \qquad \cos\frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}} \qquad \tan\frac{x}{2} = \frac{\sin x}{1 + \cos x} \qquad \tan\frac{x}{2} = \frac{1 - \cos x}{\sin x}$$

Example 3
Use half-angle identities to simplify.

a. $\cos\left(\frac{\pi}{8}\right)$ b. sin 75º c. tan (-15)

Pg 204, 1-21 odd

Product and Sum Identities
§3.6

Product-to-Sum Identities

$$\sin A \cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

Example 1
Use the product-to-sum identity to simplify.

a. cos 23 sin 15 b. $\sin \frac{3\pi}{8} \cos \frac{\pi}{4}$ c. cos 67.5 sin 112.5

Sum-to-Product Identities

$$\sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

$$\sin x - \sin y = 2 \cos\left(\frac{x + y}{2}\right)\sin\left(\frac{x - y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

$$\cos x - \cos y = -2 \sin\left(\frac{x + y}{2}\right)\sin\left(\frac{x - y}{2}\right)$$

Example 2

Use the sum-to-product identity to simplify.

a. sin 36 – sin 14 b. $\cos\frac{\pi}{6} - \cos\frac{\pi}{8}$ c. cos(112.5) + cos(67.5)

Pg 215, 1-27 odd