Chapter 1 Notes

Algebra 2

1.1-1.2 Discovery Activity: Analyzing Graphs of Quadratic Functions

Targets:

1. I can describe transformations of functions.
2. I can write functions that represent transformations of functions.

Use Desmos (desmos.com) to complete the following activity.

1. Enter $y = x^2$ into the first slot on the left in Desmos. This graph is called the *parent graph* for all quadratic equations. All quadratic functions are based on this single function and its graph. ***Compare every graph from here on to this graph.*** Leave this graph on your screen at all times throughout the activity.

Quadratic = Parabola

Section One:

Graph each of the following and compare it to the parent graph above. Describe how the new graph changed from the parent graph $y = x^2$.

Graph:	How did the graph change from the parent graph?
2. $y = x^2 + 2$	Up 2
3. $y = x^2 - 3$	Down 3
4. $y = x^2 + 1$	Up 1

5. How do you think the graph of $y = x^2 - 1$ will change (move) from the parent graph? Down 1

6. Graph it in your calculator to determine if your conjecture was true or false? True or False Circle One

This is called a Vertical Stretch (Horizontal Shrink). Answer this during class discussion.

Section Two:

Graph each of the following and compare it to the parent graph above. Describe how the new graph changed from the parent graph $f(x) = x^2$.

Graph:	How did the graph change from the parent graph?
7. $f(x) = (x + 2)^2$	Left 2
8. $f(x) = (x + 1)^2$	Left 1
9. $f(x) = (x - 1)^2$	Right 1

10. How do you think the graph of $f(x) = (x + 3)^2$ will change (move) from the parent graph? Left 3

11. Graph it in your calculator to determine if your conjecture was true or false? True or False Circle One

This is called a Vertical Shrink (Horizontal Stretch). Answer this during class discussion.

Section Three:

Graph each of the following and compare it to the parent graph above. Describe how the new graph changed from the parent graph $y = x^2$.

Graph: How did the graph change from the parent graph?

12. $y = 2x^2$ Vertical Stretch

13. $y = 3x^2$ Vert. Stretch

14. How do you think the graph of $y = 4x^2$ will change from the parent graph? Vert Stretch

This is called a Vertical Stretch. Answer this during class discussion

15. $y = \frac{1}{2}x^2$ Vertical Shrink

16. $y = \frac{2}{3}x^2$ Vert Shrink

17. How do you think the graph of $y = \frac{1}{3}x^2$ will change from the parent graph? Vert. Shrink

This is called a Vertical Shrink. Answer this during class discussion.

Section Four:

Graph each of the following and compare it to the parent graph above. Describe how the new graph changed from the parent graph $y = x^2$.

Graph: How did the graph change from the parent graph?

18. $y = -x^2$ Reflection

19. $y = -3x^2$ Vert Stretch and Reflection

20. $y = -(x+2)^2$ Left 2, Reflection

21. How do you think the graph of $y = -x^2 + 1$ will change from the parent graph? Up 1, Reflection

This is called a Vert Shift, Reflection. Answer this during class discussion.

Section Five: Other Function Families

Think about the transformations you have investigated above. Will these transformations transfer to other function families such as $y = |x|$, $y = \sqrt{x}$, and $y = \frac{1}{x}$, etc.?

Predict what transformations will occur for each function below.

22. How do you think the graph of $y = |x+2| - 4$ will change from the parent graph $y = |x|$?

Down 4, Left 2

23. How do you think the graph of $y = 2\sqrt{x-3}$ will change from the parent graph $y = \sqrt{x}$?

Right 3, Vert Stretch

24. How do you think the graph of $y = \frac{1}{x+3} - 4$ will change from the parent graph $y = \frac{1}{x}$?

Left 3, Down 4

25. How do you think the graph of $y = -\frac{1}{(x-2)^2} + 1$ will change from the parent graph $y = \frac{1}{x^2}$?

Right 2, Up 1, Reflectin

26. Do the transformations for the function family $y = x^2$ hold true for other function families? Yes

27. Are there any function families that do not follow these set of transformation rules? No

Section Six:

In the first five sections, you discovered the transformations applied to one family of functions is the same for other family of functions. What transformations will occur for each of the functions below from the parent graph?

28. $y = (x+2)^2 - 1$ Down 1, Left 2

29. $y = \frac{1}{2}|x| + 3$ Up 3, Vert Shrink

30. $y = \frac{2}{3}x - 4$ Down 4, Vert Shrink

31. $f(x) = -3\sqrt{x-1} + 5$ Up 5, Right 1, Vert Stretch, Reflecti

32. $g(x) = |x+4| - 2$ Down 2, Left 4

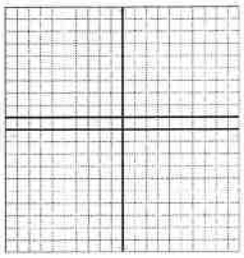
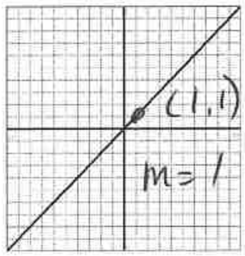
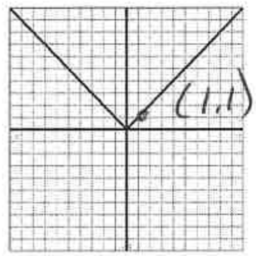
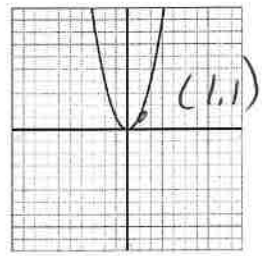
1.1 Parent Functions and Transformations

