

# **Chapter 2 Notes**

## **Algebra 2**



## 2.1 Transformations of Quadratic Functions

### Targets:

1. I can describe and graph transformations of quadratic functions.

- I can describe transformations of quadratic functions.
- I can graph transformations of quadratic functions.
- I can write functions that represent transformations of quadratic functions.

Class Opener: Using what we learned in Chapter 1 about transformations of functions, match each equation below to its graph.

a.  $g(x) = -(x - 2)^2$  **D**

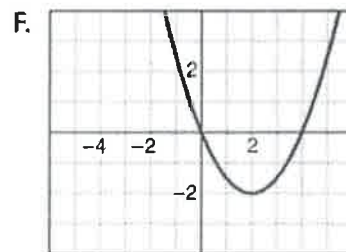
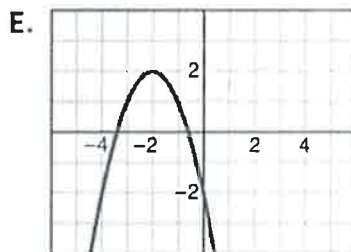
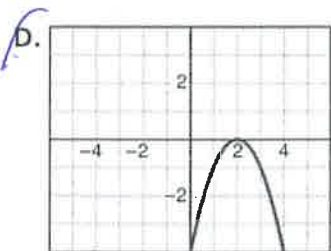
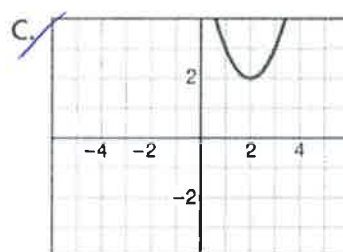
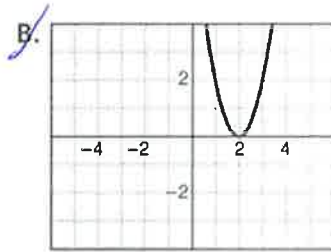
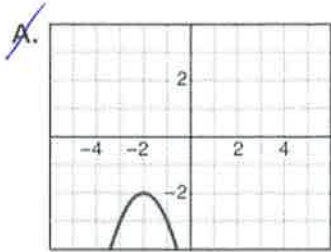
c.  $g(x) = -(x + 2)^2 - 2$  **A**

e.  $g(x) = 2(x - 2)^2$  **B**

b.  $g(x) = (x - 2)^2 + 2$  **C**

d.  $g(x) = 0.5(x - 2)^2 - 2$  **F?? off little**

f.  $g(x) = -(x + 2)^2 + 2$  **E**



In the quadratic equation below, how do a, h, and k effect the graph of  $y = x^2$ ?

$$y = a(x - h)^2 + k$$

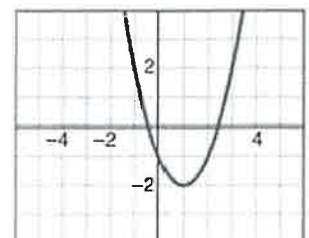
a: V. stretch/spink >1 st <1 sh

h: Left/Right - Right + Left

k: Up + / Down -

Try It! Using these rules/patterns above, write an equation for the quadratic equation graphed to the right.

$$y = (x - 1)^2 - 2$$



Describing Transformations of Quadratic Functions

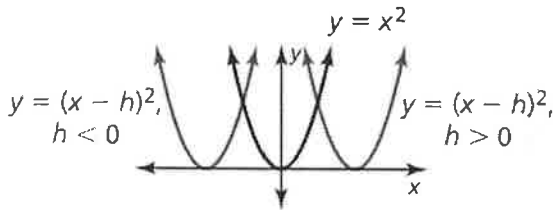
A Quadratic is a function that can be written in the form  $f(x) = a(x - h)^2 + k, a \neq 0$ .

The U-shaped graph of a quadratic function is called a Parabola.

Consider the parent function  $f(x) = x^2$ .

