Chapter 1 Notes

Equations and Inequalities

Section 1.1 Linear Equations and Absolute Value Equations

Targets: I can solve linear equations in one variable.

I can solve an absolute value equation.

Solve.

1. $3x-5=7x-11$

2. $\frac{2}{3}x+\frac{3}{5}=\frac{7}{3}-\frac{3}{10}x$

3. $\frac{x}{4}-2=19-\frac{4}{5}x$

4. $3(2x+5)-2(x+7)=17$

5. $|2x-5|=21$

6. $|3x+2|=4$

7. $|4x-1|=0$

8. $|3x+5|=-2$

9. $3|5x-7|-16=8$

Application of Linear Equations

8. Movie theater ticket prices have been increasing steadily in recent years. An equation that models the average U.S. movie theater ticket price p, in dollars is given by $p = 0.293t + 6.590$ where t is the number of years after 2006. (This means that $t = 0$ corresponds to 2006.) Use this equation to predict the year in which the average U.S. movie theater ticket price will reach \$9.00.

9. Alicia is driving along a highway that passes through Centerville. Her distance d, in miles from Centerville is given by the equations $d = |135 - 60t|$ where t is the time in hours since the start of her trip and $0 \le t \le 5$. Determine when Alicia will be exactly 15 miles from Centerville.

Section 1.3 Quadratic Equations

Targets: I can solve a quadratic equation by factoring.

I can solve quadratic equations using square roots.

Solve the quadratic equation.

1. $x^2+2x-15=0$

2. $2x^2-5x=12$

3. $4x^2-2=7x$

4. $3x^2+12=0$

5. $\left(x+1\right)^2=48$

6. $9\left(x+8\right)^2+4=85$

7. $(x+1)(x+7)+8=0$

8. $2(x+7)^2-10=90$

Section 1.3 Quadratic Equations

Targets: I can solve quadratic equations using the quadratic formula.

I can solve quadratic equation application problems using any of the previous methods I have learned.

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solve.

1. $x^2 = 3x + 5$

2. $4x^2 - 4x + 3 = 0$

3. $4x^2 - 24x = -5$

4. $x^2 + 7 = 5x$

The Discriminant and the Solutions of a Quadratic Equation

The equation $ax^2 + bx + c = 0$, with real coefficients and $a \neq 0$, has as its discriminant $b^2 - 4ac$.

- If $b^2 - 4ac > 0$, then $ax^2 + bx + c = 0$ has *two distinct real solutions.*
- If $b^2 - 4ac = 0$, then $ax^2 + bx + c = 0$ has *one real solution*. The solution is a double solution.
- If $b^2 - 4ac < 0$, then $ax^2 + bx + c = 0$ has *two distinct nonreal complex solutions*. The solutions are conjugates of each other.

Use the discriminant to determine the number of real solutions.

5. $2x^2 - 5x + 1 = 0$

6. $3x^2 + 7 = -6x$

7. $x^2 + 6x + 9 = 0$

Applications of Quadratics

1. A television screen measures 60 inches diagonally, and its *aspect ratio* is 16 to 9. This means that the ratio of the width of the screen to the height of the screen is 16 to 9. Find the width and the height of the screen.

2. A company that makes rectangular solid candy bars that measure 5 inches by 2 inches by 0.5 inch. Due to difficult financial times, the company has decided to keep the price of the candy bar fixed and reduce the volume of the bar by 20%. What should the dimensions be for the new candy bar if the company keeps the height at 0.5 inch and makes the length of the candy bar 3 inches longer than the width?

Quadratic equations are often used to determine the height (position) of an object that has been dropped or projected. For instance, the *position equation*

$$s = -16t^2 + v_0 t + s_0$$

can be used to estimate the height of a projected object near the surface of Earth at a given time t in seconds. In this equation, v_0 is the ***initial velocity*** of the object in feet per second and s_0 is the ***initial height*** of the object in feet.

3. A ball is thrown **downward** with an initial velocity of 5 feet per second from the Golden Gate Bridge, which is 220 feet above the water. How long will it take for the ball to hit the water? Round your answer to the nearest hundredth of a second.

Section 1.4 Other Types of Equations

Targets: I can solve polynomial equations.

To solve polynomial equations you must:

1. Set the equation equal to zero.
2. Use any of the factoring methods we have learned in Chapter P to solve the equation.
3. Make sure you factor it ***completely***.

Solve.

1. $x^3 + 3x^2 - 4x - 12 = 0$
2. $x^3 - 36x = 0$
3. $x^4 - 81 = 0$
4. $x^4 + 3x^3 - 8x - 24 = 0$

Section 1.4 Other Types of Equations

Targets: I can solve equations that are in quadratic form.

To solve an equation that is in quadratic form you must:

1. Set the equation equal to zero.
2. Substitute an u in for the variable.
3. Solve the equation by factoring.
4. Substitute the value for u back in and solve.

The following table shows equations that are quadratic in form. Each equation is accompanied by an appropriate substitution that will enable it to be written in the form $au^2 + bu + c = 0$.

