```
solve(2X²−X−15,X,2)
                   3
solve(2X²−X−15,X,-1)
                -2.5
```

applied. If you know that an equation has two real solutions, then you need to apply the solution procedure twice. Each time you must enter an initial guess that is close to the solution you are trying to find. The calculator display to the left indicates that the solutions of $2x^2 - x - 15 = 0$ are 3 and -2.5. To find these solutions, we first used the **solve(** feature with an initial guess of 2, and we then used the **solve(** feature with an initial guess of -1.

The chapters that follow will illustrate additional techniques and calculator procedures that can be used to solve equations.

CHAPTER 1 TEST PREP

The following test prep table summarizes essential concepts in this chapter. The references given in the right-hand column list Examples and Exercises that can be used to test your understanding of a concept.

1.1 Linear and Absolute Value Equations

Linear or first-degree equation A linear or first-degree equation in a single variable is one for which all of the variable expressions have degree 1. To solve a linear equation, apply the properties of real numbers and the properties of equality to produce equivalent equations until an equation in the form *variable* = *constant* is reached.	See Example 1, page 77, and then try Exercise 1, page 146. See Example 2, page 77, and then try Exercise 2, page 146.
Clearing fractions When solving an equation containing fractions, it is helpful to clear the equation of fractions by multiplying each side of the equation by the LCD of all denominators.	See Example 3, page 78, and then try Exercise 3, page 146.
Linear absolute value equation A linear absolute value equation in the variable x is one that can be written in the form $\lvert ax + b\rvert = c$.	See Example 5, page 79, and then try Exercise 5, page 146.

1.2 Formulas and Applications

Formula A formula is an equation that expresses known relationships between two or more variables.	See Example 1, page 84, and then try Exercise 10, page 146.
Applications Some of the applications of linear equations include	
• Geometry	See Examples 3 and 4, pages 85 and 86, and then try Exercises 58 and 60, page 147.
• Business	See Example 5, page 87, and then try Exercise 63, page 147.
• Investment	See Example 6, page 87, and then try Exercise 64, page 147.
• Uniform motion	See Example 7, page 88, and then try Exercise 65, page 147.
• Percent mixture problems	See Example 8, page 89, and then try Exercise 67, page 147.
• Value mixture problems	See Example 9, page 90, and then try Exercise 69, page 148.
• Work problems	See Example 10, page 90, and then try Exercise 71, page 148.

1.3 Quadratic Equations

Quadratic equation A quadratic equation in the variable x is one that can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$. Some ways in which a quadratic equation can be solved include	
• Factoring and using the zero product principle	See Example 1, page 96, and then try Exercise 14, page 146.
• Using the square root procedure	See Example 2, page 97, and then try Exercise 15, page 146.
• Completing the square	See Example 4, page 99, and then try Exercise 18, page 146.
• Using the quadratic formula	See Example 5, page 100, and then try Exercise 20, page 146.
Discriminant The discriminant of $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$, is the value of the expression $b^2 - 4ac$. If $b^2 - 4ac > 0$, then the quadratic equation has two distinct real solutions. If $b^2 - 4ac = 0$, then the quadratic equation has one real solution. If $b^2 - 4ac < 0$, then the quadratic equation has two distinct nonreal complex solutions.	See Example 6, page 102, and then try Exercise 22, page 146.
Pythagorean Theorem The Pythagorean Theorem states that if a and b are the measures of the legs of a right triangle and c is the measure of the hypotenuse, then $a^2 + b^2 = c^2$.	See Example 7, page 102, and then try Exercise 73, page 148.
Applications Quadratic equations can be applied to a variety of situations.	See Example 8, page 103, and then try Exercise 74, page 148. See Example 9, page 104, and then try Exercise 75, page 148.

1.4 Other Types of Equations

Polynomial equations Some polynomial equations of a degree greater than 2 can be solved by factoring.	See Example 1, page 109, and then try Exercise 26, page 146.
Rational equations A rational equation is one that contains rational expressions. These equations can be solved by multiplying each side of the equation by the LCD of the denominators of the rational expressions.	See Example 2, page 109, and then try Exercise 30, page 146.
Radical equations A radical equation is one that involves one or more radical expressions.	See Example 3, page 111, and then try Exercise 31, page 146. See Example 4, page 111, and then try Exercise 36, page 146.
Equations with a rational exponent An equation of the form $ax^{p/q} + b = c$ can be solved by isolating $x^{p/q}$ and then raising each side of the equation to the q/p power.	See Example 5, page 113, and then try Exercise 38, page 146.
Equations that are quadratic in form An equation that is quadratic in form—that is, an equation that can be written as $au^2 + bu + c = 0$—can be solved using any of the techniques used to solve a quadratic equation.	See Example 6, page 114, and then try Exercise 39, page 146.