**Horizontal Translations**

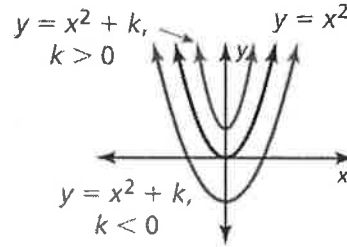
$$f(x - h) = (x - h)^2$$



- shifts left when  $h < 0$
- shifts right when  $h > 0$

**Vertical Translations**

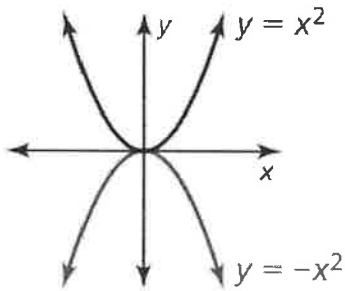
$$f(x) + k = x^2 + k$$



- shifts down when  $k < 0$
- shifts up when  $k > 0$

**Reflections in the x-Axis**

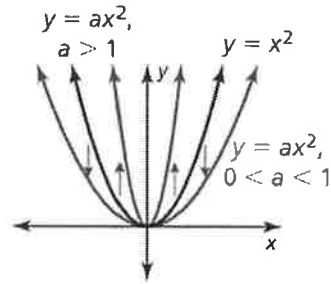
$$-f(x) = -(x^2) = -x^2$$



flips over the  $x$ -axis

**Vertical Stretches and Shrinks**

$$a \cdot f(x) = ax^2$$



- vertical stretch (away from  $x$ -axis) by a factor of  $a$  when  $a > 1$
- vertical shrink (toward  $x$ -axis) by a factor of  $a$  when  $0 < a < 1$

**Practice!**

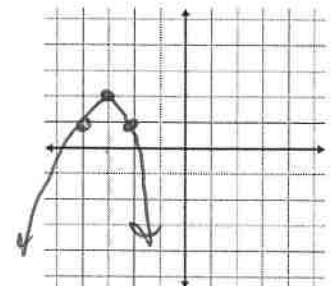
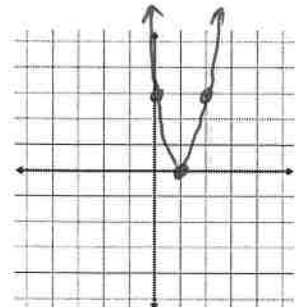
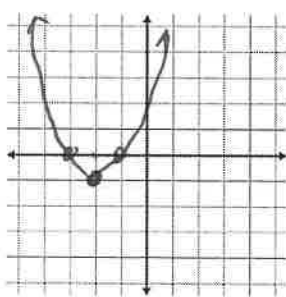
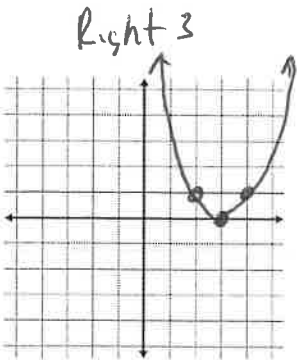
Describe in words the transformation of  $f(x) = x^2$  represented by  $g$ . Then, graph each function.

1.  $g(x) = (x - 3)^2$

2.  $g(x) = (x + 2)^2 - 1$

3.  $f(x) = 3(x - 1)^2$

4.  $g(x) = -(x + 3)^2 + 2$



### Writing Transformations of Quadratic Functions

The lowest point on a parabola that opens up or the highest point on a parabola that opens down is called the Vertex.  
The vertex form of a quadratic function is  $f(x) = a(x - h)^2 + k$ ,  $a \neq 0$ , vertex  $(h, k)$ .

Example: Let the graph of  $g$  be a vertical stretch by a factor of 2 and a reflection in the  $x$ -axis, followed by a translation 3 units down of the graph of  $f(x) = x^2$ . Write a rule for  $g$  and identify the vertex.

$$g(x) = -2x^2 - 3 \quad \text{Vertex } (0, -3)$$

~~$g(x) = -2x^2 - 3$~~

Practice: Let the graph of  $g$  be a vertical shrink by a factor of  $\frac{1}{2}$ , followed by a translation 2 units up of the graph of  $f(x) = x^2$ . Write a rule for  $g$  and identify the vertex.

$$g(x) = \frac{1}{2}x^2 + 2$$

$(0, 2)$

### Algebra Review

Simplify each expression below.

1.  $(x + 2)^2$

$$x^2 + 4x + 4$$

2.  $(x - 3)^2$

$$x^2 - 6x + 9$$

3. If  $f(x) = x^2 + 5x$ , find  $f(x + 1)$ .

$$(x+1)^2 + 5(x+1)$$
$$x^2 + 2x + 1 + 5x + 5$$
$$x^2 + 7x + 6$$

Application to Transformations

Reflection of x-axis  $-f(x)$  Reflection of y-axis  $f(-x)$

Let the graph of  $g$  be a translation 3 units right and 2 units up, followed by a reflection in the x-axis of the graph of  $f(x) = x^2 - 5x$ .

