EXERCISE SET 6.5

Concept Check

1. What is a half-plane?

2. Is it possible for a system of inequalities to have no solution? If so, give an example. If not, explain why not.

3. Is (3, 4) a solution of $-2x + 5y > 11$?

4. Is $(-2, 3)$ a solution of $6x + 4y \leq 0$?

5. Is (4, 5) a solution of the following system of inequalities?

$$\begin{cases} 3x - 2y \leq 6 \\ x + 3y < 14 \\ x \geq 2, y > 3 \end{cases}$$

6. Is (2, 1) a solution of the following system of inequalities?

$$\left\{\begin{array}{l} 2x + 7y \geq 3 \\ -x + 6y \leq 14 \\ x \geq 0, y \geq 2 \end{array}\right\}$$

In Exercises 7 to 28, sketch the graph of each inequality.

7. $y \leq -2$

8. $x + y > -2$

9. $y \geq 2x + 3$

10. $y < -2x + 1$

11. $2x - 3y < 6$

12. $3x + 4y \leq 4$

13. $4x + 3y \leq 12$

14. $5x - 2y < 8$

15. $y < x^2$

16. $x > y^2$

17. $y \geq x^2 - 2x - 3$

18. $y < 2x^2 - x - 3$

19. $(x - 2)^2 + (y - 1)^2 < 16$

20. $(x + 2)^2 + (y - 3)^2 > 25$

21. $\dfrac{(x-3)^2}{9} - \dfrac{(y+1)^2}{16} > 1$

22. $\dfrac{(x+1)^2}{25} - \dfrac{(y-3)^2}{16} \leq 1$

23. $4x^2 + 9y^2 - 8x + 18y \geq 23$

24. $25x^2 - 16y^2 - 100x - 64y < 64$

25. $y \geq |2x - 4|$

26. $y < |x|$

27. $y < 2^{x-1}$

28. $y > \log_3 x$

In Exercises 29 to 50, sketch the graph of the solution set of each system of inequalities.

29. $\begin{cases} 1 \leq x < 3 \\ -2 < y \leq 4 \end{cases}$

30. $\begin{cases} -2 < x < 4 \\ y \geq -1 \end{cases}$

31. $\begin{cases} x + y \leq 2 \\ x - y < 2 \end{cases}$

32. $\begin{cases} 2x - 5y < -6 \\ 3x + y < 8 \end{cases}$

33. $\begin{cases} 2x - y \geq -4 \\ 4x - 2y \leq -17 \end{cases}$

34. $\begin{cases} 4x + 2y > 5 \\ 6x + 3y > 10 \end{cases}$

35. $\begin{cases} 2x + 3y < 6 \\ 3x - 2y \geq -6 \end{cases}$

36. $\begin{cases} 3x + 5y \geq -8 \\ 2x - 3y \geq 1 \end{cases}$

37. $\begin{cases} y < 2x + 3 \\ y > 2x - 2 \end{cases}$

38. $\begin{cases} y > 3x + 1 \\ y < 3x - 2 \end{cases}$

39. $\begin{cases} y > x - 1 \\ y \leq -x^2 + 4 \end{cases}$

40. $\begin{cases} y \leq 2x + 7 \\ y > x^2 + 3x + 1 \end{cases}$

41. $\begin{cases} x^2 + y^2 \leq 49 \\ 9x^2 + 4y^2 \geq 36 \end{cases}$

42. $\begin{cases} y < 2x - 1 \\ y > x^2 - 2x + 2 \end{cases}$

43. $\begin{cases} (x-1)^2 + (y+1)^2 \leq 16 \\ (x-1)^2 + (y+1)^2 \geq 4 \end{cases}$

44. $\begin{cases} (x+2)^2 + (y-3)^2 > 25 \\ (x+2)^2 + (y-3)^2 < 16 \end{cases}$

45. $\begin{cases} \dfrac{x^2}{4} - \dfrac{y^2}{16} > 1 \\ \dfrac{x^2}{16} + \dfrac{y^2}{4} < 1 \end{cases}$

46. $\begin{cases} \dfrac{(x+1)^2}{36} + \dfrac{(y-2)^2}{25} < 1 \\ \dfrac{(x+1)^2}{25} + \dfrac{(y-2)^2}{36} < 1 \end{cases}$