Applications	See Example 8, page 115, and then try Exercise 66, page 147. See Example 9, page 116, and then try Exercise 72, page 148.

1.5 Inequalities

Linear or first-degree inequality A linear or first-degree inequality in a single variable is one for which all variable expressions have degree 1. To solve a linear inequality, apply the properties of real numbers and the properties of inequalities. (See pages 122–123.)	See Example 1, page 122, and then try Exercise 42, page 146.
Compound inequality A compound inequality is formed by joining two inequalities with the connective word *and* or *or*.	See Example 2, page 123, and then try Exercise 43, page 146.
Absolute value inequality An absolute value inequality can be solved by rewriting it as a compound inequality.	See Example 3, page 124, and then try Exercise 47, page 147.
Polynomial inequality A polynomial inequality can be solved by using the sign property of polynomials. (See page 127.)	See Example 4, page 127, and then try Exercise 52, page 147.
Rational inequality A rational inequality can be solved by using the critical value method. (See page 128.)	See Example 5, page 128, and then try Exercise 55, page 147.
Applications	See Example 7, page 130, and then try Exercise 79, page 148. See Example 8, page 130, and then try Exercise 80, page 149.

1.6 Variation and Applications

Direct variation The variable y varies directly as the variable x, or y is directly proportional to x, if and only if $y = kx$, where k is a constant.	See Example 1, page 135, and then try Exercise 81, page 149.
Direct variation as the *n*th power The variable y varies directly as the nth power of the variable x if and only if $y = kx^n$, where k is a constant.	See Example 2, page 136, and then try Exercise 82, page 149.
Inverse variation The variable y varies inversely as the variable x, or y is inversely proportional to x, if and only if $y = \frac{k}{x}$, where k is a constant.	See Example 3, page 137, and then try Exercise 83, page 149.
Inverse variation as the *n*th power The variable y varies inversely as the nth power of the variable x, or y is inversely proportional to the nth power of x, if and only if $y = \frac{k}{x^n}$, where k is a constant.	See Example 4, page 137, and then try Exercise 84, page 149.
Joint variation The variable z varies jointly as the variables x and y if and only if $z = kxy$, where k is a constant.	See Example 5, page 138, and then try Exercise 85, page 149.

CHAPTER 1 REVIEW EXERCISES

In Exercises 1 to 20, solve each equation.

1. $4 - 5x = 3x + 14$
2. $7 - 5(1 - 2x) = 3(2x + 1)$
3. $\frac{4x}{3} - \frac{4x - 1}{6} = \frac{1}{2}$
4. $\frac{3x}{4} - \frac{2x - 1}{8} = \frac{3}{2}$
5. $|x - 3| = 2$
6. $|x + 5| = 4$
7. $|2x + 1| = 5$
8. $|3x - 7| = 8$
9. $V = \pi r^2 h$, for h
10. $P = \frac{A}{1 + rt}$, for t
11. $A = \frac{h}{2}(b_1 + b_2)$, for b_1
12. $P = 2(l + w)$, for w
13. $x^2 - 5x + 6 = 0$
14. $6x^2 + x - 12 = 0$
15. $(x - 2)^2 = 50$
16. $2(x + 4)^2 + 18 = 0$
17. $x^2 - 6x - 1 = 0$
18. $4x^2 - 4x - 1 = 0$
19. $3x^2 - x - 1 = 0$
20. $x^2 - x + 1 = 0$

In Exercises 21 and 22, use the discriminant to determine whether the equation has real number solutions or nonreal complex number solutions.

21. $2x^2 + 4x = 5$
22. $x^2 + 4x + 7 = 0$

In Exercises 23 to 40, solve each equation.

