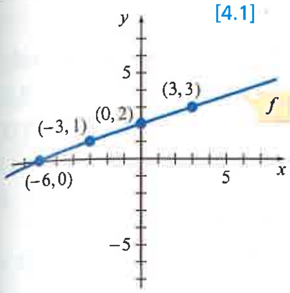
Answer graphs to Exercises 1–2, 7–10 and 25–38 are on pages AA20–AA21.

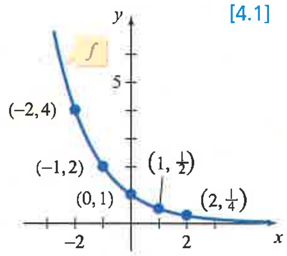
CHAPTER 4 REVIEW EXERCISES

In Exercises 1 and 2, draw the graph of the inverse of the given function.

1. [4.1]



2. [4.1]



In Exercises 3 to 6, use composition of functions to determine whether the given functions are inverse functions.

3. $F(x) = 2x - 5 \quad G(x) = \dfrac{x+5}{2}$ Yes [4.1]

4. $h(x) = \sqrt{x} \quad k(x) = x^2, \quad x \ge 0$ Yes [4.1]

5. $l(x) = \dfrac{x+3}{x} \quad m(x) = \dfrac{3}{x-1}$ Yes [4.1]

6. $p(x) = \dfrac{x-5}{2x} \quad q(x) = \dfrac{2x}{x-5}$ No [4.1]

In Exercises 7 to 10, find the inverse of the function. Sketch the graph of the function and its inverse on the same set of coordinate axes.

7. $f(x) = 3x - 4$
$f^{-1}(x) = \dfrac{x+4}{3}$ [4.1]

8. $g(x) = -2x + 3$
$g^{-1}(x) = -\dfrac{1}{2}x + \dfrac{3}{2}$ [4.1]

9. $h(x) = -\dfrac{1}{2}x - 2$
$h^{-1}(x) = -2x - 4$ [4.1]

10. $k(x) = \dfrac{1}{x}$
$k^{-1}(x) = k(x) = \dfrac{1}{x}$ [4.1]

In Exercises 11 and 12, find the inverse of the given function.

11. $f(x) = \dfrac{2x}{x-1}$, where the domain of f is $\{x \mid x > 1\}$
$f^{-1}(x) = \dfrac{x}{x-2}, \{x \mid x > 2\}$ [4.1]

12. $g(x) = x^2 + 2x$, where the domain of g is $\{x \mid x \ge -1\}$
$g^{-1}(x) = \sqrt{x+1} - 1, \{x \mid x \ge -1\}$ [4.1]

In Exercises 13 to 24, solve each equation. Do not use a calculator.

13. $\log_5 25 = x$
2 [4.3]

14. $\log_3 81 = x$
4 [4.3]

15. $\ln e^3 = x$
3 [4.3]

16. $\ln e^{\pi} = x$
π [4.3]

17. $3^{2x+7} = 27$
−2 [4.5]

18. $5^{x-4} = 625$
8 [4.5]

19. $3^x = \dfrac{1}{243}$
−5 [4.5]

20. $32(2^x) = 1024$
5 [4.5]

21. $\log x^2 = 2$
±10 [4.5]

22. $\dfrac{2}{3}\log|x| = 2$
$\pm 10^3$ [4.5]

23. $10^{\log 2x} = 14$
7 [4.5]

24. $e^{\ln x^2} = 64$
±8 [4.5]

In Exercises 25 to 36, sketch the graph of each function.

25. $f(x) = (2.5)^x$

26. $f(x) = \left(\dfrac{1}{4}\right)^x$

27. $f(x) = 3^{|x|}$

28. $f(x) = 4^{-|x|}$

29. $f(x) = 2^x - 3$

30. $f(x) = 2^{(x-3)}$

31. $f(x) = \log_5 x$

32. $f(x) = \log_{1/3} x$

33. $f(x) = \dfrac{1}{3}\log x$

34. $f(x) = 3\log x^{1/3}$

35. $f(x) = -\dfrac{1}{2}\ln x$

36. $f(x) = -\ln|x|$

In Exercises 37 and 38, use a graphing utility to graph each function.

