Chapter 3 Notes

3.1 Solving Quadratic Equations

Work with a partner.

a. Match each quadratic equation with the graph of its related function. Then use the graph to find the real solutions (if any) of each equation. Explain your reasoning. Use TI-84

i. $x^2 - 2x = 0$ E, 0, 2

ii. $x^2 - 2x + 1 = 0$ D, 1

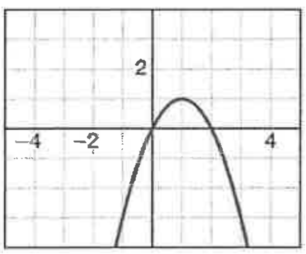
iii. $x^2 - 2x + 2 = 0$ C, $\emptyset$

iv. $-x^2 + 2x = 0$ A, 0, 2

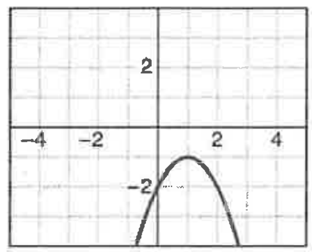
v. $-x^2 + 2x - 1 = 0$ F, 1

vi. $-x^2 + 2x - 2 = 0$ B, $\emptyset$

A.



B.



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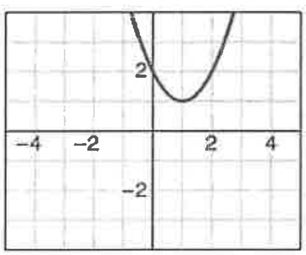
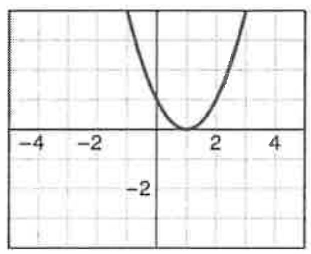
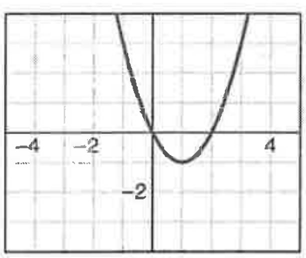


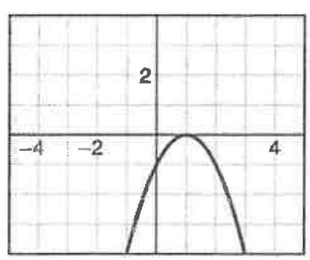
D.



E.



F.



b. How can you use a graph to determine the number of real solutions of a quadratic equation? The number of x-intercepts is the number of real solutions

c. What algebraic methods can you use to solve the equations in part (a)? Solve each equation using an algebraic method.

Factoring, Quad Formuala

A quadratic equation in one variable is an equation that can be written in standard form

$ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$. A root of an equation is a solution of the equation.

Other terms for the root of an equation:

KEY IDEA

Solving Quadratic Equations

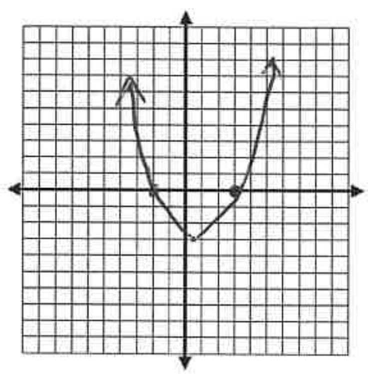
By graphing Find the x-intercepts of the graph of the related function $y = ax^2 + bx + c$.

Using square roots Write the equation in the form $u^2 = d$, where u is an algebraic expression, and solve by taking the square root of each side.

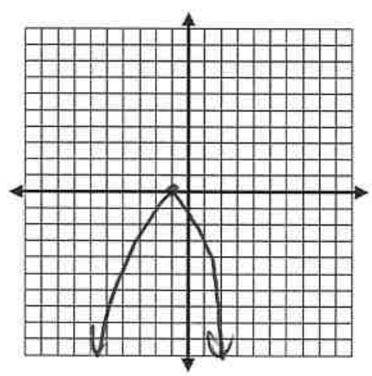
By factoring Write the quadratic equation $ax^2 + bx + c = 0$ in factored form and solve using the Zero-Product Property.

Examples: Solve each equation by graphing. TI-84; add $y=0$, 2^{nd} calc, intersect (t-chart)

a. $x^2 - x - 6 = 0$ $-2, 3$



b. $-2x^2 - 2 = 4x$ $-2x^2 - 4x - 2 = 0$

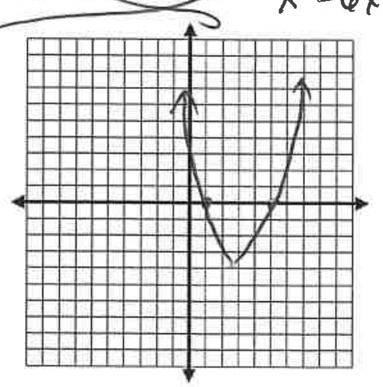


-1

Self Assessment

Solve the equation by graphing.

1. $\cancel{x^2 - x - 6 = 0}$ $x^2 - 6x + 5 = 0$



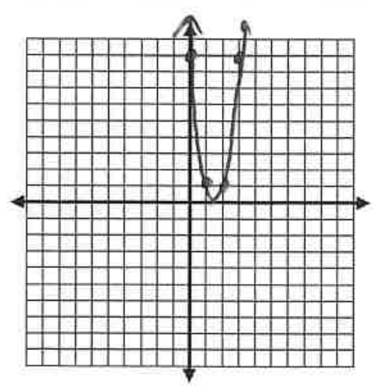
$1, 5$

2. $4x^2 - 12x = -9$ add second line

$y = 0$

2^{nd} calc

Intersect



$x = 1.5$

Solving Quadratic Equations Algebraically

You can use properties of square roots to write your solutions in different forms. When a radical is in the denominator of a fraction, you can eliminate the radical from the denominator using the process of Rationalizing the denominator.

