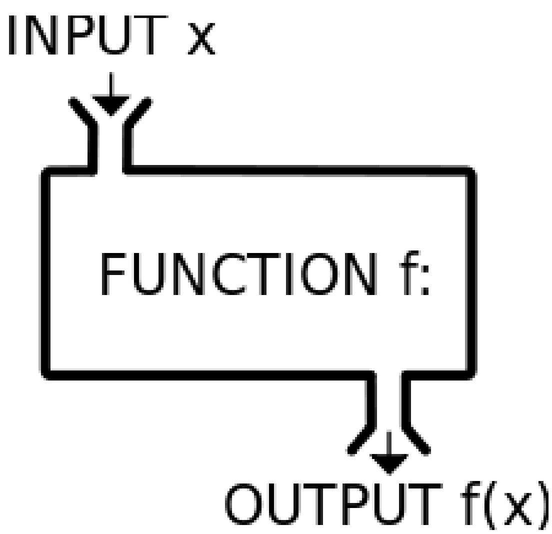
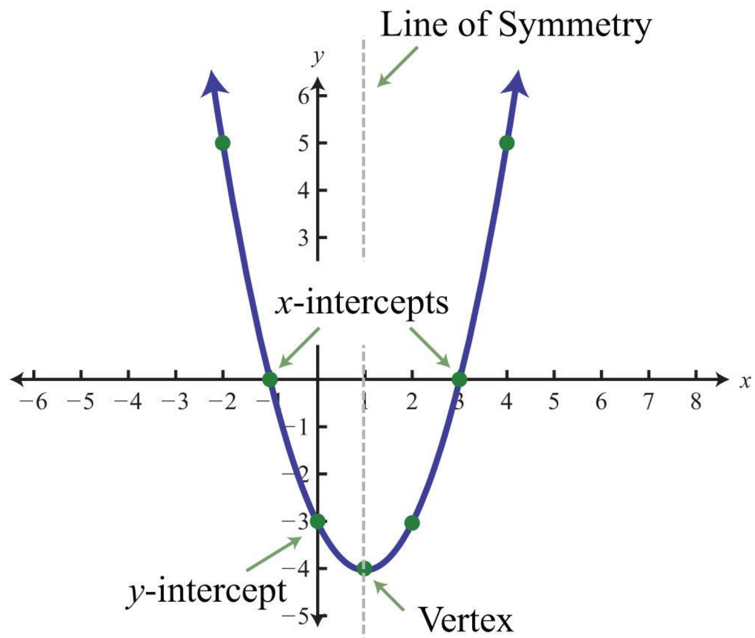
Chapter 2 Notes





Functions and Graphs

Section 2.2: Functions and Graphs

Targets: I can determine the domain of a function.
I can determine if the relation defines y as a function of x.
I can state whether the graph is a function using the vertical line test.

Function – set of ordered pairs in which no two ordered pairs have the same first coordinate ($x-coordinates$) and different second coordinate ($y-coordinates$). *For every input there is **ONLY** one output.*

Domain – set of all possible inputs ($x-coordinates$) of a function. Represents the ***independent variable***.

Range – set of all possible outputs ($y-coordinates$) of a function. Represents the ***dependent variable***.

State whether the relation defines y as a function of x.

1. $\{(2,3),\ (4,1),\ (4,5)\}$
2. $\{(0,-2),\ (4,2),\ (7,5),\ (4,2)\}$
3. $3x+y=1$
4. $-4x^2+y^2=9$

Evaluate functions

5. Let $f(x)=x^2-1$, and evaluate.

a. $f(-5)$ b. $f(3b)$ c. $f(b)$ d. $f(a+3)$ e. $f(a)+f(3)$

Determine the Domain of a Function

Determine the domain of each function.

6. $G(t)=\dfrac{t-2}{t^2-3t-10}$
7. $f(x)=\sqrt{4-x^2}$
8. $f(x)=\dfrac{x+3}{x-2}$
9. $g(x)=\sqrt{x-4}$

10. $A(s) = s^2$, where $A(s)$ is the area of a square whose side is s units.

Determine a Domain Value Given a Range Value

11. Find the values of a in the domain of $f(x) = x^2 - x - 4$ for which $f(a) = 2$.

12. Find the values of a in the domain of $f(x) = x^2 - 1$ for which $f(a) = 3$.

The solutions to a quadratic function can be referred to as **ROOTS, ZEROS,** or **X-INTERCEPTS** of the function.
Find the zeros of the function.

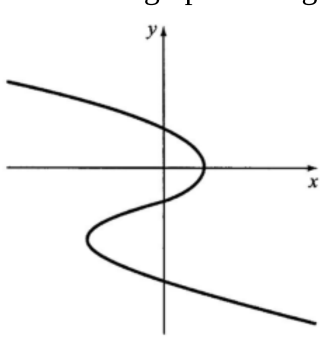
13. $f(x) = x^2 - 2x - 3$

14. $f(x) = 2x^2 - 3x - 5$

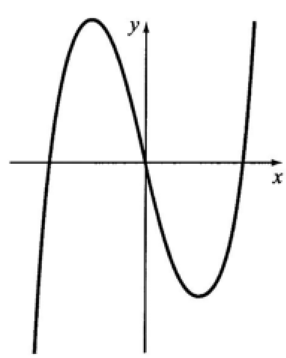
Applying Vertical Line Test

State whether the graph is the graph of a function.