47. $\begin{cases} 6x + y \geq 30 \\ x + 4y \geq 40 \\ 2x + 3y \geq 60 \\ x \geq 0, y \geq 0 \end{cases}$

48. $\begin{cases} 5x + y \leq 9 \\ 2x + 3y \leq 14 \\ x \geq -2, y \geq 2 \end{cases}$

49. $\begin{cases} x + 4y \leq 80 \\ x + y \leq 35 \\ 2x + y \leq 60 \\ x \geq 0, y \geq 0 \end{cases}$

50. $\begin{cases} 4x + y \geq 13 \\ 3x + 2y \geq 16 \\ x \leq 15, y \leq 12 \end{cases}$

51. **Physical Fitness** The instructor of an aerobics exercise class for beginners uses the following system of inequalities to find the targeted exercise heart rate ranges for the members of the class.

$$\begin{cases} y \geq 0.55(208 - 0.7x) \\ y \leq 0.75(208 - 0.7x) \\ 20 \leq x \leq 50 \end{cases}$$

In this system, y is the person's exercise heart rate in beats per minute and x is the person's age in years. Use the system of inequalities to determine the targeted exercise heart rate range for Ashley, who is 35. Round the minimum and maximum targeted heart rates to the nearest beat per minute.

52. **Physical Fitness** The sprinters on a track team use the following system of inequalities to determine their targeted exercise heart rate ranges for their workouts.

$$\begin{cases} y \geq 0.80(208 - 0.7x) \\ y \leq 0.85(208 - 0.7x) \\ 20 \leq x \leq 28 \end{cases}$$

■ Indicates Try It Exercises

In this system, y is the person's exercise heart rate in beats per minute and x is the person's age in years. Use the system of inequalities to determine the targeted exercise heart rate range for a sprinter who is 26 years old. Round the minimum and maximum targeted heart rates to the nearest beat per minute.

In Exercises 53 to 58, sketch the graph of the inequality.

53. $|y| \geq |x|$

54. $|y| \leq |x - 1|$

55. $|x + y| \leq 1$

56. $|x - y| > 1$

57. $|x| + |y| \leq 1$

58. $|x| - |y| > 1$

Enrichment Exercises

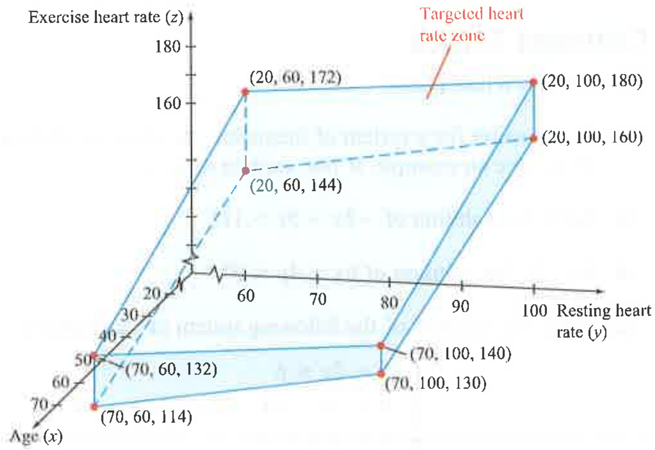
59. Sketch the graphs of $xy > 1$ and $y > \frac{1}{x}$. Note that the two graphs are not the same, yet the second inequality can be derived from the first by dividing each side by x. Explain.

60. Sketch the graph of $\frac{x}{y} < 1$ and the graph of $x < y$. Note that the two graphs are not the same, yet the second inequality can be derived from the first by multiplying each side by y. Explain.

61. Physical Fitness The *Karvonen method* is often used to find a person's targeted exercise heart rate range. This method is generally considered more reliable than the method used in Example 5 because it uses both a person's age x, in years, and the person's resting heart rate y, in beats per minute, to establish a targeted exercise heart rate range, which is displayed on a vertical z-axis. The Karvonen method is defined by the following system of inequalities.

$$\begin{cases} z \geq 0.60(220 - x) + 0.40y & (1) \\ z \leq 0.80(220 - x) + 0.20y & (2) \\ 20 \leq x \leq 70 & (3) \\ 60 \leq y \leq 100 & (4) \end{cases}$$

The graph of the Karvonen system of inequalities is the three-dimensional solid shown below.