Solve each equation using square roots.

a. $4x^2 - 31 = 49$

$4x^2 = 80$

$x^2 = 20$

$x = \pm\sqrt{20}$

$x = \pm\sqrt{4 \cdot 5}$

$x = \pm 2\sqrt{5}$

b. $3x^2 + 9 = 0$

$3x^2 = -9$

$x^2 = -3$

$x = \pm\sqrt{-3}$

$\varnothing$

c. $\frac{2}{5}(x+3)^2 = 5$

$(x+3)^2 = \frac{25}{2}$

$x + 3 = \pm\sqrt{\frac{25}{2}}$

$x = -3 \pm \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$x = -3 \pm \frac{5\sqrt{2}}{2}$

Self Assessment

6. $\frac{2}{3}x^2 + 14 = 20$

$\frac{2}{3}x^2 = 6$

$x^2 = 9$

$x = \pm 3$

7. $-2x^2 + 1 = -6$

$-2x^2 = -7$

$x^2 = \frac{7}{2}$

$x = \pm\sqrt{\frac{7}{2}}$

$x = \pm\frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$x = \pm\frac{\sqrt{14}}{2}$

8. $2(x-4)^2 = -5$

$(x-4)^2 = -\frac{5}{2}$

$x - 4 = \pm\sqrt{-\frac{5}{2}}$

$\varnothing$

When the equation is factorable, you can solve the equation using the Zero Product Property.

KEY IDEA

Zero-Product Property

Words If the product of two expressions is zero, then one or both of the expressions equal zero.

Algebra If A and B are expressions and $AB = 0$, then $A = 0$ or $B = 0$.

Solve by factoring: $x^2 - 4x = 45$.

$x^2 - 4x - 45 = 0$

$(x-9)(x+5) = 0$

$x - 9 = 0 \qquad x + 5 = 0$

$x = 9 \qquad x = -5$

Finding the Zeros of a Quadratic Function → Remember $\frac{2x^2}{-3x}$ $\frac{2x^2}{-8x}$

Find the zeros of $f(x) = 2x^2 - 11x + 12$ (opposite)

$\frac{-3}{2}$, $\frac{-8}{2}$, then switch sign

$\frac{3}{2}$, 4

Self Assessment

Solve the equation by factoring.

9. $x^2 + 12x + 35 = 0$

$(x+5)(x+7) = 0$

$x+5=0 \quad x+7=0$

$x=-5 \quad x=-7$

10. $3x^2 - 5x = 2$

$3x^2 - 5x - 2 = 0$

$\frac{1}{3}$, $\frac{-6}{3}$

$-\frac{1}{3}$, 2

6
$+1 \; -6$

Find the zero(s) of the function.

12. $f(x) = x^2 - 8x$

$0 = x(x-8)$

$x=0 \quad x=8$

14. $f(x) = 4x^2 + 28x + 49$

$\frac{14}{4}$ $\frac{14}{4}$

196
$14 \; 14$

$-\frac{7}{2}$

Modeling Real Life

A streaming service company charges \$6 per month and has 15 million subscribers. For each \$1 increase in price, the company loses 1.5 million subscribers. How much should the company charge to maximize monthly revenue? What is the maximum monthly revenue?

Revenue = # of Subsribers · Subscriptin Price

$R(x) = (15 - x) \cdot (6 + 1x)$

$R(x) = (15 - 1.5x) \cdot (6 + 1x)$

$R(x) = (-1.5x + 15)(1x + 6)$

$R(x) = -1.5(x - 10)(1x + 6)$ ∴ Zeros are $10, -6$

Avg: $\frac{10 + -6}{2} = 2$ To Max Revenue each $\$6 + 2 = \$8$

Max Monthly Revenue $-1.5(2-10)(2+6) = 96$

Self Assessment

16. A pottery store charges $10 per mug and sells 40 mugs per month. For each $0.50 decrease in price, the store sells 5 more mugs. What is the maximum monthly profit for mugs when each mug costs $3 to make?

Revenue = # of Mugs · Mug Price

$R(x) = (40 + x) \cdot (10 - .5x)$

$R(x) = (40 + 5x) \cdot (10 - .5x)$

$R(x) = (5x + 40) \cdot (-.5x + 10)$

$.5x + 10 = 0$

$x = 20$

$R(x) = 5(x + 8)(-.5x + 10)$ Zeros = $-8, 20$

$\frac{-8 + 20}{2} = 6$

Max Cost Per Mug: $(10 - .5(6)) = 7$

[illegible]

Revenue = $(40 + 5(7))(10 - .5(7))$

Revenue = \$487.5

Profit = $75 \times 3 = 225$

$487.50 - 225$

\$262.50

$\frac{-8 + 20}{2} = 6$

$-.5x + 10$

$-.5x = -10$

$x = 20$

3.2: Complex Numbers

Targets:

- Define the imaginary unit i and use it to rewrite the square root of a negative number.
- Add, subtract, and multiply complex numbers.
- Find complex solutions of quadratic equations and complex zeros of quadratic functions.

Explore!

A. Below are two model solutions for two quadratic equations. How are they solving each equation?

i.

$x^2 = 36$	Original equation
$x = \pm\sqrt{36}$	sq rt
$x = \pm 6$	take sq rt

ii.

$x^2 = -9$	Original equation
$x = \pm\sqrt{-9}$	sq. root
$x = \pm\sqrt{9}\sqrt{-1}$	neg out
$x = \pm 3\sqrt{-1}$	take sq. rt

B. Solve each quadratic equation below.

a. $x^2 = 36$

$x = \pm\sqrt{36}$

$x = \pm 6$

b. $x^2 = 25$

$x = \pm\sqrt{25}$

$x = \pm 5$

c. $x^2 = 0$

$x = \pm\sqrt{0}$

$x = 0$

d. $x^2 = -49$

$x = \pm\sqrt{-49}$

$x = \pm\sqrt{-1 \cdot 49}$

$x = \pm 7i$

In your study of mathematics, you have typically only worked with real numbers, which can be represented graphically on the real number line. Which equations above seem to have real solutions?