23. $3x^3 - 5x^2 = 0$
24. $2x^3 - 8x = 0$
25. $2x^3 + 3x^2 - 8x - 12 = 0$
26. $3x^3 - 2x^2 - 3x + 2 = 0$
27. $\frac{x}{x + 2} + \frac{1}{4} = 5$
28. $\frac{y - 1}{y + 1} - 1 = \frac{2}{y}$
29. $3x + \frac{2}{x - 2} = \frac{4x - 1}{x - 2}$
30. $\frac{x + 1}{x + 3} + \frac{2x - 1}{x - 2} = \frac{3x + 5}{x + 3}$
31. $\sqrt{2x + 6} - 1 = 3$
32. $\sqrt{5x - 1} + 3 = 1$
33. $\sqrt{-2x - 7} + 2x = -7$
34. $\sqrt{-8x - 2} + 4x = -1$
35. $\sqrt{3x + 4} + \sqrt{x - 3} = 5$
36. $\sqrt{2x + 2} - \sqrt{x + 2} = 1$
37. $x^{5/4} - 32 = 0$
38. $2x^{2/3} - 5 = 13$
39. $6x^4 - 23x^2 + 20 = 0$
40. $3x + 16\sqrt{x} - 12 = 0$

In Exercises 41 to 56, solve each inequality. Write the answer using interval notation.

41. $-3x + 4 \geq -2$
42. $-2x + 7 \leq 5x + 1$
43. $3x + 1 > 7$ or $3x + 2 < -7$
44. $5x - 4 \leq 6$ and $4x + 1 > -7$
45. $61 \leq \frac{9}{5}C + 32 \leq 95$

46. $30 < \frac{5}{9}(F - 32) < 65$

47. $|3x - 4| < 2$

48. $|2x - 3| \geq 1$

49. $0 < |x - 2| < 1$

50. $0 < |x - a| < b \ (b > 0)$

51. $x^2 + x - 6 \geq 0$

52. $x^3 + 2x^2 - 16x - 32 < 0$

53. $\frac{x + 3}{x - 4} > 0$

54. $\frac{x(x - 5)}{x + 7} \leq 0$

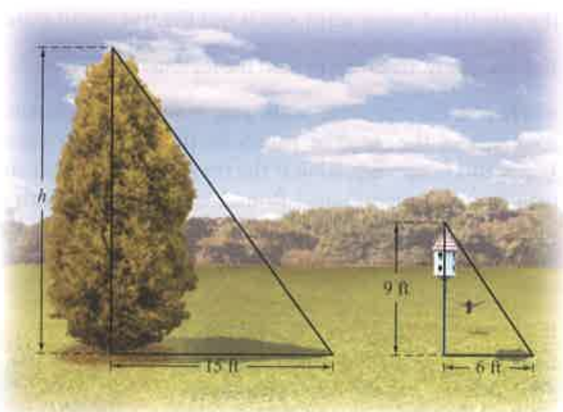
55. $\frac{2x}{3 - x} \leq 10$

56. $\frac{x}{5 - x} \geq 1$

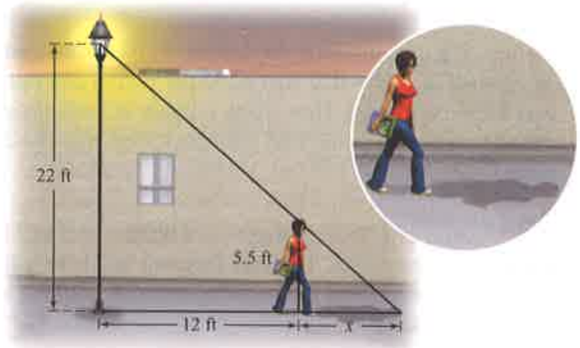
57. **Rectangular Region** The length of a rectangle is 9 feet less than twice the width of the rectangle. The perimeter of the rectangle is 54 feet. Find the width and the length.

58. **Rectangular Region** The perimeter of a rectangle is 40 inches and its area is 96 square inches. Find the length and the width of the rectangle.

59. **Height of a Tree** The height of a tree is estimated by using its shadow and the known height of a pole as shown in the figure below. Find the height of the tree.



60. **Shadow Length** A person 5 feet 6 inches tall is walking away from a lamppost that is 22 feet tall. What is the length of the person's shadow at a point 12 feet from the lamppost? See the diagram below.



61. **Diameter of a Cone** As sand is poured from a chute, it forms a right circular cone whose height is one-fourth the diameter of the base. What is the diameter of the base when the cone has a volume of 144 cubic feet? Round to the nearest foot.

62. **Individual Price** A calculator and a battery together sell for \$21. The price of the calculator is \$20 more than the price of the battery. Find the price of the calculator and the price of the battery.

63. **Maintenance Cost** Eighteen owners share the maintenance cost of a condominium complex. If six more units are sold, the maintenance cost will be reduced by \$12 per month for each of the present owners. What is the total monthly maintenance cost for the condominium complex?