15.



16.



Section 2.3: Linear Functions

Targets: I can graph a linear function in slope-intercept form and standard form.
I can write the equation of a line for all conditions given.

Slope-Intercept Form: The graph of $f(x) = mx + b$ is a linear function with slope m and *y*-intercept $(0, b)$.

Slope: $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Examples:

a. The graph of $f(x) = -2x + 3$ is a line with slope -2 and *y*-intercept $(0, 3)$.

b. The graph of $f(x) = \dfrac{2}{3}x - 4$ is a line with slope $\dfrac{2}{3}$ and *y*-intercept $(0, -4)$.

Find the slope of the line passing through P_1 and P_2.

1. $P_1(1, 2),\ P_2(3, 6)$

2. $P_1(-3, 4),\ P_2(1, -2)$

Horizontal and Vertical Lines:

The graph of $x = a$ is a vertical line through $(a, 0)$. The slope of the line is *undefined*.

The graph of $y = b$ is a horizontal line through $(0, b)$. The slope of the line is *zero*.

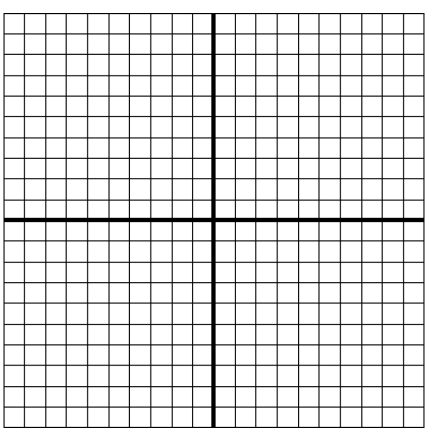
Examples:

c. The graph of $x = -2$ is a vertical line through $(-2, 0)$. The slope is *undefined*.

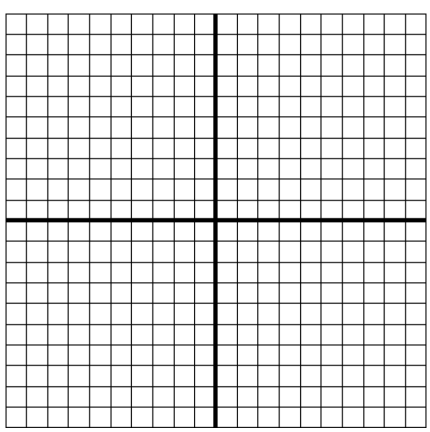
d. The graph of $y = 3$ is a horizontal line through $(0, 3)$. The slope of the line is *zero*.

Graph the linear function given in slope-intercept form.

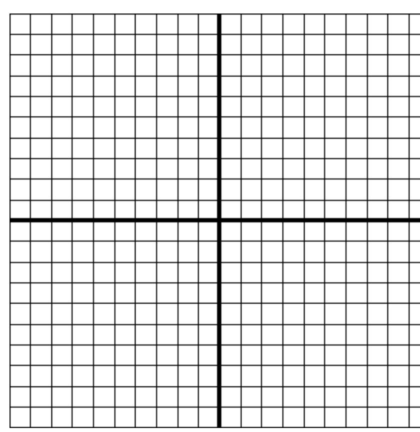
3. $y = 2x - 1$



4. $x = -2$



5. $y = 3$



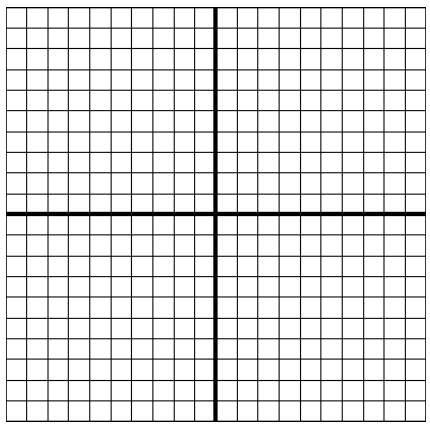
Standard Form (General Form): $Ax + By = C$

When graph a line given in standard form:

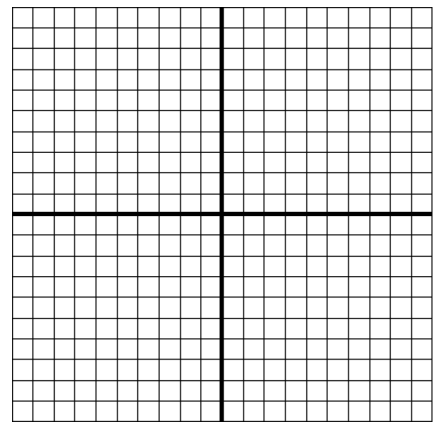
1. Put into slope-intercept form by solving for *y*.
2. Find the *x* and *y-intercepts*.
   a. To find the *x-intercept* plug 0 into the equation for *y* and solve for *x*.
   b. To find the *y-intercept* plug 0 into the equation for *x* and solve for *y*.

