7.1 Inverse Variation

Learning Target Understand inverse variation.

Success Criteria

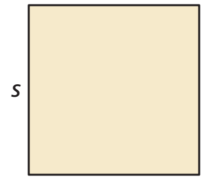
- I can identify equations and data sets that show direct variation.
- I can identify equations and data sets that show inverse variation.
- I can write inverse variation equations.
- I can solve real-life problems using inverse variation functions.

EXPLORE IT! Describing Types of Variation

Work with a partner.

a. The table represents the side length s (in inches) and the perimeter P (in inches) of a square. Complete the table. Describe the relationship between s and P. Explain why the perimeter P is said to vary *directly* with the side length s.

s	1	2	3	4	5	6
P						

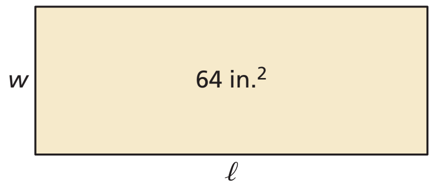


b. Make a scatter plot of the data. What are some characteristics of the graph?

c. Write an equation that represents P as a function of s.

d. The table represents the length ℓ (in inches) and the width w (in inches) of a rectangle that has an area of 64 square inches. Complete the table. Describe the relationship between ℓ and w. Explain why the width w is said to vary *inversely* with the length ℓ.

ℓ	1	2	4	8	16	32	64
w							



Math Practice

Communicate Precisely

How can you explain to a friend the difference between two quantities that vary directly and two quantities that vary inversely? Give real-life situations to support your explanation.

e. Make a scatter plot of the data. Compare the characteristics of the graph with the graph in part (b).

f. Write an equation that represents w as a function of ℓ. Compare it with the equation in part (c).

g. How can you recognize when two quantities vary directly or inversely?

Classifying Direct and Inverse Variation

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Vocabulary VOCAB

direct variation, *p. 356*
constant of variation, *p. 356*
inverse variation, *p. 356*

WORDS AND MATH

A *variation* is a change in an amount or quantity. *Direct variation* and *inverse variation* describe relationships where a change in one quantity causes a predictable change in another quantity.

Recall that two variables x and y are in a proportional relationship when $y = ax$ for some nonzero constant a. When this occurs, x and y are said to show *direct variation*. Another type of variation is *inverse* variation.

KEY IDEAS

Direct Variation

Two variables x and y show **direct variation** when they are related as follows:

$y = ax, a \neq 0$ Direct variation

The constant a is the **constant of variation**, and y is said to *vary directly* with x.

Inverse Variation

Two variables x and y show **inverse variation** when they are related as follows:

$y = \dfrac{a}{x}, a \neq 0$ Inverse variation

The constant a is the constant of variation, and y is said to *vary inversely* with x.

EXAMPLE 1 Classifying Equations

WATCH

Tell whether x and y show *direct variation*, *inverse variation*, or *neither*.

a. $xy = 5$ **b.** $y = x - 4$ **c.** $\dfrac{y}{2} = x$

SOLUTION

	Given Equation	Solved for y	Type of Variation
a.	$xy = 5$	$y = \dfrac{5}{x}$	inverse
b.	$y = x - 4$	$y = x - 4$	neither
c.	$\dfrac{y}{2} = x$	$y = 2x$	direct

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. MP **REASONING** Describe the relationship the equation in Example 1(b) represents. Explain why it does not show direct or inverse variation.

Tell whether x and y show *direct variation*, *inverse variation*, or *neither*.

2. $6x = y$ **3.** $xy = -0.25$ **4.** $y + x = 10$

The general equation $y = ax$ for direct variation can be rewritten as $\dfrac{y}{x} = a$. So, a set of data pairs (x, y) shows direct variation when the ratios $\dfrac{y}{x}$ are constant.

The general equation $y = \dfrac{a}{x}$ for inverse variation can be rewritten as $xy = a$. So, a set of data pairs (x, y) shows inverse variation when the products xy are constant.

EXAMPLE 2 Classifying Data

WATCH

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Tell whether x and y show *direct variation*, *inverse variation*, or *neither*.

a.

x	2	4	6	8
y	-12	-6	-4	-3

b.

x	1	2	3	4
y	2	4	8	16

SOLUTION

a. Find the products xy and ratios $\frac{y}{x}$.

xy	-24	-24	-24	-24
$\frac{y}{x}$	$\frac{-12}{2} = -6$	$\frac{-6}{4} = -\frac{3}{2}$	$\frac{-4}{6} = -\frac{2}{3}$	$-\frac{3}{8}$

The products are constant.

The ratios are not constant.

▶ So, x and y show inverse variation.

b. Find the products xy and ratios $\frac{y}{x}$.

xy	2	8	24	64
$\frac{y}{x}$	$\frac{2}{1} = 2$	$\frac{4}{2} = 2$	$\frac{8}{3}$	$\frac{16}{4} = 4$

The products are not constant.

The ratios are not constant.

▶ So, x and y show neither direct nor inverse variation.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

5. VOCABULARY Explain how a set of data pairs that shows direct variation is different from a set of data pairs that shows inverse variation.