~~$g(x) = (x-3)^2 + 2$~~   
 ~~$g(x) = x(x-8) + 2$~~   
 ~~$g(x) = -x(x-8) - 2$~~   
 ~~$g(x) = x^2 - 8x - 2$~~

$$h(x) = f(x-3) + 2$$

now substitute.

$$= (x-3)^2 - 5(x-3) + 2$$

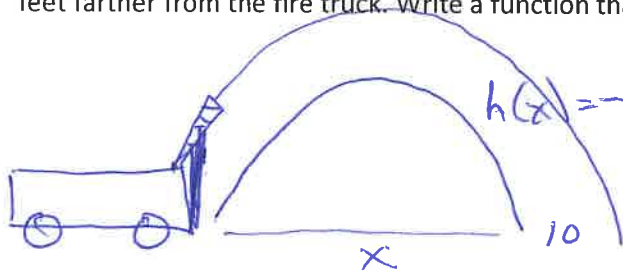
$$= x^2 - 11x + 26$$

$$g(x) = -h(x)$$

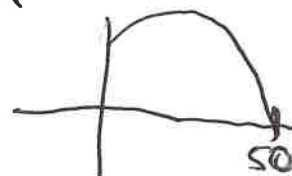
$$= -x^2 + 11x - 26$$

Modeling Real Life

The height (in feet) of water spraying from a fire hose can be modeled by  $h(x) = -0.03x^2 + x + 25$ , where  $x$  is the horizontal distance (in feet) from the fire truck. The crew raises the aerial ladder so that the water hits the ground 10 feet farther from the fire truck. Write a function that models the new path of the water.



Use TI-84 To graph  
 window  $x$   $0 \rightarrow 60$   
 $y$   $0 \rightarrow 30$



so  $+10 = \underline{60}$ , so find  $h(60)$

$$h(60) = -0.03(60)^2 + 60 + 25$$

$$= -23$$

so 23 ft up will make water go 10 more feet so add 23

$$h(x) = -0.03x^2 + x + 48$$

## 2.2 Characteristics of Quadratic Functions

### Target:

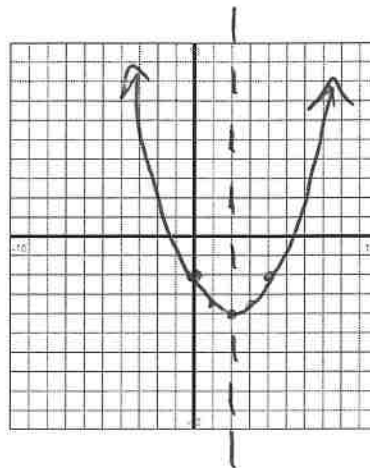
- I can graph and describe quadratic functions.
  - I can use properties of parabolas to graph quadratic functions.
  - I can identify characteristics of quadratic functions and their graphs.
  - I can use characteristics of quadratic functions to solve real-life problems.

### Explore It! Parabolas and Symmetry

#### Work with a partner.

- Sketch the graph of the function.  $f(x) = \frac{1}{2}x^2 - 2x - 2$

x	y
0	-2
1	- $\frac{7}{2}$
2	-4
3	- $\frac{7}{2}$
4	-2



- Find a vertical line on your graph so that if you folded the paper, the left portion of the graph coincides with the right portion of the graph. What is the equation of this line? How does it relate to the vertex?

$x=2$ . goes thru the vertex ( $x=h$ )

- Show that the vertex form  $f(x) = \frac{1}{2}(x-2)^2 - 4$  is equivalent to the function given in part 1.

$$\begin{aligned} & \frac{1}{2}(x^2 - 4x + 4) - 4 \\ & \frac{1}{2}x^2 - 2x + 2 - 4 \\ & \frac{1}{2}x^2 - 2x - 2 \end{aligned}$$

- Consider the graph of  $f(x) = a(x-h)^2 + k$ .

- How can you describe the symmetry of the graph?  $x=h$
- How can you determine whether the graph opens up or down? Explain.  $a > 0$  graph opens upward,  $a < 0$  graph opens downward
- What is the vertex?  $(h, k)$
- What is the least or greatest value of the function? Explain.  $k$  is the minimum or maximum of the function

Axis of Symmetry is the line that divides a parabola into mirror images and passes through the Vertex.