For the equation . $x^2 = c$,

If $c > 0$, there are 2 real solutions.

If $c = 0$, there are 1 real solutions.

If $c < 0$, there are 2 imaginary solutions.

C. The solutions to . $x^2 = -9$, solved out in part A, are called imaginary numbers and are usually written as $3i$ and $-3i$. Based on this description and the solutions in A, what do you think i represents? What do you think i^2 represents?

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -\sqrt{-1}$

$i^4 = 1$

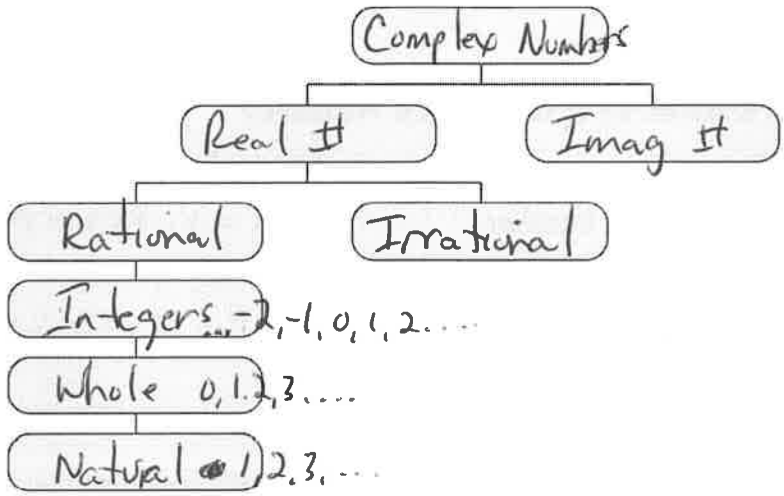
Self Assessment

16. A pottery store charges \$10 per mug and sells 40 mugs per month. For each \$0.50 decrease in price, the store sells 5 more mugs. What is the maximum monthly profit for mugs when each mug costs \$3 to make?

D. Sets of Numbers

Try to complete the word map below representing the relationships between the sets of numbers.

Word Bank: Integers, Natural Numbers, Rational Numbers, Whole Numbers, Real Numbers, Complex Numbers, Irrational Numbers, Imaginary Numbers



E. Which subsets of numbers do each of the values belong to?

i. $\sqrt{9}$	ii. $\sqrt{0}$	iii. $-\sqrt{4}$	iv. $\sqrt{\frac{4}{9}}$	v. $\sqrt{2}$	vi. $\sqrt{-1}$
N	N	I	R	I	I
W	I	R	R	C	C
I	R	R	C		
R	R	C			
R	C				
C					

The Imaginary Unit i

Not all quadratic equations have real-number solutions because there is no real number whose square is a negative number. To fix this problem, mathematicians expanded the system of numbers to include the imaginary unit, I, defined as $i = \sqrt{-1}, where\ i^2 = -1$. The imaginary unit can be used to write the square root of any negative number.

KEY IDEA

The Square Root of a Negative Number

Property	Example
1. If r is a positive real number, then $\sqrt{-r} = \sqrt{-1}\sqrt{r} = i\sqrt{r}$.	$\sqrt{-3} = \sqrt{-1}\sqrt{3} = i\sqrt{3}$
2. By the first property, it follows that $(i\sqrt{r})^2 = i^2 \cdot r = -r$.	$(i\sqrt{3})^2 = i^2 \cdot 3 = -1 \cdot 3 = -3$

Practice:

a. $\sqrt{-25}$

$\sqrt{-1 \cdot 25}$

$5i$

b. $\sqrt{-72}$

$\sqrt{-1 \cdot 36 \cdot 2}$

$6i\sqrt{2}$

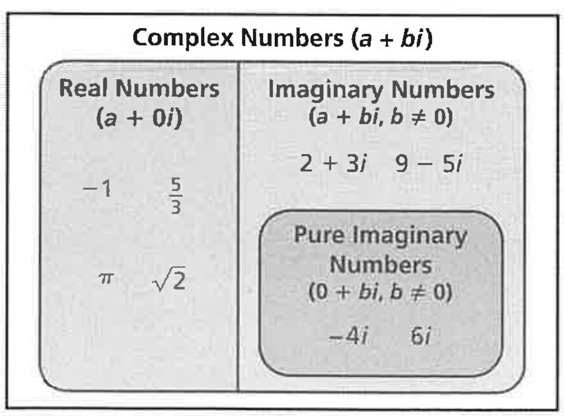
c. $-5\sqrt{-9}$

$-5\sqrt{-1 \cdot 9}$

$-15i$

A Complex Number written in standard form is a number $a + bi$, where a and b are real numbers. The number a is the real part and the number bi is the imag part.

The diagram below shows the relationship between real numbers, imaginary numbers, and pure imaginary numbers in the system of complex numbers.



Equality of Complex Numbers

Two complex numbers, $a + bi$ and $c + di$, are equal if an only if $a = c$ and $b = d$.

Find the values of x and y that satisfy the equation $2x - 7i = 10 + yi$.

$2x = 10$ $\quad$ $-7i = yi$

$x = 5$ $\quad$ $-7 = y$

Self Assessment

1. Define i and describe how you can use it. Used to help write the square root of a negative number

2. Identify the real and imaginary part of the complex number $5 - 2i$.

5 real $\quad$ $-2i$ imag

Find the square root of each number below.

3. $\sqrt{-4}$

$\sqrt{-1 \cdot 4}$

$2i$

4. $\sqrt{-12}$

$\sqrt{-1 \cdot 4 \cdot 3}$

$2i\sqrt{3}$

5. -$\sqrt{-36}$

$-\sqrt{-1 \cdot 36}$

$-6i$

Find the values of x and y that satisfy each equation.