37. $f(x) = \dfrac{4^x + 4^{-x}}{2}$

38. $f(x) = \dfrac{3^x - 3^{-x}}{2}$

In Exercises 39 to 42, change each logarithmic equation to its exponential form.

39. $\log 1000 = 3$
$10^3 = 1000$ [4.3]

40. $\log_7 2401 = 4$
$7^4 = 2401$ [4.3]

41. $\log_{\sqrt{2}} 4 = 4$
$(\sqrt{2})^4 = 4$ [4.3]

42. $\ln 1 = 0$
$e^0 = 1$ [4.3]

In Exercises 43 to 46, change each exponential equation to its logarithmic form.

43. $5^3 = 125$
$\log_5 125 = 3$ [4.3]

44. $2^{10} = 1024$
$\log_2 1024 = 10$ [4.3]

45. $10^0 = 1$
$\log_{10} 1 = 0$ [4.3]

46. $8^{1/2} = 2\sqrt{2}$
$\log_8 2\sqrt{2} = \dfrac{1}{2}$ [4.3]

In Exercises 47 to 50, expand the given logarithmic expression.

47. $\log_b\left(\dfrac{x\sqrt{y}}{z^3}\right)$
$\log_b x + \dfrac{1}{2}\log_b y - 3\log_b z$ [4.4]

48. $\log_5\left(\dfrac{25\sqrt{x}}{y^2}\right)$
$2 + \dfrac{1}{2}\log_5 x - 2\log_5 y$ [4.4]

49. $\ln xy^3$
$\ln x + 3\ln y$ [4.4]

50. $\ln \dfrac{\sqrt{xy}}{z^4}$
$\dfrac{1}{2}\ln x + \dfrac{1}{2}\ln y - 4\ln z$ [4.4]

In Exercises 51 to 54, write each logarithmic expression as a single logarithm with a coefficient of 1.

51. $2\log x + \dfrac{1}{3}\log(x+1)$
$\log(x^2\sqrt[3]{y+1})$ [4.4]

52. $5\log x - 2\log(x+5)$
$\log \dfrac{x^5}{(x+5)^2}$ [4.4]

53. $\dfrac{1}{2}\ln 2xy - 3\ln z$
$\ln \dfrac{\sqrt{2xy}}{z^3}$ [4.4]

54. $\ln x - (\ln y - \ln z)$
$\ln \dfrac{xz}{y}$ [4.4]

In Exercises 55 to 58, use the change-of-base formula and a calculator to approximate each logarithm accurate to six significant digits.

55. $\log_2 551$ 9.10591 [4.4]

56. $\log_{12} 43$ 1.51362 [4.4]

57. $\log_4 0.85$ −0.117233 [4.4]

58. $\log_8 0.3$ −0.578989 [4.4]

In Exercises 59 to 74, solve each equation for *x*. Give exact answers. Do not use a calculator.

59. $4^x = 30$ $\frac{\ln 30}{\ln 4}$ [4.5]

60. $5^{x+1} = 41$ $\frac{\log 41}{\log 5} - 1$ [4.5]

61. $\ln 3x - \ln(x - 1) = \ln 4$ 4 [4.5]

62. $\ln 3x + \ln 2 = 1$ $\frac{1}{6}e$ [4.5]

63. $e^{\ln(x+2)} = 6$ 4 [4.5]

64. $10^{\log(2x+1)} = 31$ 15 [4.5]

65. $\frac{4^x + 4^{-x}}{4^x - 4^{-x}} = 2$ $\frac{\ln 3}{2 \ln 4}$ [4.5]

66. $\frac{5^x + 5^{-x}}{2} = 8$ $\frac{\ln(8 \pm 3\sqrt{7})}{\ln 5}$ [4.5]

67. $\log(\log x) = 3$ 10^{1000} [4.5]

68. $\ln(\ln x) = 2$ $e^{(e^2)}$ [4.5]

69. $\log \sqrt{x - 5} = 3$ 1,000,005 [4.5]

70. $\log x + \log(x - 15) = 1$ $\frac{15 + \sqrt{265}}{2}$ [4.5]