### Graphing Quadratic Functions:

#### Vertex Form:

$$f(x) = a(x - h)^2 + k$$

- **a** is the **vertical stretch/shrink** of the function
  - When  $a < 0$  the parabola opens down. The graph has a **Maximum** value (y-coordinate of vertex).
  - When  $a > 0$  the parabola opens up. The graph has a **Minimum** value (y-coordinate of vertex).
- **Vertex** of the parabola is (opposite  $h$ , same as  $k$ ).
  - The  $x$ -coordinate of the vertex is **when** the minimum or maximum happens.
  - The  $y$ -coordinate of the vertex is the minimum or maximum value of the quadratic function.

#### Standard Form (Quadratic Form):

$$f(x) = ax^2 + bx + c$$

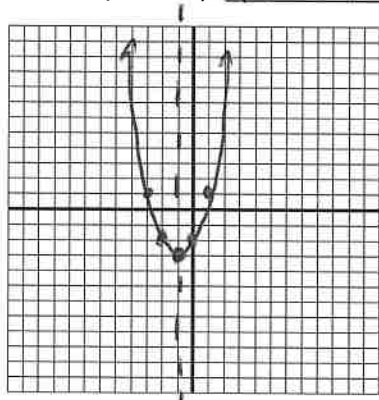
- Find the  $x$  coordinate of the vertex using:  $x = \frac{-b}{2a}$ .
- Plug the  $x$  into the original function to get the  $y$ -coordinate of the vertex  $(\frac{-b}{2a}, f(\frac{-b}{2a}))$ .
- The **y-intercept** is  $c$ . So, the point  $(0, c)$  is on the parabola.

Graph each function. Name the vertex and the axis of symmetry.