Graph the linear equation given in general form (standard form).

6. $3x + 2y = 6$



7. $3x + 2y = 4$



Point-Slope Form: $y - y_1 = m(x - x_1)$

Write the equation of a line given the slope and a point.

8. Through $(-1, 4)$, slope -3

9. *y*-intercept $(0, -7)$, slope $\frac{2}{3}$

Write the equation of a line given two points.

10. $P_1(-2, 4)$, $P_2(2, -1)$

11. $P_1(-6, -8)$, $P_2(9, 2)$

Section 2.3: Parallel and Perpendicular Liners

Targets: I can write the equation of parallel and perpendicular lines.
I can solve applications of linear functions.

Parallel Lines: Two nonintersecting lines in a plane are ***parallel***.

- Two lines that are parallel have the ***same slope***, but different ***y-intercepts***.

Perpendicular Lines: Two lines are ***perpendicular*** if and only if they intersect and form adjacent angles, each of which measures $90°$.

- Two lines that are perpendicular have ***slopes*** that are ***negative (opposite) reciprocals***.

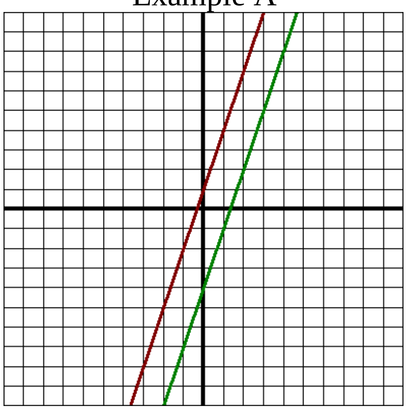
Examples:

a. If $f_1(x) = 3x + 1$ and $f_2(x) = 3x - 4$, then the slopes are equal: $m_1 = m_2 = 3$. The lines are parallel.

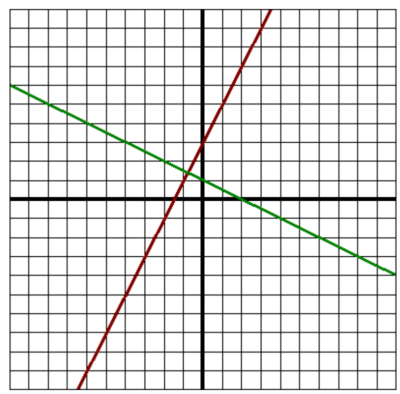
b. If $g_1(x) = 2x + 3$ and $g_1(x) = -\frac{1}{2}x + 1$, then $m_1 = 2$ and $m_2 = -\frac{1}{2}$ which are ***negative reciprocals***.
The lines are perpendicular.

Graphs for the examples.

Example A



Example B



Write the equation of a line in slope-intercept form, that satisfies the given conditions.

1. Find the equation of the line whose graph is parallel to the graph of $2x - 3y = 7$ and passes through the point $P(-6, -2)$.

2. Find the equation of a line whose graph is perpendicular to the graph of $y = \frac{4}{3}x - 2$ and passes through the point $P(-8, 3)$.

Applications of Linear Functions

3. The data in the table show the relationship between the swing speed of a golfer using a driver and the distance the ball will carry.

Swing speed, *s* (mph)	109	86	87	108	95	81	101	90	114	89
Carry distance, *d* (yards)	283	189	191	275	233	178	238	215	288	211

a. Find a linear function that models the distance *d*, in yards, a golf ball will carry in terms of the swing speed *s*, in miles per hour, of the golfer. Round the slope and *y*-intercept to the nearest tenth.

b. Using the model you found in part ***a***, how far would a golf ball carry that was hit with a swing speed of 120 mph?

4. A manufacturer finds that the costs incurred in the manufacture and sale of a particular type of calculator are $180,000 plus $27 per calculator.

a. Determine the profit function *P*, given that *x* calculators are manufactured and sold at $59 each.

b. Determine the break-even point.

If a manufacture produces *x* units of a product that sells for *p* dollars per unit, that the **cost function** *C*, the **revenue function** *R*, and the **profit function** *P* are defined as follows.

$C(x) =$ cost of producing and selling *x* units

$R(x) = xp =$ revenue from the sale of *x* units at *p* dollars each

$P(x) =$ profit from selling *x* units

Because profit equals the revenue less the cost, we have $P(x) = R(x) - C(x)$.

The value of *x* for which $R(x) = C(x)$ is called the **break-even point**. At the break-even point, $P(x) = 0$.

Section 2.4: Quadratic Functions

Targets: I can graph a quadratic function.
I can find the vertex, axis of symmetry, minimum or maximum, slope, and range of the quadratic function.
I can write a quadratic function in vertex form.