71. $\log_4(\log_3 x) = 1$ 81 [4.5]

72. $\log_7(\log_5 x^2) = 0$ $\pm\sqrt{5}$ [4.5]

73. $\log_5 x^3 = \log_5 16x$ 4 [4.5]

74. $25 = 16^{\log_4 x}$ 5 [4.5]

75. **Earthquake Magnitude** Determine, to the nearest 0.1, the Richter scale magnitude of an earthquake with an intensity of $I = 63{,}280{,}000I_0$. 7.8 [4.4]

76. **Earthquake Magnitude** A seismogram has an amplitude of 18 millimeters, and the difference in time between the s-wave and the p-wave is 21 seconds. Find, to the nearest tenth, the Richter scale magnitude of the earthquake that produced the seismogram. 5.0 [4.4]

77. **Comparison of Earthquakes** An earthquake had a Richter scale magnitude of 7.2. Its aftershock had a Richter scale magnitude of 3.7. Compare the intensity of the earthquake with the intensity of the aftershock by finding, to the nearest unit, the ratio of the larger intensity to the smaller intensity.
3162 to 1 [4.4]

78. **Comparison of Earthquakes** On March 28, 1964, an earthquake of magnitude 9.2 on the Richter scale struck Prince William Sound, Alaska. On May 2, 2008, an earthquake of magnitude 6.6 on the Richter scale struck the Aleutian Islands in Alaska. Compare the intensity of the larger earthquake to the intensity of the smaller earthquake by finding the ratio of the larger intensity to the smaller intensity.
$10^{2.6} \approx 398$ times as great [4.4]

79. **Chemistry** Find the pH of milk of magnesia that has a hydronium-ion concentration of 3.16×10^{-11} mole per liter. Round to the nearest tenth. 10.5 [4.4]

80. **Chemistry** Find the hydronium-ion concentration of lemon juice that has a pH of 2.3. ≈0.00501 mole per liter [4.4]

81. **Compound Interest** Find the balance when \$3750 is invested at an annual interest rate of 2.5% for 5 years if the interest is compounded

   a. monthly \$4248.75

   b. daily \$4249.29

   c. continuously \$4249.31 [4.6]

82. **Compound Interest** Find the balance when \$48,000 is invested at an annual interest rate of 3.75% for 25 years if the interest is compounded

   a. semiannually \$121,512.88

   b. monthly \$122,393.25

   c. daily \$122,566.39 [4.6]

83. **Depreciation** The scrap value S of a product with an expected life span of n years is given by $S(n) = P(1 - r)^n$, where P is the original purchase price of the product and r is the annual rate of depreciation. A taxicab is purchased for \$12,400 and is expected to last 3 years. What is its scrap value if it depreciates at a rate of 29% per year? \$4,438.10 [4.6]

84. **Medicine** A skin wound heals according to the function given by $N(t) = N_0 e^{-0.12t}$, where N is the number of square centimeters of unhealed skin t days after the injury and N_0 is the number of square centimeters covered by the original wound.

   a. What percentage of the wound will be healed after 10 days? 69.9%

   b. How many days, to the nearest day, will it take for 50% of the wound to heal? 6 days

   c. How long, to the nearest day, will it take for 90% of the wound to heal? 19 days [4.6]

In Exercises 85 to 88, find the exponential growth or decay function $N(t) = N_0 e^{kt}$ that satisfies the given conditions.

85. $N(0) = 1, N(2) = 5$ $N(t) \approx e^{0.8047t}$ [4.6]

86. $N(0) = 2, N(3) = 11$ $N(t) \approx 2e^{0.5682t}$ [4.6]

87. $N(1) = 4, N(5) = 5$ $N(t) \approx 3.783e^{0.0558t}$ [4.6]

88. $N(-1) = 2, N(0) = 1$ $N(t) \approx e^{-0.6931t}$ [4.6]

89. **Population Growth**

   a. Find the exponential growth function for a city whose population was 25,200 in 2007 and 26,800 in 2008. Use $t = 0$ to represent 2007. $P(t) \approx 25{,}200e^{0.061557898t}$

   b. Use the growth function to predict, to the nearest hundred, the population of the city in 2014. 38,800 [4.6]

90. **Carbon Dating** Determine, to the nearest 10 years, the age of a bone if it now contains 96% of its original amount of carbon-14. The half-life of carbon-14 is 5730 years. 340 yrs [4.6]

91. **Movie Theater Admissions** The following table shows the average number of United States movie theater admissions, per week, for the selected years from 2000 to 2011.