7. $x + 3i = 9 - yi$

$x = 9$ $\quad$ $3i = -yi$

$3 = -y$

$-3 = y$

8. $5x + 4i = 20 + 2yi$

$5x = 20$ $\quad$ $4i = 2yi$

$x = 4$ $\quad$ $4 = 2y$

$2 = y$

9. $9 + 4yi = -2x + 3i$

$9 = -2x$ $\quad$ $4yi = 3i$

$-\frac{9}{2} = x$ $\quad$ $4y = 3$

$y = \frac{3}{4}$

Operations With Complex Numbers

KEY IDEA

Sums and Differences of Complex Numbers

To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

Sum of complex numbers: $(a + bi) + (c + di) = (a + c) + (b + d)i$

Difference of complex numbers: $(a + bi) - (c + di) = (a - c) + (b - d)i$

Add or subtract. Write the answer in standard form.

a. $(8 - i) + (5 + 4i)$

$13 + 3i$

b. $(7 - 6i) - (3 - 6i)$

4

Multiply. Write the answer in standard form.

a. $4i(-6 + i)$

$-24i + 4i^2$
(-1)

$-4 - 24i$

b. $(9 - 2i)(-4 + 7i)$

$-36 + 63i + 8i - 14i^2$
(-1)

$-22 + 71i$

Pairs of complex numbers of the forms $a + bi$ and $a - bi$, where $b \neq 0$, are called complex Conjugates. Observe what happens when you multiply together the complex conjugates below.

a. $(4 + i)(4 - i)$

$16 - i^2$

17

b. $(3 + 5i)(3 - 5i)$

$9 - 25i^2$

34

When you multiply two complex conjugates together, the answer will be a real number.

Try It!

Multiply $(7 + 2i)(7 - 2i)$.

$49 - 4i^2$

53

Complex Solutions and Zeros

Solve each equation below.

a. $x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-1 \cdot 4}$

$x = \pm 2i$

b. $2x^2 - 11 = -47$

$2x^2 = -36$

$x^2 = -18$

$x = \pm\sqrt{-1 \cdot 9 \cdot 2}$

$x = \pm 3i\sqrt{2}$

Find the zeros of the function $f(x) = 4x^2 + 20$.

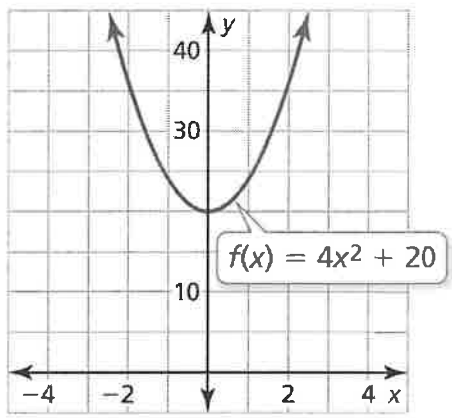
$4x^2 + 20 = 0$

$4x^2 = -20$

$x^2 = -5$

$x = \pm\sqrt{-1 \cdot 5}$

$x = \pm 5i$



Self Assessment

Solve each equation.

25. $x^2 - 8 = -36$

$x^2 = -28$

$x = \pm\sqrt{-1 \cdot 4 \cdot 7}$

$x = \pm 2i\sqrt{7}$

26. $3x^2 - 7 = -31$

$3x^2 = -24$

$x^2 = -8$

$x = \pm\sqrt{-1 \cdot 4 \cdot 2}$

$x = \pm 2i\sqrt{2}$

27. $5x^2 + 33 = 3$

$5x^2 = -30$

$x^2 = -6$

$x = \pm\sqrt{-1 \cdot 6}$

$x = \pm i\sqrt{6}$

Find the zeros of the function.

28. $f(x) = x^2 + 7$

$0 = x^2 + 7$

$-7 = x^2$

$\pm\sqrt{-1 \cdot 7} = x$

$\pm i\sqrt{7} = x$

29. $f(x) = -x^2 - 4$

$0 = -x^2 - 4$

$4 = -x^2$

$-4 = x^2$

$\pm\sqrt{-1 \cdot 4} = x$

$\pm 2i = x$

30. $f(x) = 9x^2 + 1$

$0 = 9x^2 + 1$

$-1 = 9x^2$

$-\frac{1}{9} = x^2$

$\pm\sqrt{-\frac{1}{9}} = x$

$\pm\sqrt{-1 \cdot \frac{1}{9}} = x$

$\pm i\sqrt{\frac{1}{9}} = x$

$\pm \frac{1}{3} i = x$

3.4: Using the Quadratic Formula

Targets:

- Solve quadratic equations using the Quadratic Formula.
- Find and interpret the discriminant of an equation.
- Write quadratic equations with different numbers of solutions using the discriminant.

Explore! Recall from Algebra 1, the Quadratic Formula can be used to find the solutions of any quadratic equation of the form $ax^2 + bx + c = 0$, where $a, b, and\ c\ are\ real\ numbers\ and\ a \neq 0$.

The Quadratic Formula states that the equation above has solutions $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

A. Use the Quadratic Formula to solve each equation below.

i. $x^2 + 5x + 6 = 0$

$\frac{-5 \pm \sqrt{25-4(1)(6)}}{2}$

$\frac{-5 \pm \sqrt{1}}{2}$ $\quad -2, -3$

$\frac{-5 \pm 1}{2}$

ii. $x^2 - 3x + 1 = 0$

$\frac{3 \pm \sqrt{9-4(1)(1)}}{2}$

$\frac{3 \pm \sqrt{5}}{2}$

c. $2x^2 + 3x + 3 = 0$

$\frac{-3 \pm \sqrt{9-4(2)(3)}}{4}$

$\frac{-3 \pm \sqrt{-15}}{4}$

$\frac{-3 \pm \sqrt{-1 \cdot 15}}{4}$, $\frac{-3 \pm i\sqrt{15}}{4}$

B. What part of the Quadratic Formula determines whether a quadratic equation has real solutions or imaginary solutions? When does the formula produce real solutions? When does it produce imaginary solutions?