64. **Investment** A total of \$5500 was deposited into two simple interest accounts. On one account the annual simple interest rate is 4%, and on the second account the annual simple interest rate is 6%. The amount of interest earned for 1 year was \$295. How much was invested in each account?

65. **Distance to an Island** A motorboat left a harbor and traveled to an island at an average rate of 8 knots. The average speed on the return trip was 6 knots. If the total trip took 7 hours, how many nautical miles is it from the harbor to the island?

66. **Running** Inez can run at a rate that is 2 miles per hour faster than Olivia's rate. One day, Inez gave Olivia a 25-minute head start on a run. If Inez passed Olivia 5 miles from the starting point, how fast was each running?

67. **Chemistry** A chemist mixes a 5% salt solution with an 11% salt solution. How many milliliters of each should be used to make 600 milliliters of a 7% salt solution?

68. **Pharmacy** How many milliliters of pure water should a pharmacist add to 40 milliliters of a 5% salt solution to produce a 2% salt solution?

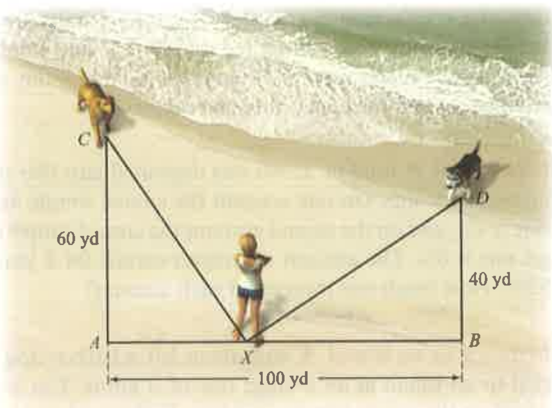
69. Alloys How many ounces of a gold alloy that costs \$460 per ounce must be mixed with 25 ounces of a gold alloy that costs \$220 per ounce to make a mixture that costs \$310 per ounce?

70. Blends A grocer makes a snack mixture of raisins and nuts by combining raisins that cost \$2.50 per pound and nuts that cost \$4.50 per pound. How many pounds of each should be mixed to make 20 pounds of this snack that costs \$3.25 per pound?

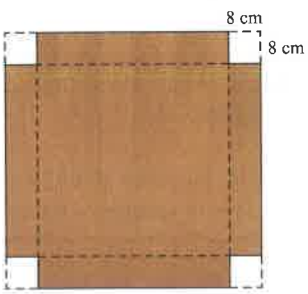
71. Construction of a Wall A mason can build a wall in 9 hours less than an apprentice. Together they can build the wall in 6 hours. How long would it take the apprentice, working alone, to build the wall?

72. Parallel Processing One computer can solve a problem 5 minutes faster than a second computer. Working together, the computers can solve the problem in 6 minutes. How long does it take the faster computer working alone to solve the problem?

73. Dogs on a Beach Two dogs start, at the same time, from points C and D on a beach and run toward their owner, who is positioned at point X. If the dogs run at the same rate and reach their owner at the same instant, what is the distance AX? See the diagram below. *Note:* Angle A and angle B are right angles.



74. Constructing a Box A square piece of cardboard is formed into a box by cutting an 8-centimeter square from each corner and folding up the sides. If the volume of the box is to be 80,000 cubic centimeters, what size square piece of cardboard is needed? (*Hint*: Volume of a box is $V = lwh$.)



75. Sports In an Olympic 10-meter diving competition, the height h, in meters, of a diver above the water t seconds after leaving the board can be given by $h = -4.9t^2 + 7.5t + 10$. In how many seconds will the diver be 5 meters above the water? Round to the nearest tenth of a second.

76. Fair Coin If a fair coin is tossed 100 times, we would expect heads to occur *about* 50 times. But how many heads would suggest that a coin is *not* fair? An inequality used by statisticians to answer this question is $\left|\frac{x-50}{5}\right| < 1.96$, where x is the actual number of heads that occurred in 100 tosses of a coin. What range of heads would suggest that the coin is a fair coin?

