

CHAPTER 6 REVIEW EXERCISES

In Exercises 1 to 30, solve each system of equations.

$$1. \begin{cases} 5x - 3y = -4 \\ 2x + 5y = 11 \end{cases} \left(\frac{13}{31}, \frac{63}{31}\right) [6.1] \quad 2. \begin{cases} 6x + 3y = 4 \\ 5x - 2y = 8 \end{cases} \left(\frac{32}{27}, \frac{28}{27}\right) [6.1]$$

$$3. \begin{cases} 7x + 2y = 4 \\ y = \frac{2}{5}x + 3 \end{cases} \left(-\frac{10}{39}, \frac{113}{39}\right) [6.1] \quad 4. \begin{cases} 3x + y = -7 \\ y = -\frac{3}{5}x + 1 \end{cases} \left(-\frac{10}{3}, 3\right) [6.1]$$

$$5. \begin{cases} y = 2x - 5 \\ x = 4y - 1 \end{cases} (3, 1) [6.1] \quad 6. \begin{cases} y = 3x + 4 \\ x = 4y - 5 \end{cases} (-1, 1) [6.1]$$

$$7. \begin{cases} 6x + 9y = 15 \\ 10x + 15y = 25 \end{cases} \left(\frac{5-3c}{2}, c\right) [6.1] \quad 8. \begin{cases} 4x - 8y = 9 \\ 2x - 4y = 5 \end{cases} \text{No solution} [6.1]$$

$$9. \begin{cases} 2x - 3y + z = -9 \\ 2x + 5y - 2z = 18 \\ 4x - y + 3z = -4 \end{cases} \left(\frac{1}{2}, 3, -1\right) [6.2] \quad 10. \begin{cases} x - 3y + 5z = 1 \\ 2x + 3y - 5z = 15 \\ 3x + 6y + 5z = 15 \end{cases} \left(\frac{16}{3}, \frac{10}{27}, -\frac{29}{45}\right) [6.2]$$

$$11. \begin{cases} x + 3y - 5z = -12 \\ 3x - 2y + z = 7 \\ 5x + 4y - 9z = -17 \end{cases} \left(\frac{7c-3}{11}, \frac{16c-43}{11}, c\right) [6.2] \quad 12. \begin{cases} 2x - y + 2z = 5 \\ x + 3y - 3z = 2 \\ 5x - 9y + 8z = 13 \end{cases} \left(\frac{74}{31}, -\frac{1}{31}, \frac{3}{31}\right) [6.2]$$

$$13. \begin{cases} 3x + 4y - 6z = 10 \\ 2x + 2y - 3z = 6 \\ x - 6y + 9z = -4 \end{cases} \left(2, \frac{3c+2}{2}, c\right) [6.2] \quad 14. \begin{cases} x - 6y + 4z = 6 \\ 4x + 3y - 4z = 1 \\ 5x - 9y + 8z = 13 \end{cases} \left(1, -\frac{2}{3}, \frac{1}{4}\right) [6.2]$$

$$15. \begin{cases} 2x + 3y - 2z = 0 \\ 3x - y - 4z = 0 \\ 5x + 13y - 4z = 0 \end{cases} \left(\frac{14c}{11}, -\frac{2c}{11}, c\right) [6.2] \quad 16. \begin{cases} 3x - 5y + z = 0 \\ x + 4y - 3z = 0 \\ 2x + y - 2z = 0 \end{cases} (0, 0, 0) [6.2]$$

$$17. \begin{cases} 2x - y + 3z = 6 \\ x + 2y + 4z = 10 \\ -2c + \frac{22}{5} - c + \frac{14}{5} = c \end{cases} \left(-\frac{2c}{5}, \frac{22}{5} - c + \frac{14}{5}, c\right) [6.2] \quad 18. \begin{cases} 2x - 3y + z = 1 \\ 4x + 2y + 3z = 21 \\ \left(\frac{65-11c}{16}, \frac{19-c}{8}, c\right) \end{cases} [6.2]$$

$$19. \begin{cases} y = x^2 - 2x - 3 \\ y = 2x - 7 \end{cases} (2, -3) [6.3] \quad 20. \begin{cases} y = 2x^2 + x \\ y = 2x + 1 \end{cases} \left(-\frac{1}{2}, 0\right), (1, 3) [6.3]$$