Inside Sq. Root
When Positive
When Negative

C. Can the Quadratic Formula produce one real and one imaginary solution?

No

D. *Without solving*, use your answer to B to determine whether each equation has real solutions or imaginary solutions. What would this mean about the graph of each equation?

i. $x^2 - 4x + 3 = 0$ $\quad 16-4(1)(3)$, 2 real, intersects x-axis twice

ii. $x^2 + 4x + 6 = 0$ $\quad 16-4(1)(6)$, 2 imag, does not intersect x-axis

iii. $x^2 + 4x + 4 = 0$ $\quad 16-4(1)(4)$, 1 real, intersects x-axis once (vertex)

iv. $x^2 - 6x + 10 = 0$ $\quad 36-4(1)(10)$, 2 imag, does not intersect x-axis

KEY IDEA

The Quadratic Formula

The solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic Formula}$$

where a, b, and c are real numbers and $a \neq 0$.

Example 1:Solve $x^2 + 3x = 5$ using the Quadratic Formula.

$$\frac{-3 \pm \sqrt{9-4(1)(-5)}}{2}$$

$$\frac{-3 \pm \sqrt{29}}{2}$$

Example 2: Solve $25x^2 - 8x = 12x - 4$ using the Quadratic Formula.

$$\frac{-20 \pm \sqrt{400-4(25)(4)}}{50}$$

$$\frac{-20 \pm \sqrt{0}}{50} \qquad -\frac{2}{5}$$

Example 3: Solve $-x^2 + 4x = 13$ using the Quadratic Formula.

$$0 = x^2 - 4x + 13$$

$$\frac{4 \pm \sqrt{16-4(1)(13)}}{2}$$

$$\frac{4 \pm \sqrt{-36}}{2}$$

$$\frac{4 \pm \sqrt{-1 \cdot 36}}{2}$$

$$\frac{4 \pm 6i}{2}$$

$$2 \pm 3i$$

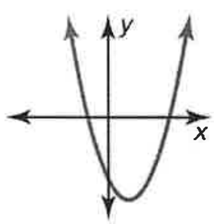
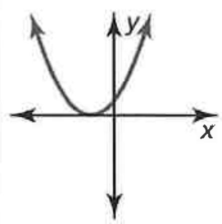
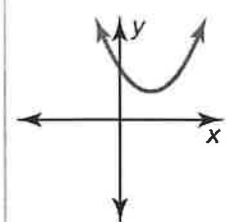
In the Quadratic Formula, the expression $b^2 - 4ac$ is called the **discriminant** of the associated equation $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ discriminant

You can analyze the discriminant of a quadratic equation to determine the number and type of solutions of the equation.

KEY IDEA

Analyzing the Discriminant of $ax^2 + bx + c = 0$

Value of discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Number and type of solutions	Two real solutions	One real solution	Two imaginary solutions
Graph of $y = ax^2 + bx + c$	 Two x-intercepts	 One x-intercept	 No x-intercept

Example 4: Find the discriminate of the quadratic equation and describe the number and type of solutions to the equation.

a. $x^2 - 6x + 10 = 0$

$36-4(1)(10)$

-4

2 imag rts

b. $x^2 - 6x + 9 = 0$

$36-4(1)(9)$

0

1 real rt.

c. $x^2 - 6x + 8 = 0$

$36-4(1)(8)$

4

2 real rts.

Writing Quadratic Equations

<u>Example 5</u>:

Find a possible pair of integer values for a *and* c so that the equation $ax^2 - 4x + c = 0$ has the given number and type of solutions. Then, write the equation.

a. one real solution

$b^2 - 4ac = 0$

$16 - 4ac = 0$

$-4ac = -16$

$ac = 4$

$\therefore$ one way $a = 1$ $c = 4$

$x^2 - 4x + 4 = 0$

b. two imaginary solutions

$b^2 - 4ac < 0$

$16 - 4ac < 0$

$-4ac < -16$

$ac > 4$

$\therefore$ one way $a = 2$, $b = 3$

$2x^2 - 4x + 3 = 0$

Modeling Real Life

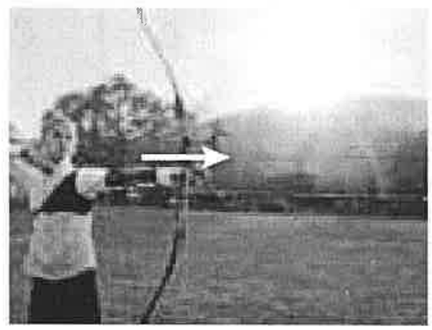
The function $h = -16t^2 + s_0$ is used to model the height of a *dropped* object, where h is the height (in feet), t is the time in motion (in seconds), and s_0 is the initial height (in feet). For an object that is *launched* or *thrown*, an extra term v_0t must be added to the model to account for the object's initial vertical velocity v_0 (in feet per second).

$h = -16t^2 + s_0$	Object is dropped.
$h = -16t^2 + v_0t + s_0$	Object is launched or thrown.

As shown below, the value of v_0 can be positive, negative, or zero depending on whether the object is launched upward, downward, or parallel to the ground.

$v_0 > 0$

$v_0 < 0$



$v_0 = 0$

Example 6: A juggler tosses a ball into the air. The ball leaves the juggler's hand 4 feet above the ground and has an initial vertical velocity of 30 feet per second. Does the ball reach a height of 10 feet? 25 feet? Explain your reasoning.

$$h = -16t^2 + v_0 t + s_0$$
$$h = -16t^2 + 30t + 4$$

$10 = -16t^2 + 30t + 4$

$0 = -16t^2 + 30t - 6$

$\dfrac{-30 \pm \sqrt{900 - 4(-16)(-6)}}{-32}$

$\dfrac{-30 \pm \sqrt{516}}{-32}$

$0.23, 1.65$

$25 = -16t^2 + 30t + 4$

$0 = -16t^2 + 30t - 21$

$\dfrac{-30 \pm \sqrt{900 - 4(-16)(-21)}}{-32}$

$\dfrac{-30 \pm \sqrt{-444}}{-32}$

$\emptyset$

Ball reaches a height of 10 feet but not 25 feet.