<u>Definition of Quadratic Function</u>

A **quadratic function** of x is a function that can be represented by an equation of the form $f(x)=ax^2+bx+c$ where a, b, and c are real numbers and $a \neq 0$.

Examples:

a. $f(x)=2x^2-3x+1$ $\quad$ $a=2,\ b=-3,\ c=1$

b. $g(x)=-x^2-5$ $\quad$ ______________________________

c. $h(x)=x^2+5x$ $\quad$ ______________________________

<u>Graphing Quadratic Functions:</u>

Vertex Form: $f(x)=a(x-h)^2+k$

- a is the ***vertical stretch/shrink*** of the function
  - When $a<0$ the parabola opens down. The graph has a ***Maximum*** value (y coordinate of vertex).
  - When $a>0$ the parabola opens up. The graph has a ***Minimum*** value (y coordinate of vertex).
- ***Vertex*** of the parabola is $(opposite\ h,\ same\ as\ k)$.

Standard Form: $f(x)=ax^2+bx+c$

- Find the x coordinate of the vertex using: $x=\dfrac{-b}{2a}$.
- Plug the x into the original function to get the y coordinate of the vertex.

Write the quadratic function in vertex form.

1. $g(x)=2x^2-12x+19$

2. $f(x)=2x^2-4x-1$

Find the zeros of f and the $x-intercepts$ of the graph of f.

3. $f(x)=x^2+12x+32$

4. $h(x)=2x^2-9x+10$

Find the maximum or minimum value of the function. State whether the value is a minimum or maximum.

5. $g(x) = -2x^2 + 8x - 1$

6. $f(x) = x^2 - 3x + 1$

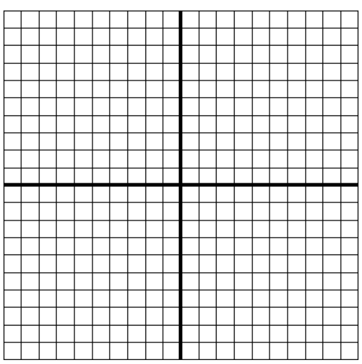
Find the range of the quadratic function.

7. $f(x) = -2x^2 - 6x - 1$

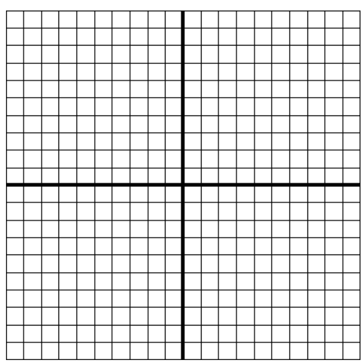
8. $h(x) = 3x^2 - 12x + 13$

Graph the quadratic function and list the axis of symmetry for the function.

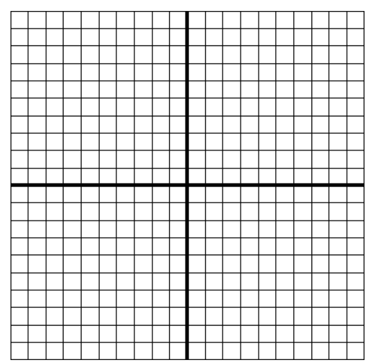
7. $f(x) = x^2$



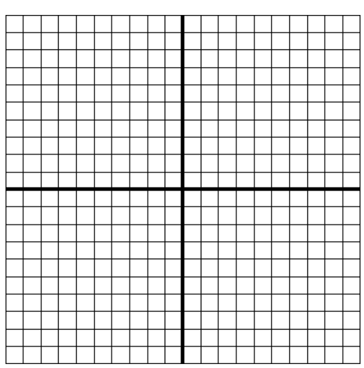
8. $f(x) = (x-4)^2 + 1$



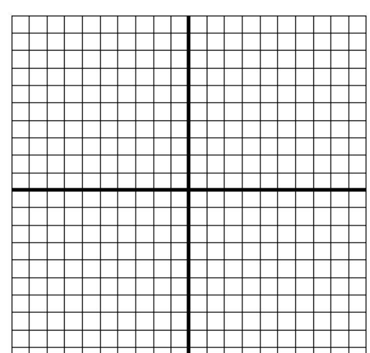
9. $f(x) = -2(x+2)^2 - 3$



10. $f(x) = 2x^2 - 8x + 3$



11. $g(x) = x^2 - 4x + 7$



Section 2.4: Quadratic Functions Application Problems

Target: I can accurately solve a quadratic function application problem.

1. The height $h(t)$, in feet, of a snowboarder t seconds after beginning a certain jump can be approximated by $h(t) = -16t^2 + 22.9t + 9$. If the snowboarder lands at a point that is 3 feet below the base of the jump, determine the *airtime* (the time the snowboarder is in the air) for this jump. Round to the nearest tenth of a second.

2. A long sheet of tin 20 inches wide is to be made into a trough by bending up two sides until they are perpendicular to the bottom. How many inches should be turned up so that the trough will achieve it maximum carrying capacity?