Tell whether x and y show *direct variation*, *inverse variation*, or *neither*.

6.

x	-4	-3	-2	-1
y	20	15	10	5

7.

x	1	2	3	4
y	60	30	20	15

Writing Inverse Variation Equations

EXAMPLE 3 Writing an Inverse Variation Equation

WATCH

The variables x and y vary inversely, and $y = 4$ when $x = 3$. Write an equation that relates x and y. Then find y when $x = -2$.

SOLUTION

$y = \frac{a}{x}$ — Write general equation for inverse variation.

$4 = \frac{a}{3}$ — Substitute 4 for y and 3 for x.

$12 = a$ — Multiply each side by 3.

▶ The inverse variation equation is $y = \frac{12}{x}$. When $x = -2$, $y = \frac{12}{-2} = -6$.

ANOTHER WAY

Because x and y vary inversely, the products xy are constant. This product equals the constant of variation a. So, you can quickly determine that $a = xy = 3(4) = 12$.

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EXAMPLE 4 Modeling Real Life

WATCH

The time t (in hours) that it takes a group of volunteers to build a playground varies inversely with the number n of volunteers. It takes a group of 10 volunteers 8 hours to build the playground.

- Make a table showing the times that it takes to build the playground when the number of volunteers is 15, 20, 25, and 30.
- What happens to the time it takes to build the playground as the number of volunteers increases?

SOLUTION

1. **Understand the Problem** You are given a description of two quantities that vary inversely and one pair of data values. You are asked to create a table that gives additional data pairs and determine what happens to the building time as the number of volunteers increases.
2. **Make a Plan** Use the time that it takes 10 volunteers to build the playground to find the constant of variation. Then write an inverse variation equation and substitute for the different numbers of volunteers to find the corresponding times.
3. **Solve and Check**

$t = \dfrac{a}{n}$ Write general equation for inverse variation.

$8 = \dfrac{a}{10}$ Substitute 8 for t and 10 for n.

$80 = a$ Multiply each side by 10.

The inverse variation equation is $t = \dfrac{80}{n}$. Make a table of values.

n	15	20	25	30
t	$\frac{80}{15} = 5$ h 20 min	$\frac{80}{20} = 4$ h	$\frac{80}{25} = 3$ h 12 min	$\frac{80}{30} = 2$ h 40 min

▶ As the number of volunteers increases, the time it takes to build the playground decreases.

Check Because the time decreases as the number of volunteers increases, the time for 5 volunteers to build the playground should be greater than 8 hours.

$t = \dfrac{80}{5} = 16$ hours ✓

Math Practice

Look for Patterns
Will the time change more when increasing from 30 to 35 volunteers or when increasing from 55 to 60 volunteers? Explain.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

The variables x and y vary inversely. Use the given values to write an equation relating x and y. Then find y when $x = 2$.

8. $x = 4, y = 5$

9. $x = 6, y = -1$

10. $x = \frac{1}{2}, y = 16$

11. **WHAT IF?** In Example 4, it takes a group of 10 volunteers 12 hours to build the playground. How long does it take a group of 15 volunteers?

12. A company determines that the demand d for one of its products varies inversely with the price p of the product. When the price is \$2.75, the demand is 550 units. When the price is doubled, is the demand for the product halved? Justify your answer.

7.1 Practice WITH CalcChat® AND CalcView®

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In Exercises 1–8, tell whether x and y show *direct variation*, *inverse variation*, or *neither*. *Example 1*

1. $y = \dfrac{2}{x}$

2. $xy = 12$

3. $\dfrac{y}{x} = 8$

4. $4x = y$

5. $y = x + 4$

6. $x + y = 6$

7. $8y = x$

8. $xy = \dfrac{1}{5}$

In Exercises 9–12, tell whether x and y show *direct variation*, *inverse variation*, or *neither*. *Example 2*

9.

x	12	18	23	29	34
y	132	198	253	319	374

10.

x	1.5	2.5	4	7.5	10
y	13.5	22.5	36	67.5	90

11.

x	4	6	8	8.4	12
y	21	14	10.5	10	7

12.

x	4	5	6.2	7	11
y	16	11	10	9	6

In Exercises 13–20, the variables x and y vary inversely. Use the given values to write an equation relating x and y. Then find y when $x = 3$. *Example 3*

13. $x = 5, y = -4$

14. $x = 1, y = 9$

15. $x = -3, y = 8$

16. $x = 7, y = 2$

17. $x = \frac{3}{4}, y = 28$

18. $x = -4, y = -\frac{5}{4}$

19. $x = -12, y = -\frac{1}{6}$

20. $x = \frac{5}{3}, y = -7$

21. **COLLEGE PREP** In which equation do the variables x and y *not* show inverse variation?

Ⓐ $y = -\dfrac{4}{x}$

Ⓑ $y = \dfrac{x}{3}$

Ⓒ $xy = 7$

Ⓓ $x = \dfrac{6}{y}$

22. **MP STRUCTURE** The variable P varies inversely with t, and $P = -8$ when $t = -2$. Find P when $t = 4$.