77. Mean Height If a researcher wanted to know the mean height (the *mean* is the sum of all the measurements divided by the number of measurements) of women in the United States, the height of every woman would have to be measured and then the mean height calculated—an impossible task. Instead, researchers find a representative sample of women and find the mean height of the sample. Because the entire population of women is not used, there is a possibility that the calculated mean height is not the true mean height. For one study, researchers used the formula $\left|\frac{63.8-\mu}{0.45}\right| < 1.645$, where μ is the true mean height, in inches, of all women, to be 90% sure of the range of values for the true mean height. Using this inequality, what is the range of mean heights of women in the United States? Round to the nearest tenth of an inch. (*Source*: Based on data from the National Center for Health Statistics)

78. Mean Waist Size If a researcher wanted to know the mean waist size (see Exercise 77 for the definition of *mean*) of men in the United States, the waist size of every man would have to be measured and then the mean waist size calculated. Instead, researchers find a representative sample of men and find the mean waist size of the sample. Because the entire population of men is not used, there is a possibility that the calculated mean waist size is not the true mean waist size. For one study, researchers used the formula $\left|\frac{39-\mu}{0.53}\right| < 1.96$, where μ is the true mean waist size, in inches, of all men to be 95% sure of the range of values for the true mean waist size. Using this inequality, what is the range of mean waist sizes of men in the United States? Round to the nearest tenth of an inch. (*Source*: Based on data from the National Center for Health Statistics)

79. Basketball Dimensions A basketball is to have a circumference of 29.5 to 30.0 inches. Find the acceptable range of diameters for the basketball. Round results to the nearest hundredth of an inch.

80. **Population Density** The population density D, in people per square mile, of a city is related to the horizontal distance x, in miles, from the center of the city by the equation

$$D = -45x^2 + 190x + 200, \qquad 0 < x < 5$$

Describe the region of the city in which the population density exceeds 300 people per square mile. Round critical values to the nearest tenth of a mile.

81. **Physics** Force F is directly proportional to acceleration a. If a force of 10 pounds produces an acceleration of 2 feet per second squared, what acceleration will be produced by a 15-pound force?

82. **Physics** The distance an object will fall on the moon is directly proportional to the square of the time it falls. If an object falls 10.6 feet in 2 seconds, how far would an object fall in 3 seconds?

83. **Business** The number of MP3 players a company can sell is inversely proportional to the price of the player. If 5000 MP3 players can be sold when the price is \$150, how many players could be sold if the price is \$125?

84. **Magnetism** The repulsive force between the north poles of two magnets is inversely proportional to the square of the distance between the poles. If the repulsive force is 40 pounds when the distance between the poles is 2 inches, what is the repulsive force when the distance between the two poles is 4 inches?

85. **Acceleration** The acceleration due to gravity on the surface of a planetary body is directly proportional to the mass of the body and inversely proportional to the square of its radius. If the acceleration due to gravity is 9.8 meters per second squared on Earth, whose radius is 6,370,000 meters and whose mass is 5.98×10^{26} grams, find the acceleration due to gravity on the moon, whose radius is 1,740,000 meters and whose mass is 7.46×10^{24} grams. Round to the nearest hundredth of a meter per second squared.

CHAPTER 1 TEST

1. $\dfrac{2x}{3} + \dfrac{1}{2} = \dfrac{x}{2} - \dfrac{3}{4}$

2. Solve: $|2x + 5| = 13$

3. Solve $ax - c = c(x - d)$ for x.

4. Solve $6x^2 - 13x - 8 = 0$ by factoring and applying the zero product principle.

5. Solve $2x^2 - 8x + 1 = 0$ by completing the square.

6. Solve $x^2 + 13 = 4x$ by using the quadratic formula.

7. Determine the discriminant of $2x^2 + 3x + 1 = 0$ and state the number of real solutions of the equation.

8. Solve: $\sqrt{x - 2} - 1 = \sqrt{3 - x}$

9. Solve: $\sqrt{3x + 1} - \sqrt{x - 1} = 2$

10. Solve: $3x^{4/5} - 7 = 41$

11. Solve: $\dfrac{3}{x + 2} - \dfrac{3}{4} = \dfrac{5}{x + 2}$

12. Solve: $2x^3 + x^2 - 8x - 4 = 0$

13. Solve: $x^3 - 64 = 0$

14. **a.** Solve the compound inequality:

$$2x - 5 \le 11 \qquad \text{or} \qquad -3x + 2 > 14$$

Write the solution set using set-builder notation.

b. Solve the compound inequality:

$$2x - 1 < 9 \qquad \text{and} \qquad -3x + 1 \le 7$$

Write the solution set using interval notation.

15. Solve $|3x - 4| > 5$. Write the answer in interval notation.

16. Solve $x^2 - 5x - 6 < 0$. Write the answer using interval notation.

17. Solve: $\dfrac{x^2 + x - 12}{x + 1} \ge 0$

Write the solution set using interval notation.