3. A ball is thrown vertically upward with an initial velocity of 48 feet per second. If the ball started its flight at a height of 8 feet, then its height at time t can be determined by $s(t) = -16t^2 + 48t + 8$, where $s(t)$ is measured in feet above ground level and t is the number of seconds of flight.

   a. Determine the time it takes the ball to attain its maximum height.
   b. Determine the maximum height the ball attains.
   c. Determine the time it takes the ball to hit the ground.

Section 2.5: Properties of Graphs

Targets: I can determine whether the function is an even function, odd function, or neither.

I can translate graphs accurately.

Definition of Even and Odd Functions

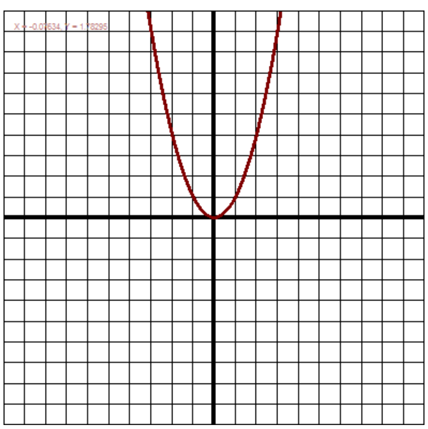
The function f is an **even function** if: $f(-x) = f(x)$ for all x in the domain of f.

The function f is an **odd function** if: $f(-x) = -f(x)$ for all x in the domain of f.

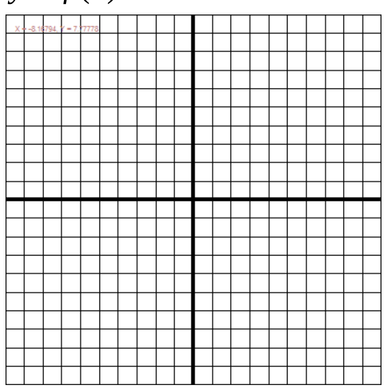
Determine whether the given function is an even function, odd function, or neither.

1. $f(x) = x^3$
2. $F(x) = |x|$
3. $h(x) = x^4 + 2x$
4. $g(x) = 4x^5 + 5x$

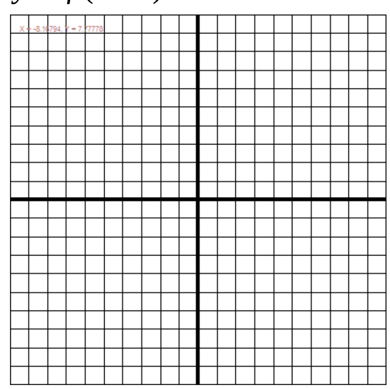
Use the graph of $f(x) = x^2$ below to sketch a graph of each function.



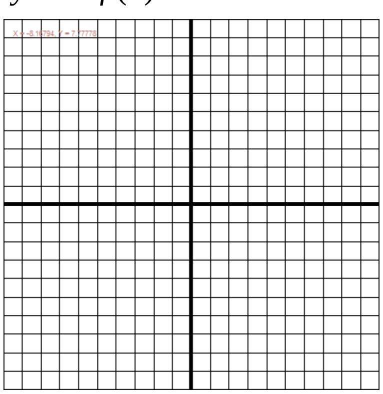
5. $y = f(x) - 4$



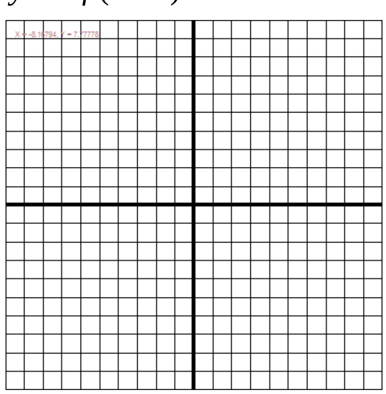
6. $y = f(x+3)$



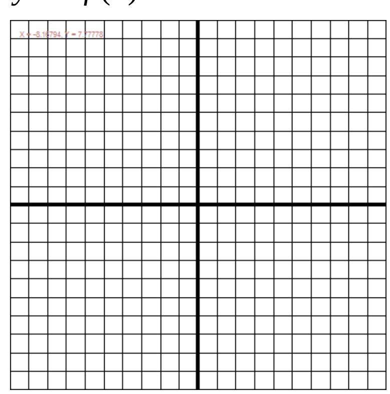
7. $y = -2f(x)$



8. $y = -f(x-2) + 3$



9. $y = 2f(x) - 1$



10. Let f be a function such that $f(-2)=5,\ f(0)=-2,\ f(1)=0$. Give the coordinates of three points on the graph of:

a. $y=f(x)-2$ b. $y=f(x-3)+2$ c. $y=2f(x)$

11. Let f be a function such that $g(-1)=3,\ \text{g}(2)=-4$. Give the coordinates of three points on the graph of:

a. $y=g(x+1)-2$ b. $y=-g(x)$ c. $y=g(-x)$

12. Let f be a function such that $g(0)=3,\ \text{g}(2)=7$. Give the coordinates of three points on the graph of:

a. $y=g(x-6)-5$ b. $y=-3g(x)$ c. $y=g(x+3)+4$

Section 2.6: Algebra of Functions

Targets: I can accurately use operations of functions and determine the domain of functions.