$$21. \begin{cases} (x-1)^2 - y = 4 \\ (x+2)^2 - y = 9 \end{cases} \left(\frac{1}{3}, \frac{32}{9}\right) [6.3] \quad 22. \begin{cases} y = 4x^2 - 2x - 3 \\ y = 2x^2 + 3x - 6 \end{cases} \left(\frac{3}{2}, 3\right), (1, -1) [6.3]$$

$$23. \begin{cases} (x+1)^2 + (y-2)^2 = 4 \\ 2x + y = 4 \end{cases} \left(\frac{1}{5}, \frac{18}{5}\right), (1, 2) [6.3]$$

$$24. \begin{cases} (x-1)^2 + (y+1)^2 = 5 \\ y = 2x - 3 \end{cases} (0, -3), (2, 1) [6.3]$$

$$25. \begin{cases} (x-2)^2 + (y-2)^2 = 1 \\ (x-1)^2 + (y+2)^2 = 16 \end{cases} (1, 2), \left(\frac{49}{17}, \frac{26}{17}\right) [6.3]$$

$$26. \begin{cases} (x-1)^2 - (y+2)^2 = 4 \\ (x-2)^2 + (y+2)^2 = 9 \end{cases} (-1, -2), (4, -2 - \sqrt{5}), (4, -2 + \sqrt{5}) [6.3]$$

$$27. \begin{cases} x^2 - 3xy + y^2 = -1 \\ 3x^2 - 5xy - 2y^2 = 0 \end{cases} (2, 1), (-2, -1) [6.3]$$

$$28. \begin{cases} 2x^2 + 2xy - y^2 = -1 \\ 6x^2 + xy - y^2 = 0 \end{cases} (1, 3), (-1, -3), \left(\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{3}\right), \left(-\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}\right) [6.3]$$

$$29. \begin{cases} 2x^2 - 5xy + 2y^2 = 56 \\ 14x^2 - 3xy - 2y^2 = 56 \end{cases} (2, -3), (-2, 3) [6.3]$$

$$30. \begin{cases} 2x^2 + 7xy + 6y^2 = 1 \\ 6x^2 + 7xy + 2y^2 = 1 \end{cases} \left(\frac{\sqrt{15}}{15}, \frac{\sqrt{15}}{15}\right), \left(-\frac{\sqrt{15}}{15}, -\frac{\sqrt{15}}{15}\right), (1, -1), (-1, 1) [6.3]$$

In Exercises 31 to 36, find the partial fraction decomposition.

$$31. \frac{5x+1}{x^2+x-6} = \frac{5x+1}{(x-2)(x+3)} = \frac{11}{5(x+3)} + \frac{11}{5(x-2)} [6.4]$$

$$33. \frac{(x^2+1)(x+2)}{(6x-2)(-6)} = \frac{5(x^2+1)}{5(x+2)} + \frac{-6}{5(x+2)} [6.4]$$

$$35. \frac{11x^2-x-2}{x^3-x} = \frac{2}{x} + \frac{4}{x-1} + \frac{5}{x+1} [6.4]$$

$$37. \frac{x^3-2x^2+5x-3}{x^2+3x} = \frac{x-5-\frac{1}{x}+\frac{21}{x+3}}{x-5-\frac{1}{x}+\frac{21}{x+3}} [6.4]$$

$$32. \frac{3x-1}{(x-5)^2} = \frac{x-5}{3(x-5)^2} + \frac{14}{(x-5)^2} [6.4]$$

$$34. \frac{5x^2-10x+9}{(x-2)^2(x+1)} = \frac{7}{3(x-2)} + \frac{1}{(x-2)^2} + \frac{8}{3(x+1)} [6.4]$$

$$36. \frac{x^4+x^3+4x^2+x+3}{(x^2+1)^2(x+2)} = \frac{1}{1+x^2} + \frac{2}{x^2+1} [6.4]$$

$$38. \frac{x^4-2x^2+5x-1}{x^3-2x^2} = x+2-\frac{9}{4x}+\frac{1}{2x^2}+\frac{17}{4(x-2)} [6.4]$$

In Exercises 39 to 50, graph the solution set of each inequality.