3.  $y = (x + 1)^2 - 3$       $a=1, h=-1, k=-3$

Vertex:  $(-1, -3)$

Axis of Symmetry:  $x = -1$

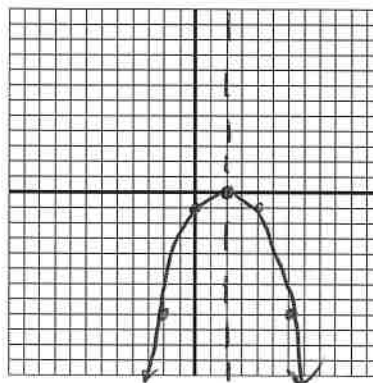


$x$	$y$
1	1
-3	1

4.  $f(x) = -\frac{1}{2}(x - 2)^2$

Vertex:  $(2, 0)$

Axis of Symmetry:  $x = 2$

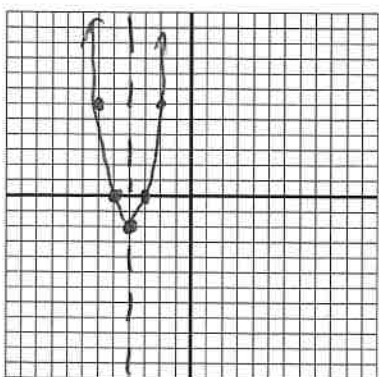


$x$	$y$
6	-8
-2	-8

5.  $f(x) = 2x^2 + 16x + 30$       $a=2, b=16, c=30$

Vertex:  $(-4, -2)$

Axis of Symmetry:  $x = -4$



$$x = \frac{-b}{2a}$$

$$x = \frac{-16}{4} = -4$$

$$f(-4) = -2$$

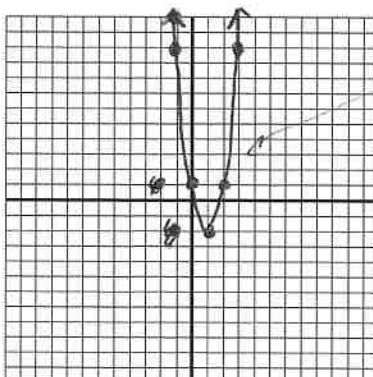
$c = y\text{-int}$

$x$	$y$
-3	0
-5	0
-2	6

6.  $f(x) = 3x^2 - 6x + 1$

Vertex:  $(1, -2)$

Axis of Symmetry:  $x = 1$



$a=3, b=-6, c=1$

$$x = \frac{-b}{2a}$$

$$x = \frac{6}{6} = 1$$

$$f(1) = -2$$

$x$	$y$
-1	10
3	10

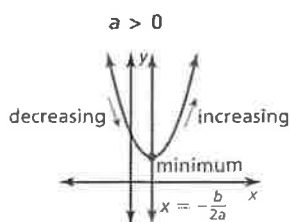




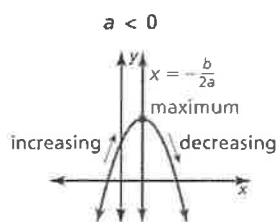
### KEY IDEA

#### Minimum and Maximum Values

For the quadratic function  $f(x) = ax^2 + bx + c$ , the  $y$ -coordinate of the vertex is the **minimum value** of the function when  $a > 0$  and the **maximum value** when  $a < 0$ . These values can be used to describe other properties of the function, as shown below.



- Minimum value:  $f\left(-\frac{b}{2a}\right)$
- Range:  $y \geq f\left(-\frac{b}{2a}\right)$
- Decreasing when  $x < -\frac{b}{2a}$
- Increasing when  $x > -\frac{b}{2a}$



- Maximum value:  $f\left(-\frac{b}{2a}\right)$
- Range:  $y \leq f\left(-\frac{b}{2a}\right)$
- Increasing when  $x < -\frac{b}{2a}$
- Decreasing when  $x > -\frac{b}{2a}$

Find the minimum value or maximum value of the function. Find the domain and the range of the function, and when the function is increasing and decreasing.

7.  $f(x) = \frac{1}{2}x^2 - 2x - 1$

$a = \frac{1}{2}, b = -2, c = -1$

$a > 0$ , opens up, minimum

$x = -\frac{b}{2a} = \frac{2}{1} = 2$

$f(2) = -3$

D:  $\mathbb{R}$

R:  $y \geq -3$

• • Min = -3

$y < 2$  decreasing

$y > 2$  increasing

8.  $f(x) = -x^2 + 5x + 9$

$a = -1, b = 5, c = 9$

$a < 0$ , opens down, maximum

$x = -\frac{b}{2a} = \frac{-5}{-2} = \frac{5}{2}$

$f\left(\frac{5}{2}\right) = -\frac{25}{4} + \frac{5}{1}\left(\frac{5}{2}\right) + \frac{9}{1}$

$-\frac{25}{4} + \frac{50}{4} + \frac{36}{4}$

$= \frac{61}{4}$

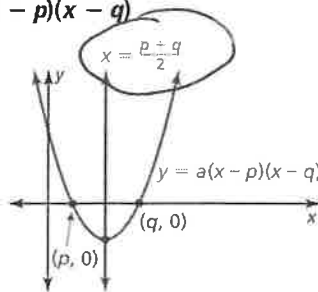
• • Max =  $\frac{61}{4}$



### KEY IDEA

#### Properties of the Graph of $f(x) = a(x - p)(x - q)$

- Because  $f(p) = 0$  and  $f(q) = 0$ ,  $p$  and  $q$  are the  $x$ -intercepts of the graph of the function.
- The axis of symmetry is halfway between  $(p, 0)$  and  $(q, 0)$ . So, the axis of symmetry is  $x = \frac{p+q}{2}$ .
- The parabola opens up when  $a > 0$  and opens down when  $a < 0$ .



D:  $\mathbb{R}$

R:  $y \leq \frac{61}{4}$

$y < \frac{5}{2}$  increasing  
 $y > \frac{5}{2}$  decreasing

Graph the Quadratic Function in Intercept Form

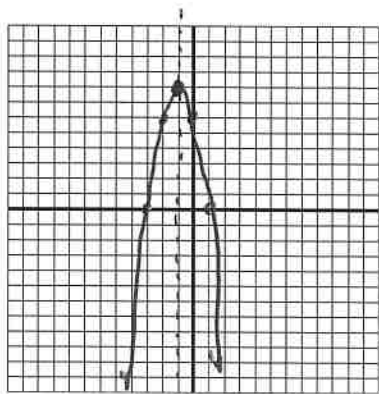
Graph the function. Identify the  $x$ -intercepts, vertex and axis of symmetry.