The coordinates of a point on the solid are given in the order (x, y, z). For instance, the two points (20, 60, 144) and (20, 60, 172) indicate that a 20-year-old person with a resting heart rate of 60 beats per minute has a targeted exercise heart rate range from 144 to 172 beats per minute. For any given age x, $20 \leq x \leq 70$, the graph of the Karvonen system of inequalities is a trapezoidal cross section of the above solid.

a. Use the Karvonen inequalities to graph the targeted exercise heart rate zone for people who are 25 years of age. (*Hint:* Use a sheet of graph paper. Substitute 25 for x to produce a system of inequalities that involves only two variables. Label the horizontal axis as the y-axis and the vertical axis as the z-axis. The domain will be $\{y \mid 60 \leq y \leq 100\}$.)

b. Tyler is 25 years old and has a resting heart rate of 80 beats per minute. Use your graph from **a** to estimate Tyler's targeted heart rate range.

c. Use the Karvonen inequalities to determine your targeted exercise heart rate range. (*Note:* Your resting heart rate is your heart rate after you have rested for 7 to 8 hours.)

SECTION 6.6

Introduction to Linear Programming
Solving Optimization Problems

Linear Programming

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A41.

PS1. Graph: $2x + 3y \leq 12$ [6.5]

PS2. Evaluate $C = 3x + 4y$ at (0, 5), (2, 3), (6, 1), and (9, 0). [P.3]

PS3. Evaluate $C = 6x + 4y + 15$ at (0, 20), (4, 18), (10, 10), and (15, 0). [P.3]

PS4. Solve: $\begin{cases} 3x + y = 6 \\ x + y = 4 \end{cases}$ [6.1]

PS5. Solve: $\begin{cases} 300x + 100y = 900 \\ 400x + 300y = 2200 \end{cases}$ [6.1]

Introduction to Linear Programming

Consider a business analyst who is trying to maximize the profit from the production of a product or an engineer who is trying to minimize the amount of energy an electric circuit needs to operate. Generally, problems that seek to maximize or minimize a situation are called **optimization problems.** One strategy for solving these problems is called **linear programming.**

A linear programming problem involves a **linear objective function,** which is the function that must be maximized or minimized. This objective function is subject to some **constraints,** which are inequalities or equations that restrict the values of the variables. To illustrate these concepts, suppose a manufacturer produces computers with a 17-inch screen and computers with a 24-inch screen. Past sales experience shows that at least twice as many 17-inch computers are sold as 24-inch computers. Suppose further that the manufacturing plant is capable of producing 12 computers per day. Let x represent the number of 17-inch computers produced per day, and let y represent the number of 24-inch computers produced per day. Then

$$\begin{cases} x \geq 2y \\ x + y \leq 12 \end{cases}$$ • These are the constraints.

TO REVIEW

Systems of Inequalities
See page 517.

These two inequalities place constraints, or restrictions, on the manufacturer. For example, the manufacturer cannot produce five 24-inch computers per day, because that would require producing at least ten 17-inch computers, and $5 + 10 \not\leq 12$.

Suppose a profit of \$50 is earned on each 17-inch computer sold and \$75 is earned on each 24-inch computer sold. Then the manufacturer's daily profit P, in dollars, is given by the equation

$$P = 50x + 75y$$ • Objective function

The equation $P = 50x + 75y$ defines the objective function. The goal of this linear programming problem is to determine how many of each computer should be produced to maximize the manufacturer's profit and satisfy the constraints.

Because the manufacturer cannot produce fewer than zero units of either computer, there are two other implied constraints: $x \geq 0$ and $y \geq 0$. Our linear programming problem now looks like

Objective function $P = 50x + 75y$

Constraints $\begin{cases} x - 2y \geq 0 \\ x + y \leq 12 \\ x \geq 0, y \geq 0 \end{cases}$

Study tip

The set of feasible solutions includes ordered pairs with whole number coordinates and fractional coordinates. For instance, the ordered pair $\left(5, 2\frac{1}{2}\right)$ is in the set of feasible solutions. During one day, the company could produce 5 17-inch computers and $2\frac{1}{2}$ 24-inch computers.