I can evaluate functions.

Definitions of Operations of Functions

If f and g are functions with domains D_f and D_g, then we define the sum, difference, product, and quotient of f and g as

Sum	$(f+g)(x)=f(x)+g(x)$	Domain: $D_f \cap D_g$
Difference	$(f-g)(x)=f(x)-g(x)$	Domain: $D_f \cap D_g$
Product	$(f \cdot g)(x)=f(x)\cdot g(x)$	Domain: $D_f \cap D_g$
Quotient	$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$	Domain: $D_f \cap D_g,\ g(x)\neq 0$

Example: Let $f(x)=3x-2$ and $g(x)=x^2-9$.

1. $(f+g)(x)$

2. $(f-g)(x)$

3. $(f\cdot g)(x)$

4. $\left(\frac{f}{g}\right)(x)$

Determine the Domain of a Function

5. If $f(x)=\sqrt{x-1}$ and $g(x)=x^2-4$, find the domains of $f+g$, $f-g$, fg, and $\frac{f}{g}$.

6. If $f(x)=x^2-25$ and $g(x)=\sqrt{x+3}$, find the domains of $f+g$, $f-g$, fg, and $\frac{f}{g}$.

Evaluate Functions

Let $f(x) = x^2 - 9$ and $g(x) = 2x + 6$. Find the following

7. $(f + g)(5)$

8. $(fg)(-1)$

9. $\left(\frac{f}{g}\right)(4)$

Composition of Functions: The composition of functions is another way functions can be combined. The function defined by $(f \circ g)(x)$ is also called the composition of f and g. We read $(f \circ g)(x)$ as "f circle g of x" and $f\left[g(x)\right]$ as "f of g of x."

If $f(x) = x^2 - 3x$ and $g(x) = 2x + 1$, find the following

10. $(g \circ f)(x)$

11. $(f \circ g)(x)$

Evaluate the Composition of Functions

If $f(x) = 2x - 7$ and $g(x) = x^2 + 4$, evaluate the following

12. $(f \circ g)(3)$

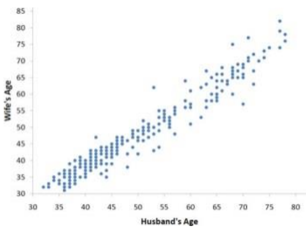
13. $(g \circ f)(-2)$

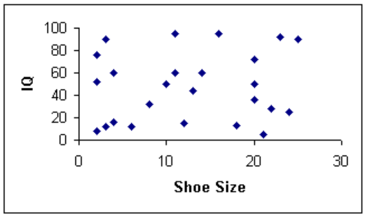
Section 2.7: Modeling Data Using Regression

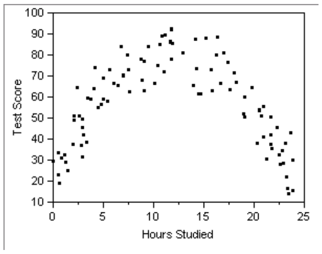
Targets: I can use a calculator to calculate the least-squares line, correlation coefficient, & coefficient of determination.

I can use the calculator to calculate the quadratic regression line.

<u>Determine the relationship between the independent variable (*x*) and dependent variable (*y*)</u>.







<u>Linear Regression:</u> To find the line that "best" approximates the data, **regression analysis** is used. This type of analysis produces the linear function whose graph is called the **line of best fit** or the **least-squares regression line**.

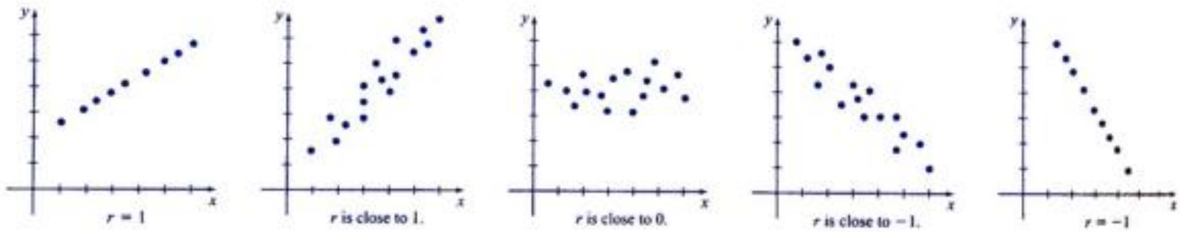
<u>Use the graphing calculator to calculate the **least-squares regression line**</u>:

1. **Stat, Edit** (enter the data into L_1 and L_2)
   a. To clear any data that is in L_1 and L_2 use the **<u>up arrow</u>** until you highlight L_1 or L_2, then hit **<u>clear</u>** and the **<u>down arrow</u>**.
2. **2nd Stat Plot, Enter** (move cursor to **ON**), **2nd Quit, Graph**
3. **Stat, Calc, LinReg** (option 4 on the list)

The data in the table given below show the population of selected states and the number of professional sports teams (MLB, NFL, NHL, NBA, and WNBA) in those states. Calculate the least-squares line for the data.