Average Number of U.S. Movie Theater Admissions per Week

Year	Admissions per Week (millions)	Year	Admissions per Week (millions)
2002	30.8	2008	26.2
2004	28.5	2010	25.8
2006	26.8	2011	24.7

Source: The World Almanac and Book of Facts 2013.

a. Find an exponential regression function and a logarithmic regression function that model the average number of admissions per week, N, as a function of the year t. Use $t = 2$ to represent 2002, $t = 4$ to represent 2004, $t = 6$ to represent 2006, $t = 10$ to represent 2010, and $t = 11$ to represent 2011. Answers on page AA21.

b. Examine the coefficients of determination to determine which function provides a better fit to the data.

c. Use the regression function you selected in part **b** to predict the average number of admissions per week in 2015 ($t = 15$). Round to the nearest tenth of a million.

92. **Internet Use** The following table shows the number of people, in the United States, that used the Internet for the years from 2001 to 2010.

Number of U.S. Internet Users

Year	Number of Users, U (in millions)	Year	Number of Users, U (in millions)
2001	140	2006	206
2002	169	2007	227
2003	179	2008	226
2004	190	2009	240
2005	201	2010	245

Source: Time Almanac 2012.

a. Find an exponential regression function and a logarithmic regression function for the data. Use $t = 1$ to represent 2001, $t = 2$ to represent 2002, . . . , and $t = 10$ to represent 2010. Answers on page AA21.

b. Use the exponential function to predict the number of U.S. Internet users in 2014. Round to the nearest million.

c. Use the logarithmic function to predict the number of U.S. Internet users in 2014. Round to the nearest million.

93. **Logistic Growth** The population of coyotes in a national park satisfies the logistic model with $P_0 = 210$ in 2001, $c = 1400$, and $P(3) = 360$ (the population in 2004).

a. Determine the logistic model. $P(t) \approx \dfrac{1400}{1 + \frac{17}{3}e^{-0.22458t}}$

b. Use the model to predict, to the nearest 10, the coyote population in 2014. 1070 coyotes [4.6]

94. **Logistic Growth** Consider the logistic function

$$P(t) = \frac{128}{1 + 5e^{-0.27t}}$$

a. Find P_0. $21\frac{1}{3}$

b. What does $P(t)$ approach as $t \to \infty$? $P(t) \to 128$ [4.6]

Answer graph for Exercise 1 and answers for Exercises 7–8, 18–19, are on page AA21.

CHAPTER 4 TEST

1. Find the inverse of $f(x) = 2x - 3$. Graph f and f^{-1} on the same coordinate axes. $f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$ [4.1]

2. Find the inverse of $f(x) = \dfrac{x}{4x - 8}$, where the domain of f is $\{x \mid x > 2\}$. State the domain and the range of f^{-1}.
Answer is below Exercise 3.

3. a. Write $\log_b(5x - 3) = c$ in exponential form. $b^c = 5x - 3$

b. Write $3^{x/2} = y$ in logarithmic form. $\log_3 y = \frac{x}{2}$ [4.3]

2. $f^{-1}(x) = \dfrac{8x}{4x - 1}$; domain of f^{-1}: $\left\{x \mid x > \frac{1}{4}\right\}$; range of f^{-1}: $\{y \mid y > 2\}$ [4.1]

4. Expand $\log_b \dfrac{z^2}{y^3\sqrt{x}}$. $2\log_b z - 3\log_b y - \frac{1}{2}\log_b x$ [4.4]

5. Write $\log(2x + 3) - 3\log(x - 2)$ as a single logarithm with a coefficient of 1. $\log \dfrac{2x + 3}{(x - 2)^3}$ [4.4]

6. Use the change-of-base formula and a calculator to approximate $\log_4 12$. Round your result to the nearest ten-thousandth. 1.7925 [4.4]