Target:

I can graph and describe transformations of functions.

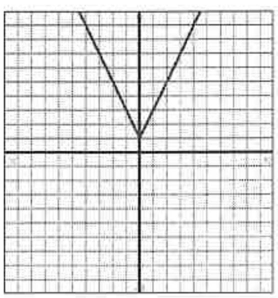
- I can identify the function family to which the function belongs.
- I can graph transformations of functions.
- I can explain how translations, reflections, stretches and shrinks affect graphs of functions.

KEY IDEA: Parent Functions

Family:	**Constant**	**Linear**	**Absolute Value**	**Quadratic**
Rule:	$f(x) = 1$	$f(x) = x$	$f(x) = \|x\|$	$f(x) = x^2$
Graph:				
Domain:	All real numbers	All real numbers	All real numbers	All real numbers
Range:	$y = 1$	All real numbers	$y \geq 0$	$y \geq 0$

IDENTIFYING A FUNCTION FAMILY

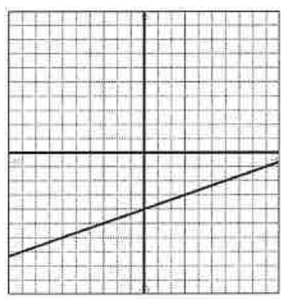
Identifying the function family to which *f* belongs. Compare the graph of *f* to the graph of its parent function. State the domain and range.

1. 

Ab Value

Up 1, V.Strekh by a factor of 2

D: R R: $y \geq 1$

2. 

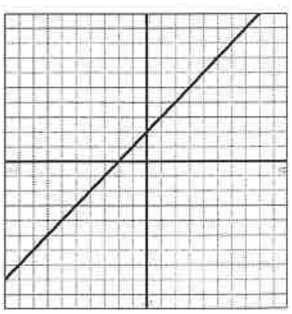
Linear

Down 4

V.Shrink $\frac{1}{3}$

D: R R: R

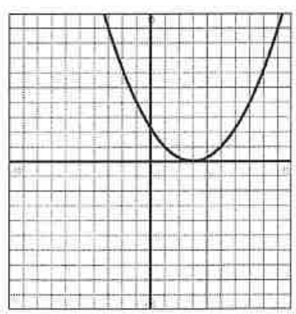
Self-Assessment:

3. 

Linear

Up 2

D: R R: R

4. 

Quadratic

Right 3

V. Shrink by a factor of $\frac{1}{2}$

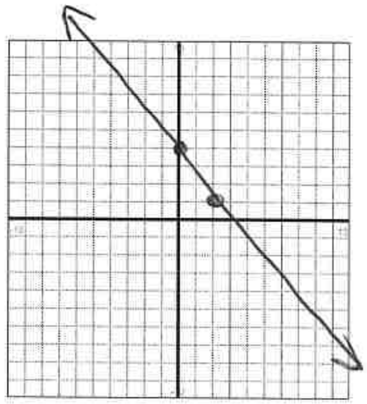
D: R R: $y \geq 0$

Remember: Slope-intercept form of a linear equations is $y = mx + b$, where *m* is the Slope and *b* is the y-int.

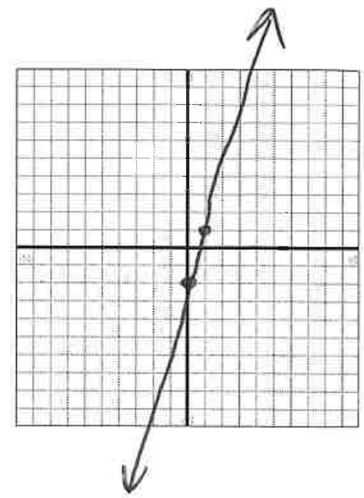
Graph.

5. $3x + 2y = 8$

$y = -\frac{3}{2}x + 4$



6. $y = 3x - 2$



Describing Transformations

Transformation - changes the size, shape, position or orientation of a graph.

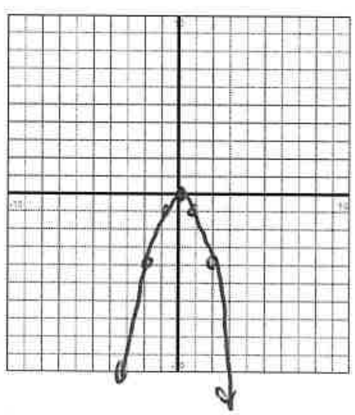
Translation - is a transformation that shifts a graph horizontally and/or vertically but does not change its size, shape, or orientation.