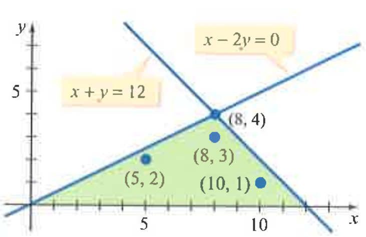


Figure 6.29

To solve this problem, we graph the solution set of the constraints. The solution set of the constraints is called the **set of feasible solutions.** Ordered pairs in this set are used to evaluate the objective function to determine which ordered pair maximizes the profit. For example, (5, 2), (8, 3), and (10, 1) are three ordered pairs in the set. See Figure 6.29. For these ordered pairs, the profits would be

$$P = 50(5) + 75(2) = 400$$ • $x = 5, y = 2$

$$P = 50(8) + 75(3) = 625$$ • $x = 8, y = 3$

$$P = 50(10) + 75(1) = 575$$ • $x = 10, y = 1$

It would be impossible to check every ordered pair in the set of feasible solutions to find which one maximizes profit. Fortunately, we can find that ordered pair by solving the objective function $P = 50x + 75y$ for y.

$$y = -\frac{2}{3}x + \frac{P}{75}$$

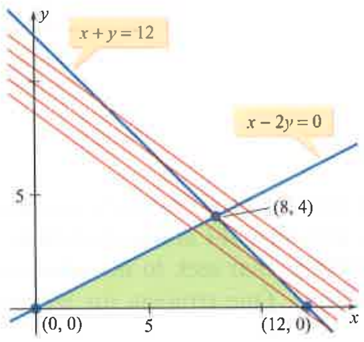


Figure 6.30

In this form, the objective function is a linear equation whose graph has a slope of $-\frac{2}{3}$ and a y-intercept of $\frac{P}{75}$. If P is as large as possible (P a maximum), then the y-intercept will be as large as possible. Thus the maximum profit will occur on the line that has a slope of $-\frac{2}{3}$, has the largest possible y-intercept, and intersects the set of feasible solutions.

From Figure 6.30, the largest possible y-intercept occurs when the line passes through the point with the coordinates $(8, 4)$. At this point, the profit is

$$P = 50(8) + 75(4) = 700$$

The manufacturer will maximize profit by producing 8 17-inch computers and 4 24-inch computers each day. The profit will be \$700 per day.

In general, the goal of any linear programming problem is to maximize or minimize the objective function, subject to the constraints. Minimization problems occur, for example, when a manufacturer wants to minimize the cost of operations.

Suppose that a cost minimization problem results in the following objective function and constraints.

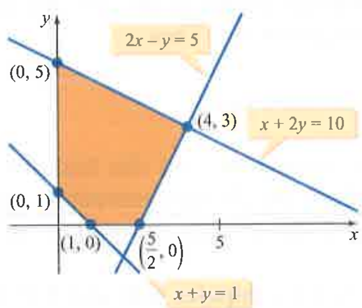


Figure 6.31

Objective function $C = 3x + 4y$

Constraints $\begin{cases} x + y \geq 1 \\ 2x - y \leq 5 \\ x + 2y \leq 10 \\ x \geq 0, y \geq 0 \end{cases}$

Figure 6.31 is the graph of the solution set of the constraints. The task is to find the ordered pair that satisfies all the constraints and gives the smallest value of C. We again could solve the objective function for y and, because we want to minimize C, find the smallest y-intercept. However, a theorem from linear programming simplifies our task even more. The proof of this theorem, omitted here, is based on the techniques we used to solve our examples.

Fundamental Linear Programming Theorem

If an objective function has an optimal solution, then that solution will be at a vertex of the set of feasible solutions.

Following is a list of the values of C at the vertices. The minimum value of the objective function occurs at the point whose coordinates are $(1, 0)$.