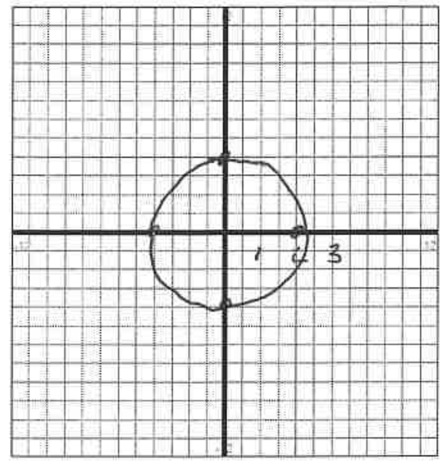
3.5 Solving Nonlinear Systems of Equations

Targets:

1. I can solve nonlinear systems graphically and algebraically.

- I can describe what a nonlinear system of equations is.
- I can solve nonlinear systems using graphing, substitution, or elimination.
- I can solve quadratic equations by graphing each side of the equation.

EXPLORE IT! Solving Systems of Equations: Work with a partner.



a. Use Desmos to graph the equation $x^2 + y^2 = 4$. Make several observations about the graph.

Circle: x & y both squared
$\sqrt{4} = 2$ radius
Center @ origin

b. How many intersection points can the graphs of a line and a circle have? Use graphs to support your answers. What do the intersection points represent?

0, 1, or 2 intersection pts.

c. Consider the system $\begin{cases} x^2 + y^2 = 4 \\ y = -\frac{1}{2}x + 1 \end{cases}$. Can you use a graph to solve the system? Explain.

Yes, The points of intersection are the solutions

d. Find the points of the intersection of the graphs of the equation in part (c). Explain your method.

$(-1.2, 1.6)$ $(2, 0)$ Estimate from the graph.

e. Write the equation of a line that intersects the graph of $x^2 + y^2 = 4$ at only one point. Explain how you found your answer.

Sample $y = 2$ Radius of circle $= 2$ so, $y = 2$
$y = -2$
$x = 2$
$x = -2$

f. Think of all the ways that a parabola can intersect the graph of a circle. How many points of intersection are possible? Use the graphs to support your answers.

0, 1, 2, 3, or 4

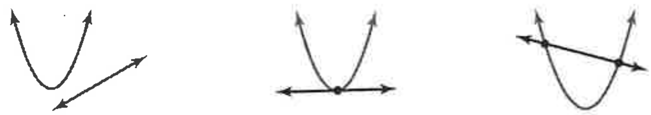
g. Use you answers in part (f) to write several equations of parabolas that have different numbers of intersection points with the graph of $x^2 + y^2 = 4$. Then compare your results with your classmates.

0: $y = x^2 + 5$ 1: $y = x^2 + 2$ 2: $y = x^2$ 3: $y = x^2 - 1$ 4: $y = x^2 - 3$

A **system of nonlinear equations** is a system in which at least __1__ of the equations is nonlinear.

Example: $\begin{cases} y = x^2 + 2x - 4 \\ y = 2x + 5 \end{cases}$

When a nonlinear system consists of a **linear equation** and a **quadratic equation**, the graphs can intersect in __0__, __1__, or __2__ points.



No real solution One real solution Two real solutions

When a nonlinear system consists of **two parabolas** that open up or open down, the graphs can intersect in __0__, __1__, or __2__ points.

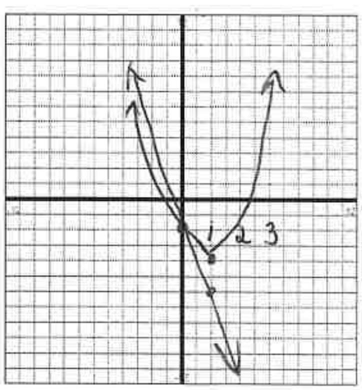
No real solution One real solution Two real solutions

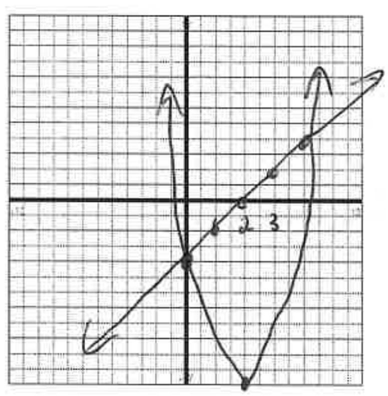
Solving Nonlinear Systems by Graphing

Solve the systems by graphing.

1. $\begin{cases} y = x^2 - 2x - 1 \\ y = -2x - 1 \end{cases}$

2. $\begin{cases} y = x^2 - 4x - 2 \\ y = x - 2 \end{cases}$





(0, -1)

(0, -2) (5, 3)

Solving Nonlinear Systems by Substitution

Solve the systems by substitution.

3. $\begin{cases} x^2 + x - y = -1 \\ x + y = 4 \end{cases}$ Par, Line 0, 1, or 2

$x^2 + x - y = -1$

$x^2 + x - (-x+4) = -1$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$-3, 1$

$x + y = 4$

$y = -x + 4$

$y = -(-3) + 4$

$y = 1$

$y = -(1) + 4$

$y = 3$

$(-3, 1)\ (1, 3)$

4. $\begin{cases} y = x^2 - 8x + 12 \\ y = 4x - 24 \end{cases}$ Par Line 0, 1, or 2

$y = x^2 - 8x + 12$

$4x - 24 = x^2 - 8x + 12$

$0 = x^2 - 12x + 36$

$(x-6)(x-6)$

6

$y = 4x - 24$

$y = 4(6) - 24$

$y = 0$

$(6, 0)$

Solving Nonlinear Systems by Elimination

Solve the systems by elimination.