7. Graph: $f(x) = 3^{-x/2}$ [4.2]

8. Graph: $f(x) = \ln(x + 1)$ [4.3]

9. Solve $5^x = 22$. Round your solution to the nearest ten-thousandth. 1.9206 [4.5]

10. Find the *exact* solution of $4^{5-x} = 7^x$. $\frac{5 \ln 4}{\ln 28}$ [4.5]

11. Solve: $\log(x + 99) - \log(3x - 2) = 2$ 1 [4.5]

12. Solve: $\ln(2 - x) + \ln(5 - x) = \ln(37 - x)$ −3 [4.5]

13. **Compound Interest** Find the balance on \$2800 invested at an annual interest rate of 2.25% for 10 years provided the interest is compounded

   a. monthly \$3505.76   b. daily \$3506.48   c. continuously \$3506.50 [4.6]

14. **Compound Interest** Find the time required for money invested at an annual interest rate of 3.75% to double in value if the investment is compounded daily. Round to the nearest hundredth of a year. 18.48 yr [4.6]

15. **Earthquake Magnitude**

   a. What, to the nearest tenth, will an earthquake measure on the Richter scale if it has an intensity of $I = 42{,}304{,}000 I_0$? 7.6

   b. Compare the intensity of an earthquake that measures 6.3 on the Richter scale with the intensity of an earthquake that measures 4.5 on the Richter scale by finding the ratio of the larger intensity to the smaller intensity. Round to the nearest natural number. 63 to 1 [4.4]

16. **Exponential Decay** From 2000 to 2010, the population of Detroit, Michigan, declined exponentially. The population of Detroit was 951,270 in 2000 and 713,777 in 2010. (*Source: The World Almanac and Book of Facts 2012.*)

   a. Find the exponential decay function that models the population of Detroit during this 10-year period. $N(t) \approx 951{,}270e^{-0.0287227345t}$

   b. Use the exponential decay function to predict Detroit's population in 2015. Use $t = 0$ to represent 2000, $t = 10$ to represent 2010, etc. Round the 2015 population prediction to the nearest thousand. 618,000 [4.6]

17. Determine, to the nearest 10 years, the age of a bone if it now contains 92% of its original amount of carbon-14. The half-life of carbon-14 is 5730 years. 690 yrs [4.6]

18. **Value of a Diamond** A diamond merchant has determined the values of several white diamonds that have different weights, measured in carats, but are similar in quality.

Value of a Diamond

Weight (in carats)	Value (in dollars)	Weight (in carats)	Value (in dollars)
0.5	3900	2.5	18,200
1.0	5100	3.0	27,400
1.5	8700	3.5	41,100
2.0	12,300	4.0	65,400

   a. Find an exponential regression function that models the value, y, of the diamonds as a function of their weight, x.

   b. Use the exponential regression function to estimate the value of a 1.75-carat diamond of similar quality. Round to the nearest \$100.

   c. Use the exponential regression function to estimate the value of a 4.25-carat diamond of similar quality. Round to the nearest \$100.

19. **Women's Javelin Throw** The following table shows the progression of the world record distances for the women's javelin throw from 1999 to 2012. (*Note:* No new world record distances were set during the years from 2009 to 2012.)

World Record Progression in the Women's Javelin Throw

Year	Distance *d* (m)
1999	67.09
2000	68.22
2000	69.48
2001	71.54
2005	71.70
2008	72.28

Source: www.nemethjavelins.hu/world-record-progression-women.

   a. Find a logarithmic model for the data. Use $t = 1$ to represent the year 1999, $t = 2$ for 2000, $t = 3$ for 2001, $t =$ [illegible] for 2005, and $t = 10$ for 2008.

   b. Assume that a new world record distance will be established in 2015. Use the model from **a** to predict the women's world record javelin throw distance for 2015, represented by $t = 17$. Round to the nearest hundredth of a meter.