$$39. 4x - 5y < 20 [6.5]$$

$$40. 2x + 7y \geq -14 [6.5]$$

$$41. y \geq 2x^2 - x - 1 [6.5]$$

$$42. y < x^2 - 5x - 6 [6.5]$$

$$43. (x-2)^2 + (y-1)^2 > 4 [6.5]$$

$$44. (x+3)^2 + (y+1)^2 \leq 9 [6.5]$$

$$45. \frac{(x-3)^2}{16} - \frac{(y+2)^2}{25} \leq 1 [6.5]$$

$$46. \frac{(x+1)^2}{9} - \frac{(y-3)^2}{4} < -1 [6.5]$$

$$47. (2x - y + 1)(x - 2y - 2) > 0 [6.5]$$

$$48. (2x - 3y - 6)(x + 2y - 4) < 0 [6.5]$$

$$49. x^2y^2 < 1 [6.5]$$

$$50. xy \geq 0 [6.5]$$

In Exercises 51 to 62, graph the solution set of each system of inequalities.

$$51. \begin{cases} 2x - 5y < 9 \\ 3x + 4y \geq 2 \end{cases} [6.5]$$

$$52. \begin{cases} 3x + y > 7 \\ 2x + 5y < 9 \end{cases} [6.5]$$

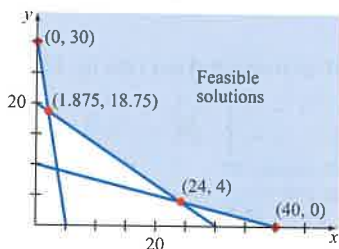
$$53. \begin{cases} 2x + 3y > 6 \\ 2x - y > -2 \\ x \leq 4 \end{cases} [6.5]$$

$$54. \begin{cases} 2x + 5y > 10 \\ x - y > -2 \\ x \leq 4 \end{cases} [6.5]$$

55.
$$\begin{cases} 2x + 3y \leq 18 \\ x + y \leq 7 \\ x \geq 0, y \geq 0 \end{cases} \quad [6.5]$$
56.
$$\begin{cases} 3x + 5y \geq 25 \\ 2x + 3y \geq 16 \\ x \geq 0, y \geq 0 \end{cases} \quad [6.5]$$
57.
$$\begin{cases} 3x + y \geq 6 \\ x + 4y \geq 14 \\ 2x + 3y \geq 16 \\ x \geq 0, y \geq 0 \end{cases} \quad [6.5]$$
58.
$$\begin{cases} 3x + 2y \geq 14 \\ x + y \geq 6 \\ 11x + 4y \leq 48 \\ x \geq 0, y \geq 0 \end{cases} \quad [6.5]$$
59.
$$\begin{cases} y < x^2 - x - 2 \\ y \geq 2x - 4 \end{cases} \quad [6.5]$$
60.
$$\begin{cases} y > 2x^2 + x - 1 \\ y > x + 3 \end{cases} \quad [6.5]$$
61.
$$\begin{cases} x^2 + y^2 - 2x + 4y > 4 \\ y < 2x^2 - 1 \end{cases} \quad [6.5]$$
62.
$$\begin{cases} x^2 - y^2 - 4x - 2y < -4 \\ x^2 + y^2 - 4x + 4y > 8 \end{cases} \quad [6.5]$$

63. Find the minimum value of the given objective function for the given set of feasible solutions. Also state where the objective function takes on its minimum value.

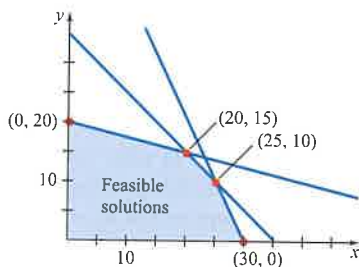
$$\text{Objective function: } C = \frac{1}{2}x + y$$



The minimum is 16 at (24, 4). [6.6]

64. Find the maximum value of the given objective function for the given set of feasible solutions. Also state where the objective function takes on its maximum value.

$$\text{Objective function: } C = 3.5x + 5y + 4$$



The maximum is 149 at (20, 15). [6.6]

In Exercises 65 to 70, solve the linear programming problem. In each problem, assume $x \geq 0$ and $y \geq 0$.

65. Objective function: $P = 2x + 2y$
- Constraints:
$$\begin{cases} x + 2y \leq 14 \\ 5x + 2y \leq 30 \end{cases}$$
- Maximize the objective function.
The maximum is 18 at (4, 5). [6.6]

66. Objective function: $P = 4x + 5y$
- Constraints:
$$\begin{cases} 2x + 3y \leq 24 \\ 4x + 3y \leq 36 \end{cases}$$

Maximize the objective function.