Reflection – is a transformation that flips a graph over a line called the *line of* symmetry.

Graph the function, then describe the transformation.

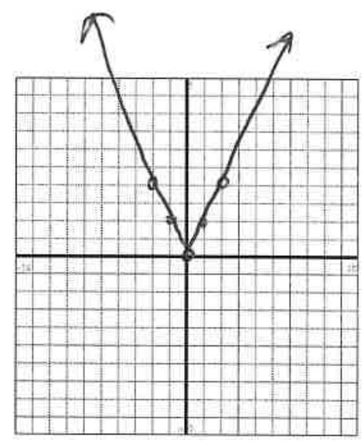
7. $f(x) = -x^2$

x	y
0	0
1	-1
-1	-1
2	-4
-2	-4



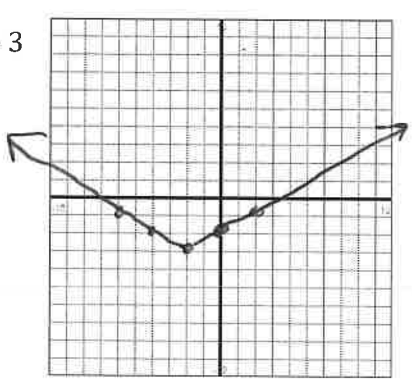
8. $y = 2|x|$

x	y
0	0
1	2
-1	2
2	4
-2	4



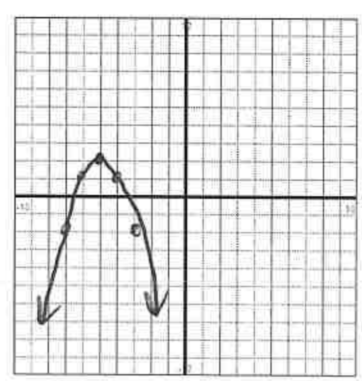
9. $g(x) = \frac{1}{2}|x + 2| - 3$

x	y
0	-2
2	-1
-2	-3
-4	-2
-6	-1



10. $y = -(x + 5)^2 + 2$

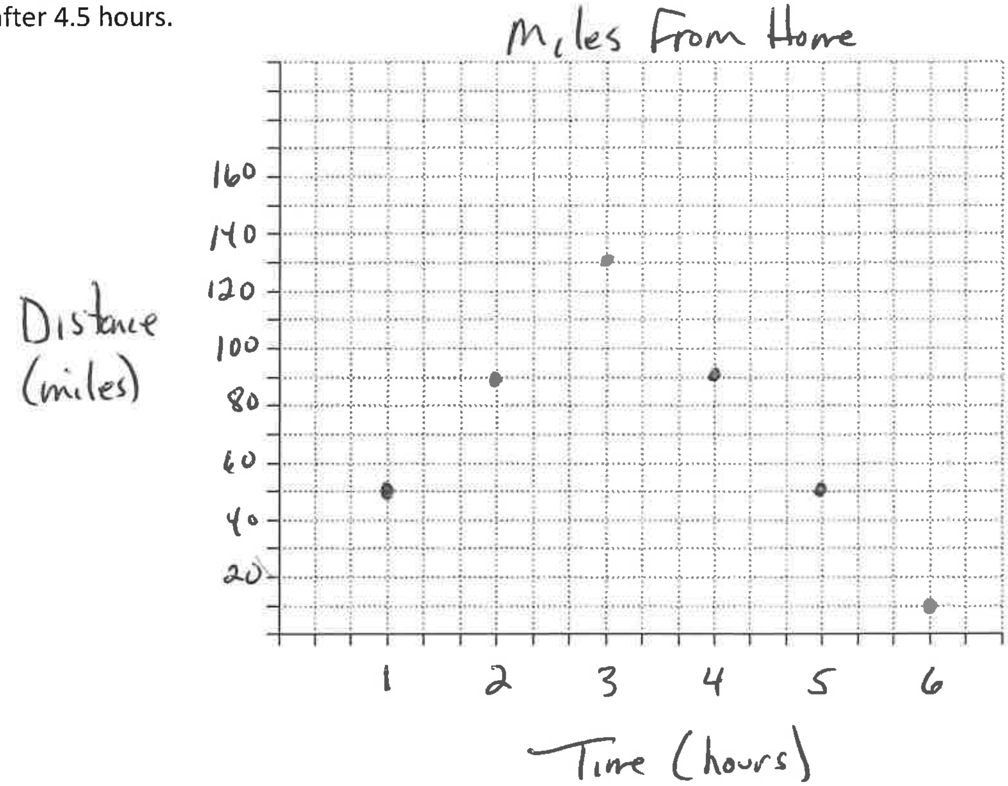
x	y
-5	2
-4	1
-6	1
-7	-2
-3	-2



Pg 8, 1-6 all, 7-23, 33 odds

11. The table shows how many miles y you are from home after x hours. What type of function can you use to model the data? Estimate the distance after 4.5 hours.