20. **Population Growth** The population of raccoons in a state park satisfies a logistic growth model with $P_0 = 160$ in 2011 and $P(1) = 190$ in 2012. A park ranger has estimated the carrying capacity of the park to be 1100 raccoons.

   a. Determine the logistic growth model for the raccoon population where t is the number of years after 2011. $P(t) \approx \frac{1100}{1 + 5.875e^{-0.20429t}}$

   b. Use the logistic model from **a** to predict the raccoon population in 2018. About 457 raccoons [4.6]

CUMULATIVE REVIEW EXERCISES

1. Solve $|x - 4| \le 2$. Write the solution set using interval notation. [2, 6] [1.5]

2. Solve $\frac{x}{2x - 6} \ge 1$. Write the solution set using set-builder notation. $\{x \mid 3 < x \le 6\}$ [1.5]

3. Find, to the nearest tenth, the distance between the points (5, 2) and (11, 7). 7.8 [2.1]

4. **Height of a Ball** The height, in feet, of a ball released with an initial upward velocity of 44 feet per second at an initial height of 8 feet is given by $h(t) = -16t^2 + 44t + 8$, where t is the time in seconds after the ball is released. Find the maximum height the ball will reach. 38.25 ft [2.4]

5. Given $f(x) = 2x + 1$ and $g(x) = x^2 - 5$, find $(g \circ f)(x)$. $4x^2 + 4x - 4$ [2.6]

6. Find the inverse of $f(x) = 3x - 5$. $f^{-1}(x) = \frac{1}{3}x + \frac{5}{3}$ [4.1]

7. **Safe Load** The load that a horizontal beam can safely support varies jointly as the width and the square of the depth of the beam. It has been determined that a beam with a width of 4 inches and a depth of 8 inches can safely support a load of 1500 pounds. How many pounds can a beam of the same material and the same length safely support if it has a width of 6 inches and a depth of 10 inches? Round to the nearest hundred pounds. 3500 lb [1.6]

8. Use Descartes' Rule of Signs to determine the number of possible positive and the number of possible negative real zeros of $P(x) = x^4 - 3x^3 + x^2 - x - 6$.
Three or one positive real zeros; one negative real zero [3.3]

9. Find the zeros of $P(x) = x^4 - 5x^3 + x^2 + 15x - 12$.
$1, 4, -\sqrt{3}, \sqrt{3}$ [3.3]

10. Find a polynomial function of lowest degree that has 2, $1 - i$, and $1 + i$ as zeros. $P(x) = x^3 - 4x^2 + 6x - 4$ [3.4]

11. Find the equations of the vertical and horizontal asymptotes of the graph of $r(x) = \frac{3x - 5}{x - 4}$.
Vertical asymptote: $x = 4$; horizontal asymptote: $y = 3$ [3.5]

12. Determine the domain and the range of the rational function $R(x) = \frac{4}{x^2 + 1}$.
Domain: all real numbers; range: $\{y \mid 0 < y \le 4\}$ [3.5]

13. State whether $f(x) = 0.4^x$ is an increasing function or a decreasing function. Decreasing function [4.2]

14. Write $\log_4 x = y$ in exponential form. $4^y = x$ [4.3]

15. Write $5^3 = 125$ in logarithmic form. $\log_5 125 = 3$ [4.3]

16. Find, to the nearest tenth, the Richter scale magnitude of an earthquake with an intensity of $I = 11{,}650{,}600I_0$. 7.1 [4.4]

17. Solve $2e^x = 15$. Round to the nearest ten-thousandth.
2.0149 [4.5]

18. Find the age of a bone if it now has 94% of the carbon-14 it had at time $t = 0$. The half-life of carbon-14 is 5730 years. Round to the nearest 10 years. 510 years old [4.6]

19. Solve $\frac{e^x - e^{-x}}{2} = 12$ for x. Round to the nearest ten-thousandth. 3.1798 [4.5]

20. **Population Growth** The wolf population in a national park satisfies a logistic growth model with $P_0 = 160$ in 2008 and $P(3) = 205$ (the population in 2011). It has been determined that the maximum population the park can support is 450 wolves. $P(t) \approx \frac{450}{1 + 1.8125e^{-0.13882t}}$
   a. Determine the logistic growth model for the data.
   b. Use the logistic growth model to predict, to the nearest 10, the wolf population in 2018. 310 wolves [4.6]