5. $\begin{cases} 2x^2 - 5x - y = -2 \\ x^2 + 2x + y = 0 \end{cases}$ +

$3x^2 - 3x = -2$

$3x^2 - 3x + 2 = 0$

does not factor 6 ^ - -

$\dfrac{3 \pm \sqrt{9 - 4(3)(2)}}{6}$

$\dfrac{3 \pm \sqrt{-15}}{6}$

$\varnothing$

6. $\begin{cases} 3x^2 + 2x - 2y = 10 \\ x^2 - 6x + 2y = -12 \end{cases}$ +

$4x^2 - 4x = -2$

$4x^2 - 4x + 2 = 0$

$2(2x^2 - 2x + 1) = 0$

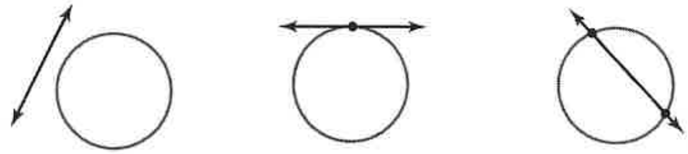
does not factor 2 ^

$\dfrac{4 \pm \sqrt{16 - 4(4)(2)}}{8}$

$\dfrac{4 \pm \sqrt{-16}}{8}$

$\varnothing$

A equation in the form of $x^2 + y^2 = r^2$ is the standard form of a circle with the center (0,0) and radius r. When a nonlinear system consists of the equation of a circle and a linear equation, the graph can intersect in __0__, __1__, or __2__ points.



No real solution | One real solution | Two real solutions

Solving Nonlinear Systems Involving a Circle

7. Solve the system $\begin{cases} x^2 + y^2 = 10 \\ y = -3x + 10 \end{cases}$.

$x^2 + y^2 = 10$

$x^2 + (-3x+10)^2 = 10$

$x^2 + 9x^2 - 60x + 100 = 10$

$10x^2 - 60x + 90 = 0$

$10(x^2 - 6x + 9) = 0$

$10(x-3)(x-3) = 0$

$x = 3$

$y = -3x + 10$

$y = -3(3) + 10$

$y = 1$

$(3, 1)$

KEY IDEA

Solving Equations by Graphing

Step 1 To solve the equation $f(x) = g(x)$, first write functions to represent each side of the equation, $y = f(x)$ and $y = g(x)$.

Step 2 Graph the functions $y = f(x)$ and $y = g(x)$. The x-value of an intersection point of the graphs of the functions is a solution of the equation $f(x) = g(x)$.

Solving Quadratic Equations by Graphing

Graphing Calculator Steps:

1. Y=, then put the functions into Y_1 and Y_2.
2. Graph, 2nd Trace, 5: intersection
3. Move the cursor close to where the graph intersect(s), then hit Enter 3 times. Repeat this step if there is more than point of intersection.

Solve each equation by graphing.

8. $3x^2 - 5x - 1 = -x^2 + 2x + 1$

(-0.25, 0.4375)

(2, 1)

9. $-(x - 1.5)^2 + 2.25 = 2x(x + 1.5)$

(0, 0)

Pg 132,1-43 odds

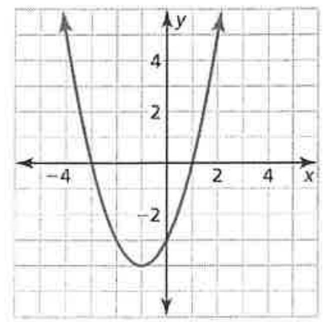
3.6 Quadratic Inequalities

Targets:

1. I can graph quadratic inequalities in two variables and solve quadratic inequalities in one variable.

- I can describe the graph of a quadratic inequality.
- I can graph quadratic inequalities.
- I can graph systems of inequalities.
- I can solve quadratic inequalities algebraically.

EXPLORE IT! Solving Systems of Equations: Work with a partner.



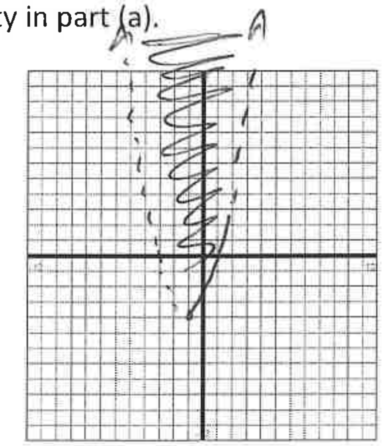
The figure shows the graph of $f(x) = x^2 + 2x - 3$.

a. Explain how you can use the graph to solve the inequality $0 > x^2 + 2x - 3$. Then use Desmos to graph the solution of the inequality.

Find the x-values between the x-intercepts of the graph

b. Explain how the inequality $y > x^2 + 2x - 3$ is different from the inequality in part (a).

The inequality has 2 variables



c. Explain how you can use the graph above to represent the solutions of $y > x^2 + 2x - 3$. Then graph the inequality.

The solution is represented by the region inside the parabola

d. Compare the graphs of the solutions of quadratic inequalities in one variable to the graphs of the solutions of quadratic inequalities.

The graphs of quadratic inequalities in one and two variables have more than one solution, so the graphs are shaded to include all possible solutions.

A **quadratic inequality in two variables**, x and y, can be written in one of the following forms, where a, b, and c are real numbers and $a \neq 0$.

$$y < ax^2 + bx + c \qquad y > ax^2 + bx + c$$

$$y \leq ax^2 + bx + c \qquad y \geq ax^2 + bx + c$$

KEY IDEA

Graphing a Quadratic Inequality in Two Variables

Step 1 Graph the parabola with the equation $y = ax^2 + bx + c$. Make the parabola *dashed* for inequalities with $<$ or $>$ and *solid* for inequalities with $\leq$ or $\geq$.