Time (hours), x	Distance (miles), y
1	50
2	90
3	130
4	90
5	50
6	10



Absolute Value

4.5 hours ≈ 70 miles

1.2 Transformations of Linear and Absolute Value Functions

Target:

I can write functions that represent transformations of functions.

- I can write functions that represent transformations of linear functions.
- I can write functions that represent transformations of absolute value functions.

1.1-1.2 Discovery Activity Review

Describe the transformations for the functions below from the parent graph.

1. $f(x) = 2x + 1$ Up 1, Vert Stretch
2. $y = (x-1)^2 + 4$ Up 4, Right 1
3. $g(x) = \frac{1}{3}|x+5|$ Left 5, Vertical Shrink
4. $y = 3f(x-2) - 1$ Down 1, Right 2, Vert Stretch

Write the Translations of Functions

5. Let $f(x) = 2x + 1$.
   a. Write a function *g* whose graph is a translation 3 units down of the graph of *f*.
   b. Write a function *h* whose graph is a translation 2 units left of the graph of *f*.

a. $g(x) = 2x - 2$

b. $h(x) = 2(x+2) + 1$

OR

$= 2x + 5$

Write a function *g* whose graph represents the indicated transformation of the graph of *f*. Use Desmos to check your answer.

6. $f(x) = 3x$; translation 5 units up

$g(x) = 3x + 5$

7. $f(x) = |x| - 3$; translation 4 units right

$g(x) = |x-4| - 3$

$-f(x)$

Writing Reflections of Functions

8. Let $f(x) = |x+3| + 1$. Write a function g whose graph is a reflection about the x-axis of f.

$$g(x) = -f(x)$$
$$= -\left[|x+3| + 1\right]$$
$$= -|x+3| - 1$$

9. Let $f(x) = -|x+2| - 1$. Write a function g whose graph is a reflection across the x-axis of f. Use Desmos to check your answer.

$$g(x) = |x+2| - 1$$
$$g(x) = -f(x)$$
$$g(x) = -\left[-|x+2| - 1\right]$$
$$= |x+2| + 1$$

Writing Stretches and Shrinks of Functions

St or Sh $\times f(x)$

10. Let $f(x) = |x-3| - 5$. Write a function g whose graph is a vertical stretch of the graph f by a factor of 2.

$$g(x) = 2f(x)$$
$$= 2(|x-3| - 5)$$
$$= 2|x-3| - 10$$

11. Let $f(x) = x^2 - 3$. Write a function g whose graph is a vertical shrink of the graph f by a factor of $\frac{1}{3}$. Use Desmos to check your answer.

$$g(x) = \frac{1}{3}x^2 - 3$$
$$g(x) = \frac{1}{3}f(x)$$
$$g(x) = \frac{1}{3}(x^2 - 3)$$
$$= \frac{1}{3}x^2 - 1$$

Combinations of Transformations

Must go in order!!

12. Let the graph of g be a vertical shrink by a factor of 0.25 followed by a translation 3 units up of the graph $f(x) = x$. Write a rule for g.

$$g(x) = \frac{1}{4}x + 3$$

4 up and vert shrink of $\frac{1}{2}$

Add: of the graph $f(x) = 2x + 2$

$$g(x) = 2x + 6$$
$$g(x) = \frac{1}{2}(2x + 6)$$
$$= x + 3$$

first

13. Let the graph of g be a translation 6 units down followed by a reflection about the x-axis of the graph $f(x) = |x|$. Write a rule for g. Use Desmos to check your answer.

second

$$g(x) = -|x| - 6$$
$$g(x) = |x| - 6$$
$$g(x) = -\left[|x| - 6\right]$$
$$g(x) = -|x| + 6$$

Modeling Real Life

14. You design a computer game. Your revenue (in dollars) for x downloads is given by $f(x) = 2x$ and your profit is \$50 less than 90% of the revenue. What is your profit for 100 downloads?

$y = 2x$

$y = 2(100)$

$y = 200$

$200 \times .90 = 180$

$180 - 50 = \$130$

Pg 16, 1-10, 15, 16, 27, 28, 33, 34

1.3 Modeling with Linear Functions: Day 1

Target:

I can use linear functions to model and analyze real-life situations.

- I can write equations of linear functions.
- I can compare linear equations to solve real-life problems.
- I can determine a line of best fit.

MODELING REAL LIFE

A company purchases a demolition robot for $87,000. The spreadsheet shows how the robot depreciates over an 8-year period.

	A	B
1	Year, *t*	Value, *V*
2	0	$87,000
3	1	$79,750
4	2	$72,500
5	3	$65,250
6	4	$58,000
7	5	$50,750
8	6	$43,500
9	7	$36,250
10	8	$29,000
11		

A. How could you use the values in the spreadsheet to write a linear function to represent the value, *V*, of the robot as a function of the number of years, *t*?