State	Population (millions)	Number of Teams
Arizona	6.4	5
California	37.3	16
Colorado	5	4
Florida	18.8	9
Illinois	12.8	6
Indiana	6.5	3
Michigan	10	4
Minnesota	5.3	5
New Jersey	8.8	3
New York	19.4	9
North Carolina	9.5	3
Ohio	11.5	7
Pennsylvania	12.7	7
Texas	25.1	10

Linear Correlation Coefficient: The **linear correlation coefficient (r)** is the measure of how close the points of a data set can be modeled by a straight line. If $r = -1$, then the points of the data set can be modeled *exactly* by a straight line with negative slope. If $r = 1$, then the data set can be modeled *exactly* by a straight line with positive slope. For all data sets, $-1 \le r \le 1$.



Coefficient of Determination: The **coefficient of determination (r^2)** measure the proportion of the variation in the dependent variable that is explained by the regression equation.

Example: For the population and sports team data, $r^2 \approx 0.8989$. This means that approximately 90% of the total variation in the number of teams (dependent variable) can be attributed to the state population.

Quadratic Regression Models:

Use the graphing calculator to calculate the **least-squares regression line**:

1. **Stat, Edit** (enter the data into L_1 and L_2)
   a. To clear any data that is in L_1 and L_2 use the **up arrow** until you highlight L_1 or L_2, then hit **clear** and the **down arrow**.
2. **2nd Stat Plot, Enter** (move cursor to **ON**), **2nd Quit, Graph**
3. **Stat, Calc, quadReg** (option 5 on the list)

Example:

The height h, in feet, of a basketball d feet from a player after the basketball has been released toward the basket is given in the table below. Find a regression model for the data.

Distance (in feet)	1	3	5	7	9	11	13	15	17	19	21	23
Height (in feet)	8	10.3	12.6	13.9	15	15.9	15.8	15.9	15.7	14.4	12.4	11.1

a) Construct a scatter plot for the data.
b) Find the regression equation using the calculator.
c) Examine the coefficient of determination.

Graphing Absolute Value, Piecewise, and Greatest Integer Functions

Target: I can accurately graph the absolute value, piecewise, and greatest integer functions.

I can evaluate a piecewise function accurately.

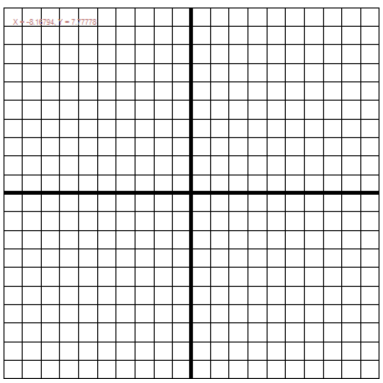
Absolute Value Function: $f(x) = a|x-h|+k$

- Vertex is $(opposite\ h, same\ as\ k)$.
- Slope is a.

Graph the Absolute value function.