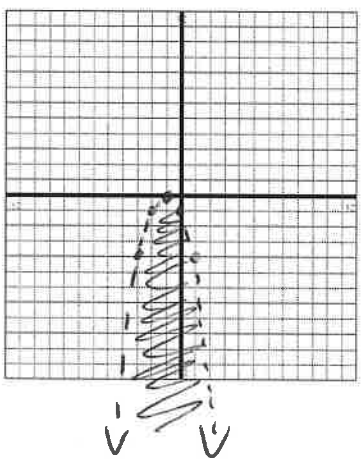
Step 2 Test a point (x, y) that does not lie on the parabola to determine whether the point is a solution of the inequality.

Step 3 When the test point is a solution, shade the region of the plane that contains the point. When the test point is not a solution, shade the region that does not contain the point.

Graphing a Quadratic Inequality in Two Variables

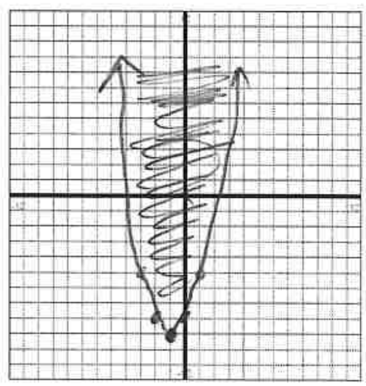
Graph.

1. $y < -x^2 - 2x - 1$



Test Point
$(-1,-1)$
$-1 < -1 + 2 - 1$
$-1 < 0$
True

2. $y \geq x^2 + 2x - 8$

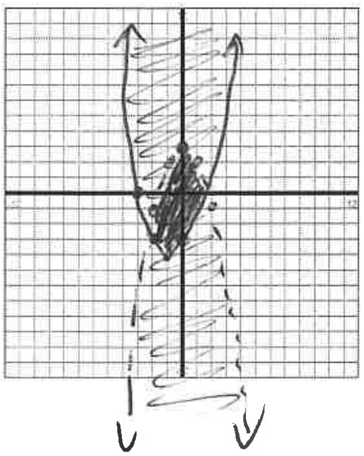


Test Point
$(-1,0)$
$0 \geq 1 - 2 - 8$
$0 \geq -9$

Graphing a System of Quadratic Inequalities

Graph the system of quadratic inequalities.

3. $\begin{cases} y < -x^2 + 3 \\ y \geq x^2 + 2x - 3 \end{cases}$



(0,0)

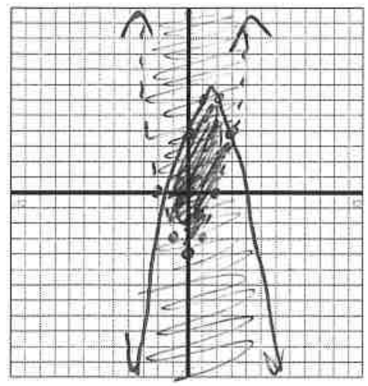
0 < 0 + 3

0 < 3

(0,0)

0 ≥ -3

4. $\begin{cases} y > x^2 - 4 \\ y \leq -x^2 + 3x + 4 \end{cases}$



(0,0)

0 > -4

(0,0)

0 ≤ 0 + 0 + 4

0 ≤ 4

Solving Quadratic Inequalities in One Variable

A **quadratic inequality in one variable**, x, can be written in one of the following forms, where a, b, and c are real numbers and $a \neq 0$.

$ax^2 + bx + c < 0 \qquad ax^2 + bx + c > 0 \qquad ax^2 + bx + c \leq 0 \qquad ax^2 + bx + c \geq 0$

Solve.

5. $x^2 - 3x - 4 < 0$

Negative

$(x-4)(x+1) < 0$

$x = 4 \quad x = -1$

$+ \quad -1 \quad - \quad 4 \quad +$

$-1 < x < 4$

GO CALI

6. $x^2 + 4x - 32 > 0$

+

$(x+8)(x-4) > 0$

$x = -8 \quad x = 4$

$+ \quad -8 \quad - \quad 4 \quad +$

$x < -8$ OR $x > 4$

Solving a Quadratic Inequality by Graphing/Quadratic Formula

Solve.

7. $3x^2 - x - 5 \geq 0$ +

GO LA !!

$$\frac{1 \pm \sqrt{1-4(3)(-5)}}{6}$$

$$\frac{1 \pm \sqrt{61}}{6}$$

$+ \quad \frac{1-\sqrt{61}}{6} \quad - \quad \frac{1+\sqrt{61}}{6} \quad +$

$x < \frac{1-\sqrt{61}}{6}$ OR $x > \frac{1+\sqrt{61}}{6}$

$3x^2 - x - 5 \geq y$

$y \leq 3x^2 - x - 5$

$y \geq 0$

Intersect: $-1.14,\ 1.47$

$x < -1.14$ or $x > 1.47$

8. $2x^2 + 4x - 3 \leq 0$ −

$$\frac{-4 \pm \sqrt{16-4(2)(-3)}}{4}$$

$$\frac{-4 \pm 2\sqrt{10}}{4}, \quad \frac{-2 \pm \sqrt{10}}{2}$$

$0.58, \ -2.58$

$+ \quad -2.58 \quad - \quad 0.58 \quad +$

$-2.58 < x < 0.58$

Modeling Real Life

9. An archaeologist is roping off a rectangular region of land to dig for artifacts. The region must have a perimeter of 440 feet and an area of at least 8000 square feet. Describe the possible lengths of the archaeological region.

Per = 440

$2l + 2w = 440$

$2w = -2l + 440$

$w = -l + 220$

Area ≥ 8000

$lw \geq 8000$

$l(-l + 220) \geq 8000$

$-l^2 + 220l \geq 8000$

$-l^2 + 220l - 8000 \geq 0$ +

TI-84

$l = 45.97 \quad l = 174.03$

$- \quad 45.97 \quad + \quad 174.03 \quad -$

$45.97 \leq x \leq 174.03$

The approximate length of the region are at least 46 feet and at most 174 feet.