Graphing Calculator

Stat Edit

Stat Calc

Lin Reg

$\boxed{y = -7250x + 87000}$

B. Sketch a graph of the linear function in the grid below.

C. What is the slope of the graph? What does it represent in this problem?

−7250, depreciation

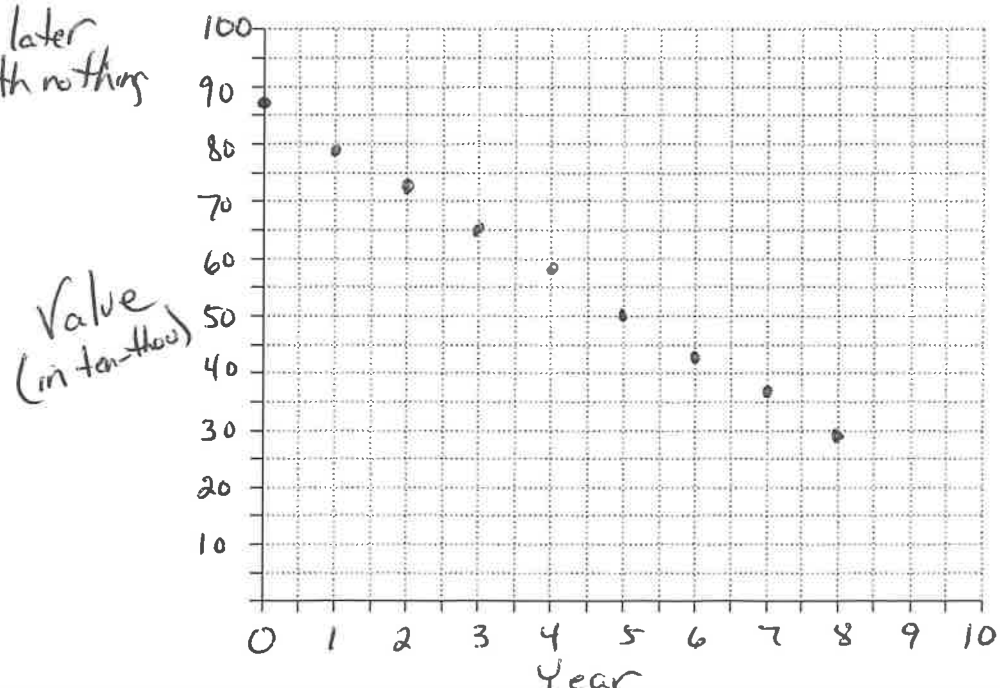
D. What are the x- and y-intercepts of the graph? What do they represent in this problem?

y-int = 87000 : initial value

x-int = 12 : 12 yrs later worth nothing

Add: y = 0

Intersect



Point Slope Form of a Line: $y - y_1 = m(x - x_1)$

Write the linear equation in slope intercept form for each given set of information.

1. (2,4), (-1,7)

$\frac{4-7}{2--1} = \frac{-3}{3} = -1$

$y-4 = -1(x-2)$

$y = -x + 6$

2. Slope = 2/3, y-intercept = 5 $(0,5)$

$y-5 = \frac{2}{3}(x-0)$

$y = \frac{2}{3}x + 5$

3. (-2, -3), x-intercept = -1 $(-1,0)$

$\frac{-3-0}{-2--1} = \frac{-3}{-1} = 3$

$y + 3 = 3(x+2)$

$y = 3x + 3$

4. (5, 1), (5,-3)

$\frac{1--3}{5-5} = \frac{4}{0} =$ Undefined

only hits x axis so:

$x = 5$

5. (2, 6), m = 0

only hits y-axis so:

$y = 2$

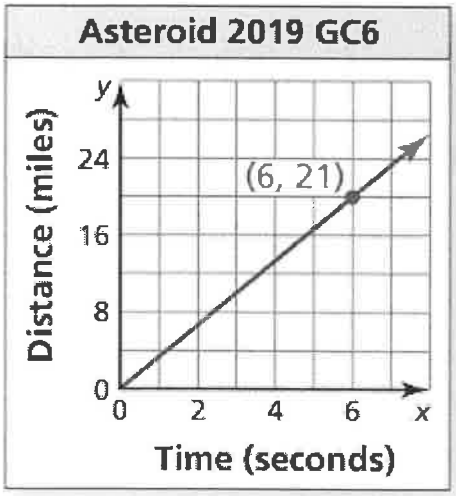
6. The graph to the right shows the distance Asteroid 2019 GC6 travels in x seconds.

A. Write an equation of the line. $(0,0)(6,21)$

$\frac{21-0}{6-0} = \frac{21}{6}$

$y - 0 = \frac{21}{6}(x-0)$

$y = \frac{21}{6}x$



B. Interpret the slope.

This indicates the asteroid travels $\frac{21}{6}$ (3.5) miles per hour

C. The asteroid came within 136,000 miles of Earth in April of 2019. How long does it take the asteroid to travel that distance?