9.  $f(x) = -2(x+3)(x-1)$   $x+3=0$   $x-1=0$

$x$ -intercepts:  $(-3, 0)(1, 0)$

Vertex:  $(-1, 8)$   $x = \frac{-3+1}{2} = -1$

Axis of Symmetry:  $x = -1$   $f(-1) = 8$



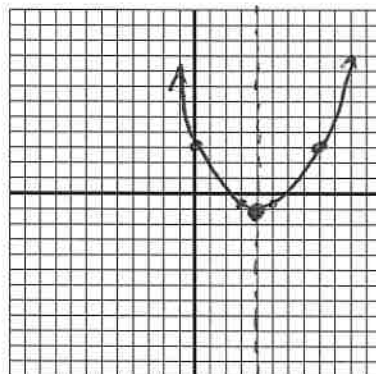
$x$	$y$
0	6
-2	6
1	0
-3	0

10.  $f(x) = \frac{1}{4}(x-6)(x-2)$

$x$ -intercepts:  $(6, 0)(2, 0)$

Vertex:  $(4, -1)$

Axis of Symmetry:  $x = 4$



$x-6=0$   $x-2=0$

$x = \frac{6+2}{2} = 4$

$f(4) = -1$

$x$	$y$
6	0
3	$-\frac{3}{4}$
0	3
8	3

Also use TI-84,

①  $y =$

② 2<sup>nd</sup> Graph for T-chart

Pg 59, ~~21, 23, 25~~

1-7, 13, ~~15, 17, 19~~ - 23, 27, 31, 33, 45, 47 odds

## 2.4 Modeling With Quadratic Functions

### Explore It!

a. Explain what the graph represents.

The graph represents the distance from earth a comet has traveled each year since being discovered

b. What do you know about the value of  $a$ ? How does the graph increased? decreased? What does this mean in this context? reasoning.

$a$  is positive because the graph opens up  
 $a$  increased, graph narrower  
 $a$  decreased, graph wider

If  $a$  increases, the distance from earth increases at faster rate, and if  $a$  decreases, the distance from earth increases at a slower rate.

c. Write an expression that represents the year  $t$  when the comet is closest to the Earth.

$$t = -\frac{b}{2a}$$

d. The comet is the same distance away from Earth in 2012 and 2020. Estimate the year when the comet is closest to the Earth. Explain your reasoning.

$$2016, \text{ middle of } 2012 \text{ and } 2020$$

$$\frac{2012 + 2020}{2}$$

e. What does  $c$  represent in this context? How does the graph change if  $c$  is increased? decreased? Explain.

$c$  represents the distance the comet was from earth the year it was discovered.  $c$  increased, the graph shifts upward.

$c$  decreased, graph shifts downward. Comet follows same path, distance changes, graph moves up or down

f. Assume that the model is still valid today. Is the comet's distance from Earth currently increasing, decreasing, or constant? Explain.

Increasing; the right side of the graph is increasing so the distance from earth is increasing

g. The table shows the approximate distances  $y$  (in millions of miles) from Earth for a planetary object  $m$  months after being discovered. Can you use a quadratic function to model the data? How do you know? Is this the only type of function you can use to model the data? Explain your reasoning.

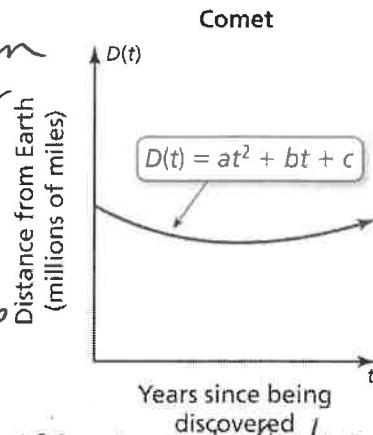
Months, $m$	0	1	2	3	4	5	6	7	8	9
Distance (millions of miles), $y$	50	57	65	75	86	101	115	130	156	175

Yes. Data lies on a curve.

No. Exponential function

h. Explain how you can find a quadratic model for the data. How do you know your model is a good fit?

Show TI-84 window  $x$   $0 \rightarrow 20$   
 $y$   $0 \rightarrow 200$  (20) plot ON



change if  $a$  is  
 Explain your

Target:

Write equations of quadratic functions using given characteristics.

- I can write equations of quadratic functions using vertices, points, and x-intercepts.
- I can write quadratic equations to model data sets.
- I can use technology to find a quadratic model for a set of data.