The maximum is 44 at (6, 4). [6.6]

67. Objective function: $P = 4x + y$
- Constraints:
$$\begin{cases} 5x + 2y \geq 16 \\ x + 2y \geq 8 \\ x \leq 20, y \leq 20 \end{cases}$$

Minimize the objective function.

The minimum is 8 at (0, 8). [6.6]

68. Objective function: $P = 2x + 7y$
- Constraints:
$$\begin{cases} 4x + 3y \geq 24 \\ 4x + 7y \geq 40 \\ x \leq 10, y \leq 10 \end{cases}$$

Minimize the objective function.

The minimum is 20 at (10, 0). [6.6]

69. Objective function: $P = 6x + 3y$
- Constraints:
$$\begin{cases} 5x + 2y \geq 20 \\ x + y \geq 7 \\ x + 2y \geq 10 \\ x \leq 15, y \leq 15 \end{cases}$$

Minimize the objective function.

The minimum is 27 at (2, 5). [6.6]

70. Objective function: $P = x + y$
- Constraints:
$$\begin{cases} x + 2y \leq 1000 \\ 3x + y \leq 900 \\ 2x + y \leq 1000 \end{cases}$$

Maximize the objective function.

The maximum is 580 at (160, 420). [6.6]

71. **Maximize Profit** An engine reconditioning company works on 4- and 6-cylinder engines. Each 4-cylinder engine requires 1 hour for cleaning, 5 hours for overhauling, and 3 hours for testing. Each 6-cylinder engine requires 1 hour for cleaning, 10 hours for overhauling, and 2 hours for testing. The cleaning station is available for at most 9 hours. The overhauling equipment is available for at most 80 hours, and the testing equipment is available for at most 24 hours. For each reconditioned 4-cylinder engine, the company makes a profit of \$150. A reconditioned 6-cylinder engine yields a profit of \$250. The company can sell all the reconditioned engines it produces. How many of each type should be produced to maximize profit? What is the maximum profit? Two 4-cylinder engines and seven 6-cylinder engines yields a maximum profit of \$2050. [6.6]

72. **Maximize Profit** A manufacturer makes two types of golf clubs: a starter model and a professional model. The starter model requires 4 hours in the assembly room and 1 hour in the finishing room. The professional model requires 6 hours in the assembly room and 1 hour in the finishing room. The total number of hours available in the assembly room is 108. There are 24 hours available

in the finishing room. The profit for each starter model is \$35, and the profit for each professional model is \$55. Assuming all the sets produced can be sold, find how many of each set should be manufactured to maximize profit. 0 starter sets and 18 pro sets [6.6]

In Exercises 73 to 79, solve each exercise by solving a system of equations.

73. Find an equation of the form $y = ax^2 + bx + c$ whose graph passes through the points (1, 0), (-1, 5), and (2, 3). $y = \frac{11}{6}x^2 - \frac{5}{2}x + \frac{2}{3}$ [6.2]
74. Find an equation of the circle that passes through the points (4, 2), (0, 1), and (3, -1). $x^2 + y^2 - \frac{47}{11}x - \frac{21}{11}y + \frac{10}{11} = 0$ [6.2]
75. Find an equation of the plane that passes through the points (2, 1, 2), (3, 1, 0), and (-2, -3, -2). Use the equation $z = ax + by + c$. $z = -2x + 3y + 3$ [6.2]

76. **Chemistry** How many liters of a 20% acid solution should be mixed with 10 liters of a 10% acid solution so that the result is a 16% acid solution? 15 L [6.1]

~~77.~~ **Uniform Motion** Flying with the wind, a small plane traveled 855 miles in 5 hours. Flying against the wind, the same plane traveled 575 miles in the same time. Find the rate of the wind and the rate of the plane in calm air.

Wind: 28 mph; plane: 143 mph [6.1]

78. **Commerce** A collection of 10 coins has a value of \$1.25. The collection consists of only nickels, dimes, and quarters. How many of each coin are in the collection? (Hint: There is more than one solution.)

4 nickels, 3 dimes, 3 quarters; 1 nickel, 7 dimes, 2 quarters [6.2]

79. Consider the ordered triple (a, b, c) . Find all real number values for a , b , and c so that the product of any two numbers equals the remaining number.

(0, 0, 0), (1, 1, 1), (1, -1, -1), (-1, -1, 1), (-1, 1, -1) [6.2]

Answer graphs to Exercises 9–11 and 13–14 are on page AA32.

CHAPTER 6 TEST

In Exercises 1 to 8, solve each system of equations.