136,000 miles is distance (y)

$136{,}000 = \frac{21}{6}x$

$38{,}857 \approx x$

38,857 seconds

$\div$ 3600 in 1 hr.

≈ 11 hours

1.3 Modeling with Linear Functions: Day 2

Target:

I can use linear functions to model and analyze real-life situations.

- I can write equations of linear functions.
- I can compare linear equations to solve real-life problems.
- I can determine a line of best fit.

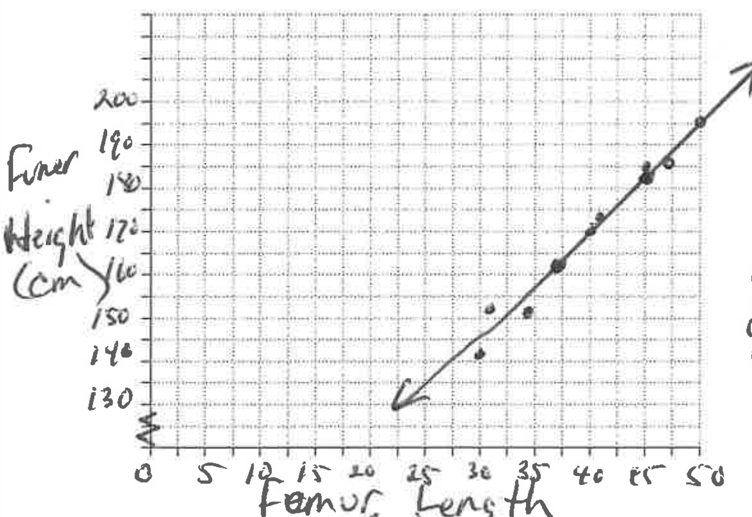
FINDING A LINE OF FIT

The table shows the femur length, in centimeters, and heights, in centimeters, of several people. Do the data show linear relationship? If so, write an equation of a line of fit and use it to estimate the height of a person whose femur is 35 cm long.

How do we approach such a problem?

Step 1: Make a scatterplot of the data.

Femur length (cm), x	Height (cm), y
40	170
45	183
32	151
50	195
37	162
41	174
30	141
34	151
47	185
45	182



$(37, 162)$
$(45, 182)$

$\frac{182-162}{45-37} = \frac{20}{8} = 2.5$

$y - 162 = 2.5(x - 37)$

$y - 162 = 2.5x - 92.5$

$y = 2.5x + 69.5$

Step 2: Draw the line that most closely appears to follow the trend given by the data points (a line of fit).

Step 3: Choose two points on the line and estimate the coordinates of each point. These points DO NOT have to be original data points.

Coordinates: $(37, 162)$ $(45, 182)$

Step 4: Write an equation of the line that passes through the two points from Step 3. This equation is a model for the data.

$y = 2.5x + 69.5$

$\frac{182-162}{45-37} = \frac{20}{8} = 2.5$

$y - 162 = 2.5(x - 3.7)$

$y - 162 = 2.5x - 92.5$

Equation: ______________________

Estimated Height for a femur length of 35cm: 157

Line of Best Fit

The line equation we found on the previous page was our best estimation of the linear pattern we saw in the scatterplot. The line of __best fit__ is the line that best models a set of data and lies as close as possible to all of the data points. Many calculators and other tools can use a linear regression to find this best fit line.

To find a line of best fit using technology:

Step 1: Enter the data into two lists (one for x, one for y)

On **TI Graphing Calculator**, Stat → Edit

On **Desmos**, + Table

Step 2: Find an equation for the line of best fit

On **TI Graphing Calculator**, Choose Stat → Calc → LinReg

On **Desmos**, type y1 ~ mx1 + b

Check your work from the previous page using technology.

Best Fit Line Equation: __~~[illegible]~~ $y = 2.6x + 65.0$__

Step 3: Graph the best fit line on your coordinate grid.

How does the best fit line compare to your line of fit?

Close same

Step 4: Find the value of y when x = 35 for your best fit line.

~~[illegible]~~ 156

How does this prediction compare to the estimate from your line of fit?

~~[illegible]~~ close

Would it make sense to use our equation to estimate the height of someone with a femur length of 100 cm?

No, rarely someone will be that big

Using a model to predict values outside of the interval of x-values from your data is known as __outliers__.

Correlation Coefficient

If data have a linear relationship in a scatterplot, a helpful additional piece of information is the correlation coefficient, which helps tell you how strong the linear relationship is between x and y. The correlation coefficient, denoted by the letter __r__, is a number between -1 and 1.

r close to 1 → The data has a __strong__ relationship.

r close to 0 → The data has a __no__ relationship.

r close to -1 → The data has a __strong__ relationship.

~~[illegible]~~

Diagnostics must be on stat, calc, lin reg.

