College Prep Algebra

Chapter P Notes

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Section P.1: The Real Number System

Targets: I can answer questions in set notation accurately.

I can answer questions in interval notation accurately.

The Real Number System

Natural Numbers - $\{1, 2, 3, 4, 5, 6, \ldots\}$ Counting #s

Whole Numbers – All the natural numbers including 0; $\{0, 1, 2, 3, 4, 5, 6, \ldots\}$

Integers - $\{\ldots, -1, -2, -3, 0, 1, 2, 3 \ldots\}$

Rational Numbers = $\left\{\frac{p}{q}, \text{where } p \text{ and } q \text{ are intergers and } q \neq 0\right\}$

- Rational numbers can be written as a fraction or a decimal. If written as a decimal it will be either a terminating decimal such as 0.65 or a repeating decimal such as 0.218181818...

Irrational Numbers – numbers that *cannot* be expressed as terminating or repeating decimals.

Properties of Real Numbers

Let *a*, *b*, and *c* be real numbers.

	Addition Properties	Multiplication Properties
Closure	$a+b$ is a unique real number.	ab is a unique real number.
Commutative Change Order	$a+b=b+a$	$ab=ba$
Associative	$(a+b)+c=a+(b+c)$	$(ab)c=a(cb)$
Identity	There exists a unique real number 0 such that $a+0=0+a=a$.	There exists a unique real number 1 such that $a \bullet 1 = 1 \bullet a = a$.
Inverse	For each real number *a*, there is a unique real number $-a$ such that $a+(-a)=(-a)+a=0$.	For each *nonzero* real number *a*, there is a unique real number $\frac{1}{a}$ such that $a\left(\frac{1}{a}\right)=\left(\frac{1}{a}\right)a=1$.
Distributive	$a(b+c)=ab+ac$	

Properties of Equality

Let *a*, *b*, and *c* be real numbers.

Reflexive	$a=a$ ex: $a+b=a+b$
Symmetric Switch Sides	If $a=b$, then $b=a$.
Transitive	If $a=b$ and $b=c$, the $a=c$.
Substitution: If $a=b$ and $c=b$, then $a=c$.	If $a=b$, then a may be replaced by b in any expression that involves a.

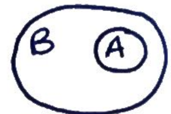
Set Notation

Element ($\in$) – every member of a set is called an element. Ex: If $C=\{1,5,7\}$, then the elements of C are 1, 5, and 7.

The notation $\boxed{1 \in C}$ is read "1 is an element of C."

Subset ($\subseteq$) – Set A is a subset of B if every $\in$ in A is also an $\in$ of B. The notation $\boxed{A \subseteq B}$ is read "A is a subset of B."

Ex: $A=\{1,2,3\}$ and $B=\{\text{natural numbers}\}$



Empty Set or **Null Set** ($\varnothing$) is a set that contains no elements. Ex: The set of people who have run a 2-minute mile is the empty set.

Finite Set – all $\in$ of the set can be listed. Ex: The set of natural numbers less than 6 is $\{1,2,3,4,5\}$.

Infinite Set – All the elements of the set cannot be listed. Ex: The set of all integers.

☆ **Set-builder Notation** – The set of real numbers greater than 2 is written; $\{x \mid x>2, x \in \text{real numbers}\}$ and is read "the set of x such that x is greater than 2 and *x* is an element of real numbers.

Shortened form: $\{x \mid x>2\}$ for this we assume that *x* is a real number.

List the four smallest elements of each set. ☆ Always try −1, 0, 1

1. $\{n^3 \mid n \in \text{natural numbers}\}$ 1, 2, 3, 4 — 1, 8, 27, 64 — $1^3, 2^3, 3^3, 4^3$
2. $\{y \mid \boxed{y=x^2-1}, x \in \text{integers}\}$ — −1, 0, 3, 8

Union and Intersection of Sets

Union ($\cup$) OR – Written $A \cup B$, is the set of all elements that belong to either A or B. In set-builder notation, this is written $A \cup B=\{x \mid x \in A \text{ or } x \in B\}$.

Intersection ($\cap$) AND - Written $A \cap B$, is the set of all elements that are common to both A and B. In set-builder notation, this is written $A \cap B=\{x \mid x \in A \text{ and } x \in B\}$.

Examples:

Find the intersection or union given $A=\{0,2,4,6,10,12\}$, $B=\{0,3,6,12,15\}$, $C=\{1,2,3,4,5,6,7\}$, and $D=\{18,20,22\}$.

3. $A \cup C$ $\{0,1,2,3,4,5,6,7,10,12\}$

4. $B \cap D$ $\varnothing$

5. $A \cap (B \cup C)$ $\{0,2,4,6,12\}$

$(B \cup C)=\{0,1,2,3,4,5,6,7,12,15\}$

6. $B \cup (A \cap C)$ $\{0,2,3,4,6,12,15\}$

$A \cap C=\{2,4,6\}$

Interval Notation

$<$ or $>$ we will now us (or) instead of an open circle. $\leq$ or $\geq$ we will not us $[or]$ instead of a closed circle.

- (a,b) represents all real numbers between *a* and *b*. This is an **open interval**. In set-builder notation, we write $\{x|a<x<b\}$.
- $[a,b]$ represents all real number between *a* and *b*, including *a* and *b*. This is a **closed interval**. In set-builder notation, we write $\{x|a\leq x\leq b\}$.
- $(a,b]$ represents all real numbers between *a* and *b*, not including *a* but including *b*. This is a **half-open interval**. In set-builder notation, we write $\{x|a<x\leq b\}$.
- $[a,b)$ represents all real numbers between *a* and *b*, including *a* but not including *b*. This a **half-open interval**. In set-builder notation, we write $\{x|a\leq x<b\}$.

$(-\infty,a)$ represents all real numbers less than *a*.

(b,∞) represents all real numbers greater than *b*.

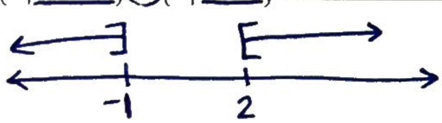
$(-\infty,a]$ represents all real numbers less than or equal to *a*.

$[b,\infty)$ represents all real numbers greater than or equal to *b*.

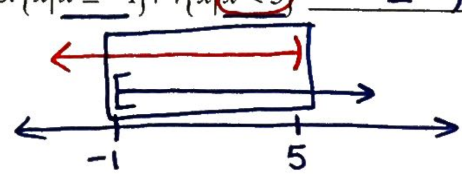
Graph Intervals

Graph the following. Write 7 and 8 using interval notation . Write 9 and 10 using set-builder notation.

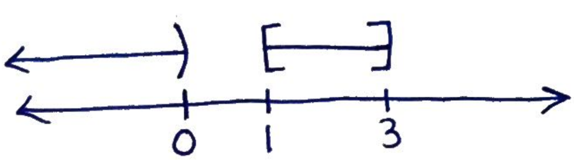
7. $\{x|\underline{x\leq -1}\}\cup\{x|\underline{x\geq 2}\}$ $(-\infty,-1]\cup[2,\infty)$



8. $\{x|\underline{x\geq -1}\}\cap\{x|x<5\}$ $[-1,5)$



9. $(-\infty,0)\cup[1,3]$ $\{x|x<0\}\cup\{x|1\leq x\leq 3\}$



10. $[-1,3]\cap(1,5)$ $\{x|1<x\leq 3\}$