(x, y)	$C = 3x + 4y$	
$(1, 0)$	$C = 3(1) + 4(0) = 3$	• Minimum
$\left(\frac{5}{2}, 0\right)$	$C = 3\left(\frac{5}{2}\right) + 4(0) = 7.5$	
$(4, 3)$	$C = 3(4) + 4(3) = 24$	• Maximum
$(0, 5)$	$C = 3(0) + 4(5) = 20$	
$(0, 1)$	$C = 3(0) + 4(1) = 4$	

The maximum value of the objective function can also be determined from the list. It occurs at (4, 3).

It is important to realize that the maximum or minimum value of an objective function depends on the objective function *and* on the set of feasible solutions. For example, using the set of feasible solutions in Figure 6.31 but changing the objective function to $C = 2x + 5y$ changes the maximum value of C to 25, at the ordered pair (0, 5).

Question • What is the minimum value of the objective function $C = 2x + 5y$ for the set of feasible solutions in Figure 6.31?

Solving Optimization Problems

EXAMPLE 1 Solve a Minimization Problem

Minimize the objective function $C = 4x + 7y$ with the constraints

$$\begin{cases} 3x + y \geq 6 \\ x + y \geq 4 \\ x + 3y \geq 6 \\ x \geq 0, y \geq 0 \end{cases}$$

Solution

Determine the set of feasible solutions by graphing the solution set of the inequalities. See Figure 6.32. Note that in this instance the set of feasible solutions is an unbounded region with four vertices. The vertex on the y-axis is the y-intercept of the line $3x + y = 6$, which is (0, 6). The vertex on the x-axis is the x-intercept of the line $x + 3y = 6$, which is (6, 0).

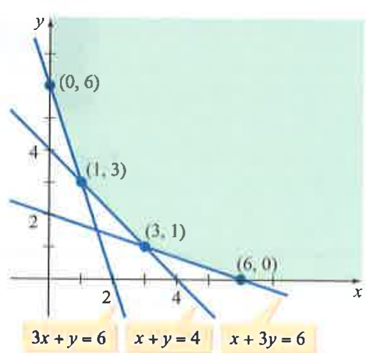


Figure 6.32

To find the other two vertices of the region, solve the following systems of equations.

$$\begin{cases} 3x + y = 6 \\ x + y = 4 \end{cases} \qquad \begin{cases} x + 3y = 6 \\ x + y = 4 \end{cases}$$

The solutions of the two systems are (1, 3) and (3, 1), respectively.

(continued)

Answer • 2.

Evaluate the objective function at each of the four vertices of the set of feasible solutions.

(x, y)	$C = 4x + 7y$	
$(0, 6)$	$C = 4(0) + 7(6) = 42$	
$(1, 3)$	$C = 4(1) + 7(3) = 25$	
$(3, 1)$	$C = 4(3) + 7(1) = 19$	• Minimum
$(6, 0)$	$C = 4(6) + 7(0) = 24$	

The minimum value of the objective function is 19 at $(3, 1)$.

▶ Try Exercise 18, page 530

Linear programming can be used to determine the best allocation of the resources available to a company. In fact, the word *programming* refers to a "program to allocate resources."

EXAMPLE 2 Solve an Applied Minimization Problem

A manufacturer of animal food makes two grain mixtures, G_1 and G_2. Each mixture contains vitamins, proteins, and carbohydrates, in the proportions shown below.

Each kilogram contains
300 grams of vitamins
400 grams of protein
100 grams of carbohydrates

Each kilogram contains
100 grams of vitamins
300 grams of protein
200 grams of carbohydrates

Minimum nutritional guidelines require that a feed mixture made from these grains contain at least 900 grams of vitamins, 2200 grams of protein, and 800 grams of carbohydrates. G_1 costs \$2.00 per kilogram to produce, and G_2 costs \$1.25 per kilogram to produce. Find the number of kilograms of each grain mixture that should be produced to minimize cost.

Solution

Let

$$x = \text{the number of kilograms of } G_1$$
$$y = \text{the number of kilograms of } G_2$$

The objective function is the cost function $C = 2x + 1.25y$.