1.
$$\begin{cases} 3x + 2y = -5 & (-3, 2) \text{ [6.1]} \\ 2x - 5y = -16 \end{cases} \quad 2. \begin{cases} x - \frac{1}{2}y = 3 & \left(\frac{6+c}{2}, c\right) \\ 2x - y = 6 \end{cases} \text{ [6.1]}$$

3.
$$\begin{cases} 2x - 3y + z = 9 \\ x - 5y + 3z = 5 \\ 3x - y - 2z = 8 \end{cases} \quad 4. \begin{cases} 3x - 2y + z = 2 \\ x + 2y - 2z = 1 \\ 4x - z = 3 \end{cases} \text{ [6.2]}$$

5.
$$\begin{cases} 2x - 3y + z = -1 \\ x + 5y - 2z = 5 \end{cases} \quad 6. \begin{cases} 4x + 2y + z = 0 \\ x - 3y - 2z = 0 \\ 3x + 5y + 3z = 0 \end{cases} \text{ [6.2]}$$

7.
$$\begin{cases} 8x = 3y + 10 \\ y = x^2 - 2x - 5 \end{cases} \quad 8. \begin{cases} x^2 + y^2 = 16 \\ y = 2x^2 - 4 \end{cases} \text{ [6.3]}$$

In Exercises 9 and 10, graph each inequality.

9. $x^2 + 4y^2 \geq 16$ [6.5] 10. $x + y^2 < 0$ [6.5]

In Exercises 11 to 14, graph each system of inequalities.

11.
$$\begin{cases} 2x - 5y \leq 16 \\ x + 3y \geq -3 \end{cases} \text{ [6.5]} \quad 12. \begin{cases} x^2 + y^2 > 9 \\ x^2 + y^2 < 4 \end{cases}$$

13.
$$\begin{cases} x + y \geq 8 \\ 2x + y \geq 11 \\ x \geq 0, y \geq 0 \end{cases} \text{ [6.5]} \quad 14. \begin{cases} 2x + 3y \leq 12 \\ x + y \leq 5 \\ 3x + 2y \leq 11 \\ x \geq 0, y \geq 0 \end{cases} \text{ [6.5]}$$

In Exercises 15 to 17, find the partial fraction decomposition.

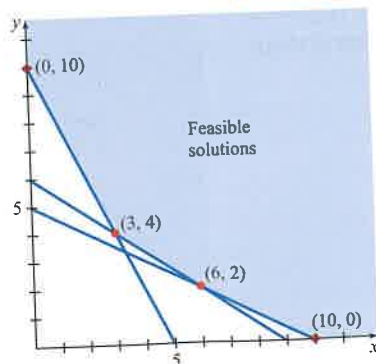
15.
$$\frac{3x - 5}{x^2 - 3x - 4} = \frac{1}{x - 4} + \frac{2}{x + 1} \text{ [6.4]}$$

16.
$$\frac{2x + 1}{x(x^2 + 1)} = \frac{1}{x} + \frac{-x + 2}{x^2 + 1} \text{ [6.4]}$$

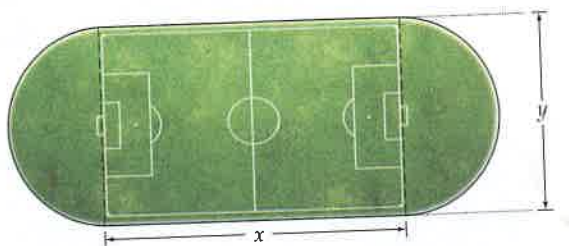
17.
$$\frac{x^3 + x - 2}{x^2 + x - 12} = x - 1 + \frac{4}{x - 3} + \frac{10}{x + 4} \text{ [6.4]}$$

18. Find the minimum value of the given objective function for the given set of feasible solutions. Also state where the objective function takes on its minimum value.