ERROR ANALYSIS In Exercises 23 and 24, the variables x and y vary inversely. Describe and correct the error in writing an equation relating x and y.

23. $x = 8, y = 5$

$y = ax$

$5 = a(8)$

$\frac{5}{8} = a$

So, $y = \frac{5}{8}x$.

24. $x = 5, y = 2$

$xy = a$

$5 \cdot 2 = a$

$10 = a$

So, $y = 10x$.

25. **MODELING REAL LIFE** The number y of apps that can be stored on a tablet varies inversely with the average size x of an app. A certain tablet can store 200 apps when the average size of an app is 28 megabytes (MB). *Example 4*

a. Make a table showing the numbers of apps that will fit on the tablet when the average size of an app is 20 MB, 25 MB, 30 MB, and 50 MB.

b. What happens to the number of apps as the average app size increases?

26. **MODELING REAL LIFE** When you stand on snow, the average pressure P (in pounds per square inch) that you exert on the snow varies inversely with the total surface area A (in square inches) of the soles of your footwear. Write an equation that gives P as a function of A when a hiker wears the snowshoes shown and the pressure is 0.43 pound per square inch. Then find the pressure when the hiker wears the boots shown.

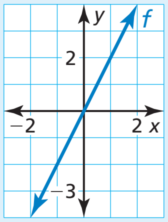
Snowshoes:
$A = 360$ in.^{2}

Boots:
$A = 60$ in.^{2}

27. **MP PROBLEM SOLVING** Computer chips are etched onto silicon wafers. The table compares the areas A (in square millimeters) of computer chips with the number c of chips that can be obtained from a silicon wafer. Write a model that gives c as a function of A. Then predict the number of chips per wafer when the area of a chip is 81 square millimeters.

Area (mm^2), A	58	62	66	70
Number of chips, c	448	424	392	376

28. HOW DO YOU SEE IT? Does the graph of f represent inverse variation or direct variation? Explain your reasoning.



29. MAKING AN ARGUMENT You download y movies for x dollars each. You have enough money to download 5 movies for \$8 each or 8 movies for \$5 each. Can this situation be represented by direct variation or inverse variation? Explain.

30. OPEN-ENDED Describe a real-life situation that can be modeled by an inverse variation equation.

31. CONNECTING CONCEPTS Consider the formula for the volume of a rectangular prism, $V = Bh$. How does the area of the base vary with the height of the prism? What is the constant of variation? Explain your reasoning.

32. THOUGHT PROVOKING The weight w (in pounds) of an object varies inversely with the square of the distance d (in miles) of the object from the center of Earth. At sea level (3978 miles from the center of the Earth), an astronaut weighs 210 pounds. How much does the astronaut weigh 200 miles above sea level?

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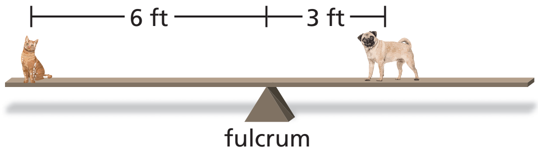
33. CRITICAL THINKING Suppose x varies inversely with y and y varies inversely with z. How does x vary with z? Justify your answer.

34. DIG DEEPER To balance the board in the diagram, the distances (in feet) of the animals from the fulcrum must vary directly. How far is each animal from the fulcrum when they move so they are 6 feet apart and the board remains balanced? Justify your answer.



REVIEW & REFRESH

WATCH

In Exercises 35 and 36, divide.

35. $(x^2 + 2x - 99) \div (x + 11)$

36. $(3x^4 - 13x^2 - x^3 + 6x - 30) \div (3x^2 - x + 5)$

37. MODELING REAL LIFE The table shows the heights y of a skateboard x seconds after jumping off the ground. What type of function can you use to model the data? Estimate the height after 0.75 second.

Time (seconds), x	Height (inches), y
0	0
0.5	12
1.0	16
1.5	12
2.0	0

In Exercises 38 and 39, solve the equation.

38. $64^x = \left(\frac{1}{4}\right)^{2x + 15}$

39. $\log(x + 6) = \log 7x$

40. MP REASONING Which properties of logarithms can you use to condense the expression $3 \ln 0.5x - \ln 6$?

41. Tell whether x and y show *direct variation*, *inverse variation*, or *neither*.

x	−5	−1	2	8	11
y	40	8	−16	−64	−88

In Exercises 42–45, graph the function. Then state the domain and range.

42. $f(x) = 5^x + 4$

43. $g(x) = e^{x-1}$

44. $y = \log 3x - 6$

45. $h(x) = 2 \ln(x + 9)$

46. Create a scatter plot of the points $(x, \ln y)$ to determine whether an exponential model fits the data. If so, find an exponential model for the data.

x	2	5	8	11	14
y	1.8	5.4	16.2	48.6	145.8

In Exercises 47–50, simplify the expression.

47. $8^{3/2} \cdot 8^{1/4}$

48. $5\sqrt{48} - 9\sqrt{3}$

49. $\dfrac{\sqrt[3]{54}}{\sqrt[3]{2}}$

50. $\sqrt[4]{\dfrac{a^{12}}{81b^{16}}}$