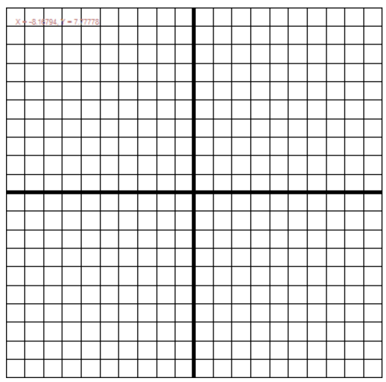
1. $f(x) = |x|$

Vertex: ____________



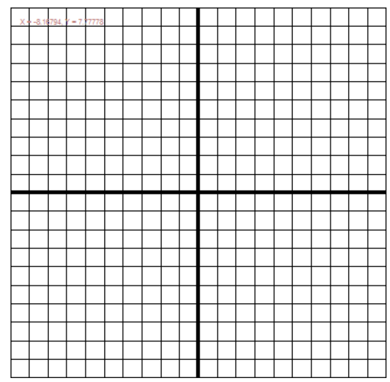
2. $f(x) = |x|+3$

Vertex: ____________



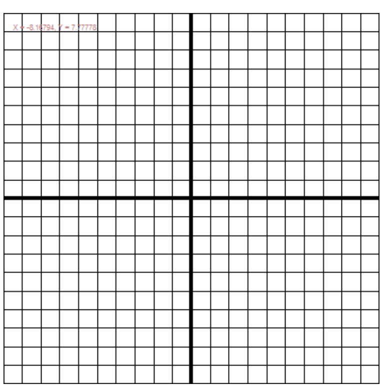
3. $f(x) = |x-4|$

Vertex: ____________



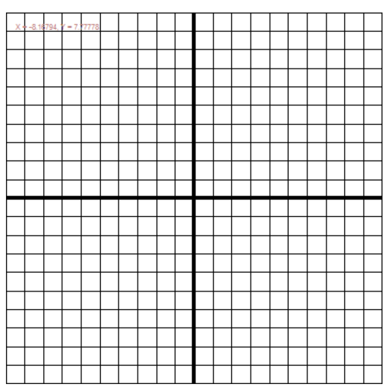
4. $f(x) = -|x+5|-2$

Vertex: ____________



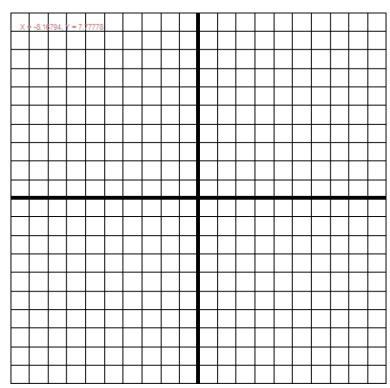
5. $f(x) = 2|x-3|+1$

Vertex: ____________



6. $f(x) = -3|x|+8$

Vertex: ____________



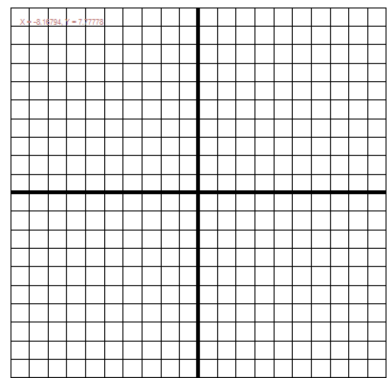
Piecewise functions are functions that are represented by more than one expression. For instance, the function *f* defined below consists of three pieces, $2x+1,\ x^2-1,$ and $4-x.$

$$f(x)=\begin{cases}2x+1, & x<-2\\ x^2-1, & -2\le x\le 3\\ 4-x, & x>3\end{cases}$$

To evaluate this function at *x*, determine the interval at which *x* lies and then use the expression that corresponds to that interval to evaluate the function.

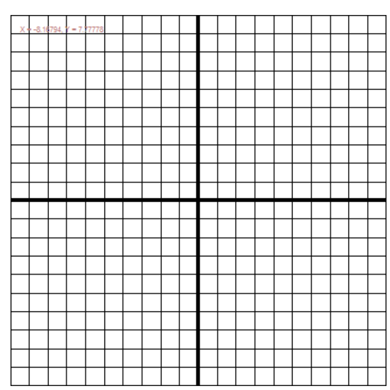
7\. Let $f(x)=\begin{cases}2x+1, & x<-2\\ x^2-1, & -2\le x\le 3\\ 4-x, & x>3\end{cases}$ **Find:** a. $f(4)$ b. $f(5)$ c. $f(2)$ d. Graph f

a. b. c. d.



8\. Let $f(x)=\begin{cases}x+6, & x\le -2\\ x^2, & -2<x\le 3\\ -2x+15, & x>3\end{cases}$ **Find:** a. $f(4)$ b. $f(0)$ c. $f(-3)$ d. Graph f

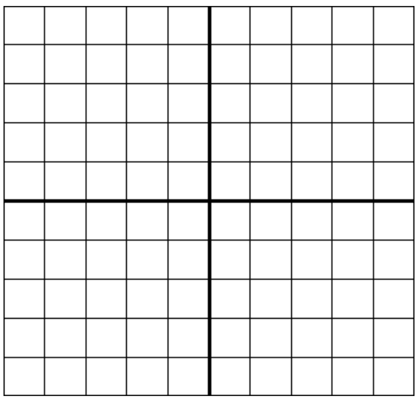
a. b. c. d.



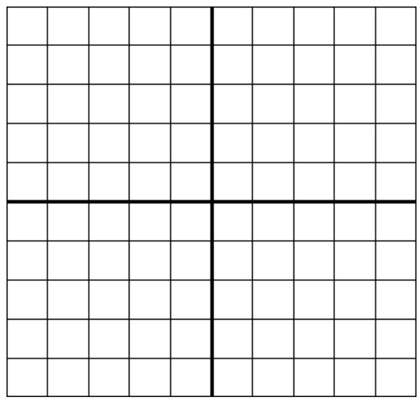
Greatest Integer Functions: Greatest integer functions or floor functions can be expressed as "$f(x) = [\![x]\!]$, $f(x) = \lfloor x \rfloor$, and $f(x) = \text{int}(x)$." The greatest integer function returns the largest integer less than or equal to x, for all real numbers x. For example, [2] = 2; [1.5] = 1; [-3.1] = -4; [-6.9] = -7. Features of the intervals on the greatest integer function can be expressed as $[n, n+1)$. The value of the function on these intervals will be n. The function is constant in each interval.

$$\boldsymbol{f(x) = a[\![b\,x - h]\!] + k}$$

9. $f(x) = [\![x]\!]$



10. $f(x) = [\![x - 2]\!] + 3$



11. $f(x) = -2[\![x]\!] + 1$