**KEY IDEA**  
**Writing Quadratic Equations**

Given a point and the vertex  $(h, k)$  Use vertex form:  
 $y = a(x - h)^2 + k$

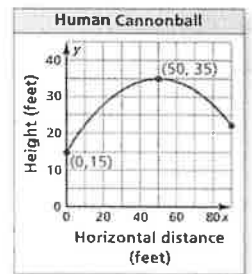
Given a point and the x-intercepts  $p$  and  $q$  Use intercept form:  
 $y = a(x - p)(x - q)$

Given three points Write and solve a system of three equations in three variables.  
 $y = ax^2 + bx + c$

*Must find a*

*Writing an Equation Using Vertex and a Point*

The graph shows the parabolic path of a performer who is shot out of a cannon, where  $y$  is the height (in feet) and  $x$  is the horizontal distance traveled (in feet). The performer lands in a net 90 feet from the cannon. What is the height of the net?



$$y = a(x - h)^2 + k$$

$$15 = a(0 - 50)^2 + 35$$

$$15 = 2500a + 35$$

$$-20 = 2500a$$

$$-0.008 = a$$

$$y = -0.008(x - 50)^2 + 35$$

$$y = -0.008(90 - 50)^2 + 35$$

$$y = -12.8 + 35$$

$$y = 22.2 \approx 22 \text{ feet}$$

What If?

If the vertex of the parabola was  $(50, 37.5)$ , what is the height of the net?

~~$$y = -0.008(x - 50)^2 + 37.5$$

$$15 = -0.008(90 - 50)^2 + 37.5$$

$$15 = -12.8 + 37.5$$

$$y = 24.7 \approx 25 \text{ feet}$$~~

$\approx 23 \text{ feet}$

$$y = a(x - h)^2 + k$$

$$15 = a(0 - 50)^2 + 37.5$$

$$15 = 2500a + 37.5$$

$$-22.5 = 2500a$$

$$-0.009 = a$$

$$y = -0.009(x - 50)^2 + 37.5$$

$$y = -0.009(90 - 50)^2 + 37.5$$

$$y = 23.1$$

Try It!

Write an equation of the parabola in vertex form based on the given information.

1. Passes through (1,-7) and has vertex (-2, 5)

$$y = a(x-h)^2 + k$$

$$-7 = a(1+2)^2 + 5$$

$$-7 = 9a + 5$$

$$-12 = 9a$$

$$-\frac{4}{3} = a$$

$$y = -\frac{4}{3}(x+2)^2 + 5$$

2. Passes through (0,8) and has vertex (-10, -3)

$$y = a(x-h)^2 + k$$

$$8 = a(0+10)^2 - 3$$

$$8 = 100a - 3$$

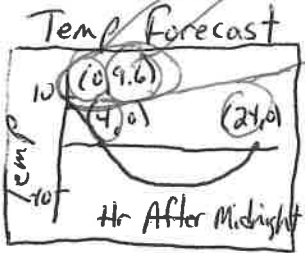
$$\frac{11}{100} = a$$

$$y = \frac{11}{100}(x+10)^2 - 3$$

Writing an Equation Using a Point and x-intercepts

A meteorologist creates a parabola to predict the temperature tomorrow, where  $x$  is the number of hours after midnight and  $y$  is the temperature (in degrees Celsius).

a. Write a function  $f$  that models the temperature over time. What is the coldest temperature?



$$y = a(x-p)(x-q)$$

$$9.6 = a(0-4)(0-24)$$

$$9.6 = 96a$$

$$0.1 = a$$

$$f(x) = 0.1(x-4)(x-24)$$

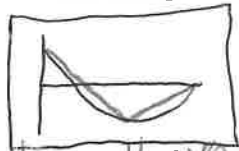
$$f(14) = 0.1(14-4)(14-24)$$

$$f(14) = -10$$

$-10^{\circ}\text{C}$  at 14 after midnight  
or  
2 p.m.

$$\frac{4+24}{2} = 14$$

b. What is the average rate of change in temperature over the interval in which the temperature is decreasing? Increasing? Compare the average rates of change.



AS:  $x=14$  dec:  $0 < x < 14$

inc:  $14 < x < 24$

slope is the avg

dec:  $0 < x < 14$

$$\text{Avg rate} \frac{f(14) - f(0)}{14 - 0} = \frac{-10 - 9.6}{14} = -1.4 = |-1.4| = 1.4$$

inc:  $14 < x < 24$

$$\text{Avg rate} \frac{f(24) - f(14)}{24 - 14} = \frac{0 - (-10)}{10} = 1$$

Try It!