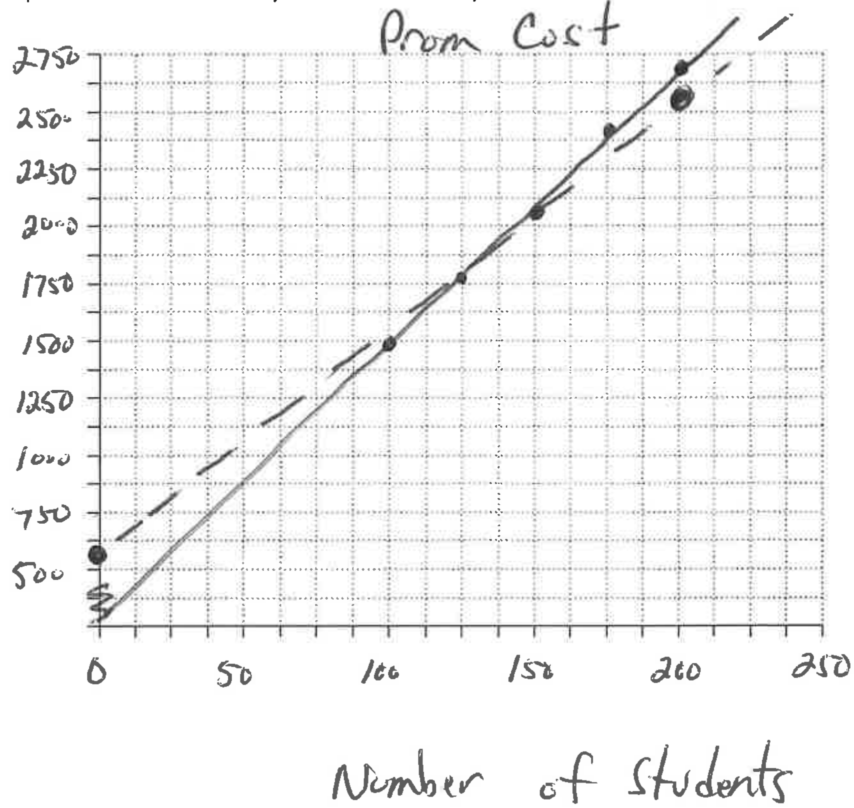
$r = 0.9945$

1.3 Part B: Comparing Linear Equations, Systems of Equations in Two Variables

COMPARING LINEAR EQUATIONS EXPLORATION

Two prom venues charge a rental fee plus a fee per student. The table shows the total costs (in dollars) for different numbers of students at Lakeside Inn. The total cost y (in dollars) for x students at Sunview Resort is represented by the equation $y = 10x + 600$. Which venue charges less per student? How many students must attend for the total costs to be the same? Use the graph or open space as needed to try to answer the questions.

Lakeside Inn	
Number of students, *x*	**Total cost, *y***
100	\$1500
125	\$1800
150	\$2100
175	\$2400
200	\$2700



Sunview = Lakeside

$10x + 600 = 12x + 300$

$300 = 2x$

$150 = x$

150 students, the cost is the same.

<150 Lakeside cheaper

>150 Sunview cheaper

Sunview - - - - -

Lakeside ————

(100, 1500) (200, 2700)

$\frac{2700 - 1500}{200 - 100} = \frac{1200}{100} = 12$

$y - 1500 = 12(x - 100)$

$y - 1500 = 12x - 1200$

$y = 12x + 300$

Solving Systems of Equations

Solve for the solution to each system of equations. Your answer should be a coordinate point (one solution), no solution(0 solutions), or the equation of the line (infinitely many solutions).

Elimination: Opposites then add

Substitution: coefficient of 1

* Do both ways

1. $-3x + y = 0$
   $3x - 3y = -6$ +

   $-2y = -6$
   $y = 3$
   $-3x + 3 = 0$
   $3 = 3x$
   $1 = x$
   $(1, 3)$

2. $3x - 4y = 4$
   -3 $x + y = 6$

   $-3x - 3y = -18$
   $-7y = -14$
   $y = 2$
   $x + 2 = 6$
   $x = 4$
   $(4, 2)$

3. $-3(2x + y = 6)$
   $6x + 3y = 18$

   $-6x - 3y = -18$
   $0 = 0$
   True
   $\mathbb{R}$

4. $y = 3x - 5$
   $-1(y = 3x + 1)$

   $-y = -3x - 1$
   $0 = -6$
   False
   $\varnothing$

5. 3 $2x - 3y = -14$
   -2 $3x - 2y = -6$

If (x, y) is a solution to the system of equations above, what is the value of $x - y$?