Objective function: $C = 5x + 4y$
The minimum is 31 at (3, 4). [6.6]



19. **Field Dimensions** A soccer stadium has a grass field in the shape of a rectangle with semicircles at the two ends. See the following figure. The perimeter of the entire grass field is approximately 554.16 meters, and the distance x is 20 meters longer than the distance y . Use a system of equations to find the x and y dimensions of the field. Round to the nearest meter. x : 120 m; y : 100 m [6.1]




20. **Parking Rates** A parking garage charges its customers a certain amount for the first hour and another amount for each additional half-hour or part of the half-hour. One day Nicole parked her car for 3 hours and 50 minutes. The parking fee was \$14.50. The next day she parked her car for 4 hours and 45 minutes. The parking fee was \$18.00. Determine the fee the parking garage charges for the first hour and the fee the garage charges for each additional half-hour or a portion of the half-hour. Fee for first hour: \$4.00; fee for each additional half-hour or portion of the half-hour: \$1.75 [6.1]

21. **Maximize Profit** A farmer has 160 acres available on which to plant oats and barley. It costs \$15 per acre for oat seed and \$13 per acre for barley seed. The labor cost is \$15 per acre for oats and \$20 per acre for barley. The farmer has \$2200 available to purchase seed and has set aside \$2600 for labor. The profit per acre for oats is \$120, and the profit per acre for barley is \$150. How many acres of oats and how many acres of barley should the farmer plant to maximize profit? $\frac{680}{7}$ acres of oats and $\frac{400}{7}$ acres of barley [6.6]

22. **Curve Fitting** Find an equation of the circle that passes through the points (3, 5), (-3, -3), and (4, 4). (Hint: Use $x^2 + y^2 + ax + by + c = 0$.) $x^2 + y^2 - 2y - 24 = 0$ [6.2]

CUMULATIVE REVIEW EXERCISES

- Find the slope of the line that passes through the points $\left(-\frac{1}{2}, 2\right)$ and $\left(4, -\frac{1}{3}\right)$. $-\frac{14}{27}$ [2.3]
- Find the range of $f(x) = -x^2 + 2x - 4$. $\{y \mid y \leq -3\}$ [2.4]
- Evaluate $3x^4 - 4x^3 + 2x^2 - x + 1$ for $x = -2$. 91 [P.1]
- Write $\log_6(x - 5) + 3 \log_6(2x)$ as a single logarithm with a coefficient of 1. $\log_6[8x^3(x - 5)]$ [4.4]
- Find the equation in standard form of the parabola that has the vertex $(4, 2)$, has an axis of symmetry parallel to the y -axis, and passes through the point $(-1, 1)$.
 $(x - 4)^2 = -25(y - 2)$ [5.1]
- Solve $\frac{1}{F} = \frac{1}{d_0} + \frac{1}{d_1}$ for d_0 . $d_0 = \frac{Fd_1}{d_1 - F}$ [1.2]
- Find the equation of the line that passes through $P_1(-4, 2)$ and $P_2(2, -1)$. $y = -\frac{1}{2}x$ [2.3]
- Let $f(x) = \frac{x^2 - 1}{x^4}$. Is f an even function, an odd function, or neither? Even [2.5]
- Solve: $\log x - \log(2x - 3) = 2$ $\frac{300}{199}$ [4.5]
- Find the equation in standard form of the hyperbola with vertices $(2, 2)$ and $(10, 2)$ and an eccentricity of 3.
 $\frac{(x - 6)^2}{16} - \frac{(y - 2)^2}{128} = 1$ [5.3]
- Given $g(x) = \frac{x - 2}{x}$, find $g\left(-\frac{1}{2}\right)$. 5 [2.2]
- Given $f(x) = x^2 - 1$ and $g(x) = x^2 - 4x - 2$, find $(f \cdot g)(-2)$. 30 [2.6]
- Evaluate: $\log_{0.25} 0.015625$ 3 [4.3]
-  Find the quadratic regression model for the data $\{(1, 1), (2, 3), (3, 10), (4, 17), (5, 26)\}$.
 $y = x^2 + 0.4x - 0.8$ [2.7]
- Find the polynomial of lowest degree that has zeros of -2 , $3i$, and $-3i$. $x^3 + 2x^2 + 9x + 18$ [3.4]
- Find the inverse function of $Q(r) = \frac{2}{1 - r}$. $Q^{-1}(r) = \frac{r - 2}{r}$ [4.1]
- Find the slant asymptote of the graph of
 $H(x) = \frac{2x^3 - x^2 - 2}{x^2 - x - 1}$ $y = 2x + 1$ [3.5]
- Given that $f(x) = 2^x$ and $g(x) = 3^{2x}$, find $g[f(1)]$. 81 [4.2]
- Sketch the graph of $F(x) = \frac{2^x - 2^{-x}}{3}$.
Answer on page AA32. [4.2]
- Compound Continuously** How long will it take \$2000 to double if it is invested at an annual interest rate of 6.5% compounded continuously? Round to the nearest year.
11 years [4.6]