Equations that are in Quadratic Form

Original Equation	**Substitution**	$au^2 + bu + c = 0$ **Form**
$x^4 - 8x^2 + 15 = 0$	$u = x^2$	$u^2 - 8u + 15 = 0$
$x^6 + x^3 - 12 = 0$	$u = x^3$	$u^2 + u - 12 = 0$
$x^{1/2} - 9x^{1/4} + 20 = 0$	$u = x^{1/4}$	$u^2 - 9u + 20 = 0$
$2x^{2/3} + 7x^{1/3} - 4 = 0$	$u = x^{1/3}$	$2u^2 + 7u - 4 = 0$
$15x^{-2} + 7x^{-1} - 2 = 0$	$u = x^{-1}$	$15u^2 + 7u - 2 = 0$

Solve.

1. $x^4 + 5x^2 - 36 = 0$

2. $3x^{2/3} - 5x^{1/3} - 2 = 0$

3. $4x^{2/3} - 4x^{1/3} - 3 = 0$

4. $4x^4 - 25x^2 = -36$

Section 1.4 Other Types of Equations

Targets: I can solve rational equations.

To solve polynomial equations you should multiply the equation by the common denominator, then solve.

Solve.

1. $\frac{2x+1}{x+4}+3=\frac{-2}{x+4}$

3. $3x+\frac{4}{x-2}=\frac{-4x+12}{x-2}$

3. $\frac{x}{x-5}=1-\frac{5}{x-5}$

4. $\frac{2x}{x+1}+\frac{x+1}{x-4}=\frac{x-1}{x+1}$

5. $\dfrac{2}{x^2+8x}=\dfrac{1}{x+8}+\dfrac{1}{x^2+8x}$

6. $\dfrac{1}{x-6}+\dfrac{5}{x^2+2x-48}=\dfrac{1}{x^2+2x-48}$

Section 1.4 Other Types of Equations

Targets: I can solve radical equations.

To solve a radical equation you must isolate a radical on one side of the equation, then square both sides and solve.

Solve.

1. $2\sqrt{3x-2}=x+1$

2. $\sqrt{x+1}-\sqrt{2x-5}=1$

3. $\sqrt{x-2}=\sqrt{x}-2$

4. $\sqrt{5x-1}-\sqrt{3x-2}=1$

Section 1.4 Other Types of Equations

Targets: I can solve rational exponent equations.

To solve a rational exponent equation you must isolate the variable raised to the rational exponent, then multiply by the reciprocal exponent.

Solve.

1. $x^{2/3} = 16$
2. $x^{3/4} = 8$
3. $2x^{4/5} - 47 = 115$
4. $5x^{3/4} + 4 = 44$
5. $3x^{2/5} - 5 = 22$
6. $2x^{3/5} + 7 = 23$

1.5 Inequalities

Targets: I can solve an inequality accurately.

I can solve compound inequalities accurately.

Solve each of the following inequalities and graph the solution. Write your answer in set-builder and interval notation.

1. $2x+1<7$

2. $-3x-2\leq 10$

3. $-2x+8\geq 14$

Solve each compound inequality. Write your answer in set-builder notation.

Remember: Union ($\cup$) is for an ***OR*** statement.

Intersection ($\cap$) is for an ***AND*** statement.

4. $2x<10$ or $x+1>9$

6. $x+3>4$ and $2x+1>15$

7. $-4<2x+6\leq 12$

1.5 Absolute Value Inequalities

Targets: I can solve an absolute value inequality accurately.

Solve each of the following inequalities and graph the solution(s). Write each solution in interval notation.

Less than ($<, \leq$)= AND statements

Greater than ($>, \geq$) = OR statements

1. $|2-3x|<7$

2. $|4x-3|\geq 5$

3. $|2x+5|>3$

4. $|5-2x|<7$

5. $|8-2x|\geq 6$

1.5 Polynomial Inequalities and Rational Inequalities

Targets: I can solve a polynomial inequality accurately.

I can solve a rational inequality accurately.

To solve a polynomial inequality you must:

1. Set the equation set it equal to zero.
2. Solve for the critical values by factoring using any method learned in Chapter P.
3. To determine the intervals for which the original inequality is true, use the critical values with a sign diagram.
4. Write your answer in the notation listed in the directions.

Use the critical value method to solve each polynomial inequality. Write your answer in interval notation.

1. $x^3 + 3x^2 - 4x - 12 \geq 0$

2. $x^3 + x^2 - 16x - 16 < 0$

Rational Inequalities

To solve a rational inequality you must:

1. Set the equation set it equal to zero.
2. Write left side as one rational equation, then solve for the critical values.
3. To determine the intervals for which the original inequality is true, use the critical values with a sign diagram.
4. Write your answer in the notation listed in the directions.

Use the critical value method to solve each rational inequality. Write your answer in interval notation.

3. $\frac{3x+4}{x+1} \le 2$

4. $\frac{(x-3)(x-5)}{x+2} \ge 0$

5. $\frac{x^2+5x-24}{x+2} \le 0$

6. $\frac{x^2-10x+21}{x+4} > 0$