Write an equation of the parabola that passes through the point (2, 5) and has x-intercepts -2 and 4.

$$y = a(x-p)(x-q)$$

$$5 = a(x+2)(x-4)$$

$$5 = -8a$$

$$-\frac{5}{8} = a$$

$$y = -\frac{5}{8}(x+2)(x-4)$$

## Writing Equations to Model Data

When data have equally spaced inputs, you can analyze patterns in the differences of the outputs to determine what type of function can be used to model the data. Linear data have constant First differences. Quadratic data have constant Second differences. The first and second differences of  $f(x) = x^2$  are shown below.

Equally-spaced  $x$ -values

$x$	-3	-2	-1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9

first differences:

-5   -3   -1   1   3   5

second differences:

2   2   2   2   2

### Writing a Quadratic Equation Using Three Points

NASA can create a weightless environment by flying a plane in parabolic paths. The table shows the heights  $h(t)$  (in feet) of a plane  $t$  seconds after starting the flight path. After about 20.8 seconds, passengers begin to experience a weightless environment. Write and evaluate a function to approximate the height at which this occurs.

Time, $t$	Height, $h(t)$
10	26,900
15	29,025
20	30,600
25	31,625
30	32,100
35	32,025
40	31,400

$2125$   
 $1575$   
 $1025$   
 $475$   
 $-75$   
 $-625$

$\left. \begin{array}{l} 2125 \\ 1575 \\ 1025 \\ 475 \\ -75 \\ -625 \end{array} \right\} -550$

$$f(x) = ax^2 + bx + c$$

Use any 3 points:

$$26900 = 100a + 10b + c$$

$$29025 = 225a + 15b + c$$

$$30600 = 400a + 20b + c$$

Subtract  $\begin{array}{r} 102 \\ 203 \end{array}$

$$\begin{array}{r} 2125 = 125a + 5b \\ 1575 = 175a + 5b \\ \hline 550 = -50a \end{array}$$

$$550 = -50a$$

$$-11 = a$$

$$1575 = -1925 + 5b$$

$$3500 = 5b$$

$$700 = b$$

$$26900 = -1100 + 7000 + c$$

$$21000 = c$$

$$f(x) = -11x^2 + 700x + 21000$$

$$f(20.8) = -11(20.8)^2 + 700(20.8) + 21000$$

$$= 30,800.96$$

$$\approx 30,801 \text{ Feet}$$

## Modeling Real Life Data Using Quadratic Regression

Real-life data that show a quadratic relationship usually do not have constant second differences because data are not exactly quadratic, they are approximately quadratic. Therefore, the data will have second differences that are very similar in value and you can use a quadratic regression to find a quadratic function that best models the data.

### Example

The table shows fuel efficiencies of a vehicle at different speeds. Write a function that models the data. Use the model to approximate the best gas mileage.

Miles per hour, $x$	Miles per gallon, $y$
20	14.5
24	17.5
30	21.2
36	23.7
40	25.2
45	25.8
50	25.8
56	25.1
60	24.0
70	19.5

Not  
Equally  
Spaced

### Desmos:

1. + Table, Enter Data
2.  $y_1 \sim ax_1^2 + bx_1 + c$
3. Write out equation using  $a$ ,  $b$ , and  $c$ .

### TI-84

1. Stat  $\rightarrow$  Edit  $\rightarrow$  Enter  $x$  values into L1,  $y$  values into L2
2. Stat  $\rightarrow$  Calc  $\rightarrow$  QuadReg

$$y = -0.014x^2 + 1.366x - 7.144$$



Practice!

Write an equation of the parabola that passes through the points (-1, 4), (0, 1), and (2, 7).

$$y = ax^2 + bx + c$$

$$4 = a - b + c$$

$$1 = c$$

$$7 = 4a + 2b + c$$

$$4 = a + 1 + 1$$

$$2 = a$$

TI-84

$$4 = a - b + 1$$

$$7 = 4a + 2b + 1$$

$$y = 2x^2 - x + 1$$

$$y = 2x^2 - x + 1$$

$$3 = a - b - 4$$

$$6 = 4a + 2b$$

$$-12 = -4a + 4b$$

$$-6 = 6b$$

$$-1 = b$$

The table shows the estimated profits  $y$  (in dollars) for a concert when the charge is  $x$  dollars per ticket. Write and evaluate a function to determine the maximum profit.

Ticket price, $x$	2	5	8	11	14	17
Profit, $y$	2600	6500	8600	8900	7400	4100

$$y = -38.79x^2 + 646.98x + 5025.56$$