A) -20
B) -8
C) -4
D) 8

$6x - 9y = -42$
$-6x + 4y = 12$
$-5y = -30$
$y = 6$
$2x - 18 = -14$
$2x = 4$
$x = 2$

$2 - 6 = -4$

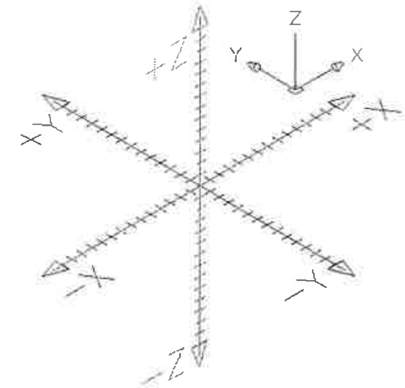
6. $kx - 3y = 4$
   $4x - 5y = 7$

In the system of equations above, k is a constant and x and y are variables. For what value of k will the system of equations have no solution?

A) $\frac{12}{5}$
B) $\frac{16}{7}$
C) $-\frac{16}{7}$
D) $-\frac{12}{5}$

Need False Statement
Look @ y's
$3 \rightarrow 5$
$3 \cdot \boxed{5/3} = 5$
So,
$k \cdot \frac{5}{3} = 4$
$k = \frac{12}{5}$

1.4 Solving Linear Systems



Target:

I can solve systems in three variables.

- I can visualize solutions of linear systems in three variables.
- I can solve linear systems in three variables algebraically.
- I can solve real-life problems using systems of equations in three variables.

The solution to a system of equations in three variables *x, y,* and *z* is called an **ordered triple** (x, y, z).

Solve the system of equations.

1. $$\begin{aligned} x + 2y + z &= 9 \\ 3y - z &= -1 \\ 3z &= 12 \end{aligned}$$

$z = 4$

$3y - 4 = -1$

$3y = 3$

$y = 1$

$x + 2 + 4 = 9$

$x = 3$

$(3, 1, 4)$

2. $$\begin{aligned} -9a - 12b + 6c &= 84 \\ 4a + 12b - 16c &= -124 \\ 2a + 3c &= 11 \end{aligned}$$

2 make 1&2 look like 3

1&2 | $-9a - 12b + 6c = 84$
$4a + 12b - 16c = -124$

2 $\quad -5a - 10c = -40$

5 $\quad 2a + 3c = 11$

$-10a - 20c = -80$

$10a + 15c = 55$

$-5c = -25$

$c = 5$

$2a + 15 = 11$

$2a = -4$

$a = -2$

$18 - 12b + 30 = 84$

$-12b = 36$

$b = -3$

$(-2, -3, 5)$

3. $$\begin{aligned} 5x + 3y + 2z &= 2 \\ 2x + y - z &= 5 \\ x + 4y + 2z &= 16 \end{aligned}$$

1+2 $10x + 6y + 4z = 4$

$-10x - 5y + 5z = -25$

$y + 9z = -21$

2+3 $2x + y - z = 5$

$-2x - 8y - 4z = -32$

$-7y - 5z = -27$

7 $y + 9z = -21$

$7y + 63z = -147$

$58z = -174$

$z = -3$

$y - 27 = -21$

$y = 6$

$5x + 18 - 6 = 2$

$5x = -10$

$x = -2$

$(-2, 6, -3)$

4. $$\begin{aligned} 2x + 4y - 5z &= 18 \\ -3x + 5y + 2z &= -27 \\ -5x + 3y - z &= -17 \end{aligned}$$

1+2 $6x + 12y - 15z = 54$

$-6x + 10y + 4z = -54$

$22y - 11z = -0$

2+3 $-15x + 25y + 10z = -135$

$15x - 9y + 3z = 51$

11 $16y + 13z = -84$

13 $22y - 11z = -0$

$176y + 143z = -924$

$286y - 143z = 0$

$462y = -924$

$y = -2$

$-44 - 11z = 0$

$-11z = 44$

$z = -4$

$2x - 8 + 20 = 18$

$2x = 6$

$x = 3$

$(3, -2, -4)$

5. $$\begin{aligned} a + 4c &= -7 \\ a - 3b &= -8 \\ b + c &= 1 \end{aligned}$$

1+2 $a + 4c = -7$

-1 $a - 3b = -8$

$-a + 3b = 8$

$3b + 4c = 1$

-3 $(b + c = 1)$

$-3b - 3c = -3$

$c = -2$

$b - 2 = 1$

$b = 3$

$a - 8 = -7$

$a = 1$

$(1, 3, -2)$

6. $$\begin{aligned} 3x - 2y + 4z &= 35 \\ -4x + y - 5z &= -36 \\ 5x - 3y + 3z &= 31 \end{aligned}$$

1+2 $12x - 8y + 16z = 140$

$-12x + 3y - 15z = -108$

$-5y + z = 32$

2+3 $-20x + 5y - 25z = -180$

$20x - 12y + 12z = 124$

$-7y - 13z = -56$

13 $(-5y + z = 32)$

$-65y + 13z = 416$

$-72y = 360$

$y = -5$

$35 - 13z = -56$

$-13z = -91$

$z = 7$

$3x + 10 + 28 = 35$

$3x = -3$

$x = -1$

$(-1, -5, 7)$