18. **Automotive** A radiator contains 6 liters of a 20% antifreeze solution. How much should be drained and replaced with pure antifreeze to produce a 50% antifreeze solution?

19. **Paving** A worker can cover a parking lot with asphalt in 10 hours. With the help of an assistant, the work can be done in 6 hours. How long would it take the assistant, working alone, to cover the parking lot with asphalt?

20. **Shadow Length** Geraldine is 6 feet tall and walking away from a lamppost that is 20 feet tall. What is the length of Geraldine's shadow when she is 10 feet from the lamppost? Round to the nearest tenth of a foot.

21. **Mixtures** A market offers prepackaged meatloaf that is made by combining ground beef that costs \$3.45 per pound with ground sausage that costs \$2.70 per pound. How many pounds of each should be used to make 50 pounds of a meatloaf mixture that costs \$3.15 per pound?

22. **Rockets** A toy rocket is launched from a platform that is 4 feet above the ground. The height h, in feet, of the rocket t seconds after launch is given by $h = -16t^2 + 160t + 4$. How many seconds after launch will the rocket be 100 feet above the ground? Round to the nearest tenth of a second.

23. **Running** Zoey can run at a rate that is 4 miles per hour faster than Tessa's rate. One day, Zoey gave Tessa a 1-hour head start on a run. Assuming that each ran at a constant rate and Zoey passed Tessa 15 miles from the starting point, what was Zoey's rate?

24. **Pass Completions** One part of the NFL quarterback rating formula requires that $0 < 0.05p - 1.5 < 2.375$, where $p\%$ is the percent of completed passes. What is the range of p used in the formula?

25. **Astronomy** A meteorite approaching the moon has a velocity that varies inversely as the square root of its distance from the center of the moon. If the meteorite has a velocity of 4 miles per second at 3000 miles from the center of the moon, find the velocity of the meteorite when it is 2500 miles from the center of the moon. Round to the nearest tenth of a mile per second.

CUMULATIVE REVIEW EXERCISES

1. Evaluate: $4 + 3(-5)$

2. Write 0.00017 in scientific notation.

3. Perform the indicated operations and simplify: $(3x - 5)^2 - (x + 4)(x - 4)$

4. Factor: $8x^2 + 19x - 15$

5. Simplify: $\dfrac{7x - 3}{x - 4} - 5$

6. Simplify: $a^{2/3} \cdot a^{1/4}$

7. Simplify: $(2 + 5i)(2 - 5i)$

8. Solve: $2(3x - 4) + 5 = 17$

9. Solve $2x^2 - 4x = 3$ by using the quadratic formula.

10. Solve: $|2x - 6| = 4$

11. Solve: $x = 3 + \sqrt{9 - x}$

12. Solve: $x^3 - 36x = 0$

13. Solve: $2x^4 - 11x^2 + 15 = 0$

14. Solve the compound inequality:

$$3x - 1 > 2 \quad \text{or} \quad -3x + 5 \geq 8$$

Write the solution set using set-builder notation.

15. Solve $|x - 6| \geq 2$. Write the solution set using interval notation.

16. Solve $\dfrac{x - 2}{2x - 3} \geq 4$. Write the solution set using set-builder notation.

17. **Dimensions of a Field** A fence built around the border of a rectangular field measures a total of 200 feet. If the length of the fence is 16 feet longer than the width, what are the dimensions of the fence?

18. **Business** The revenue R, in dollars, earned by selling x inkjet printers is given by $R = 200x - 0.004x^2$. The cost C, in dollars, of manufacturing x inkjet printers is given by the equation $C = 65x + 320{,}000$. How many printers should be manufactured and sold to earn a profit of at least \$600,000?

19. **Course Grade** An average score of at least 80, but less than 90, in a history class receives a B grade. Rebecca has scores of 86, 72, and 94 on three tests. Find the range of scores she could receive on the fourth test that would give her a B grade for the course. Assume that the highest test score she can receive is 100.

20. **Ticketing Speeding Drivers** A highway patrol department estimates that the cost of ticketing p percent of the speeders who travel on a freeway is given by

$$C = \frac{600p}{100 - p}, \quad 0 < p < 100$$

where C is in thousands of dollars. If the highway patrol department plans to fund its program to ticket speeding drivers with \$100,000 to \$180,000, what is the range of the percent of speeders the department can expect to ticket? Round your percents to the nearest 0.1%.