Because x kilograms of G_1 contains $300x$ grams of vitamins and y kilograms of G_2 contains $100y$ grams of vitamins, the total amount

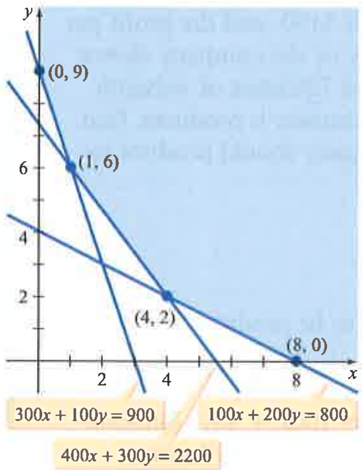


Figure 6.33

of vitamins contained in x kilograms of G_1 and y kilograms of G_2 is $300x + 100y$. At least 900 grams of vitamins are necessary, so $300x + 100y \geq 900$. Following similar reasoning, we have the constraints

$$\begin{cases} 300x + 100y \geq 900 \\ 400x + 300y \geq 2200 \\ 100x + 200y \geq 800 \\ x \geq 0, y \geq 0 \end{cases}$$

Two of the vertices of the set of feasible solutions (see Figure 6.33) can be found by solving two systems of equations. These systems are formed by the equations of the lines that intersect in Quadrant I.

$$\begin{cases} 300x + 100y = 900 \\ 400x + 300y = 2200 \end{cases}$$ • The vertex is (1, 6).

$$\begin{cases} 100x + 200y = 800 \\ 400x + 300y = 2200 \end{cases}$$ • The vertex is (4, 2).

The vertices on the x- and y-axes are $(8, 0)$, the x-intercept of $100x + 200y = 800$, and $(0, 9)$, the y-intercept of $300x + 100y = 900$.

Substitute the coordinates of the vertices into the objective function.

(x, y)	$C = 2x + 1.25y$	
$(0, 9)$	$C = 2(0) + 1.25(9) = 11.25$	
$(1, 6)$	$C = 2(1) + 1.25(6) = 9.50$	• Minimum
$(4, 2)$	$C = 2(4) + 1.25(2) = 10.50$	
$(8, 0)$	$C = 2(8) + 1.25(0) = 16.00$	

The minimum value of the objective function is \$9.50. It occurs when the company produces a feed mixture that contains 1 kilogram of G_1 and 6 kilograms of G_2.

▶ **Try Exercise 30, page 531**

EXAMPLE 3 Solve an Applied Maximization Problem

A company manufactures two types of cleansers. One is an all-purpose cleanser (*AP*) and the other is an industrial strength cleanser (*IS*). Each cleanser is a mixture of three chemicals, as shown below.

Each kiloliter requires
12 liters of surfactants
9 liters of enzymes
30 liters of solvents

Each kiloliter requires
24 liters of surfactants
5 liters of enzymes
30 liters of solvents

(continued)

The profit per kiloliter from the *AP* cleanser is \$100, and the profit per kiloliter from the *IS* cleanser is \$85. The inventory of the company shows 480 liters of surfactants, 180 liters of enzymes, and 720 liters of solvents available. Assuming the company can sell all the cleanser it produces, find the number of kiloliters of each cleanser the company should produce to maximize profit. What is the maximum profit?

Solution

Let

x = the number of kiloliters of *AP* to be produced

y = the number of kiloliters of *IS* to be produced

The objective function is the profit function $P = 100x + 85y$. Because x kiloliters of *AP* requires $12x$ liters of surfactants and y kiloliters of *IS* requires $24y$ liters of surfactants, the total amount of surfactants needed is $12x + 24y$. There are 480 liters of surfactants in inventory, so $12x + 24y \le 480$. Following similar reasoning, we have the constraints

$$\begin{cases} 12x + 24y \le 480 \\ 9x + 5y \le 180 \\ 30x + 30y \le 720 \\ x \ge 0, y \ge 0 \end{cases}$$

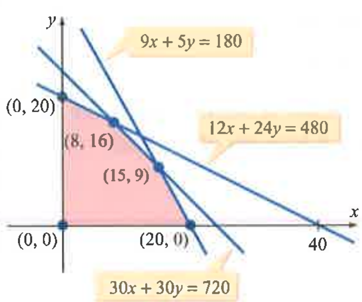


Figure 6.34

Two of the vertices of the set of feasible solutions (see Figure 6.34) can be found by solving two systems of equations. These systems are formed by the equations of the lines that intersect in Quadrant I.

$$\begin{cases} 12x + 24y = 480 \\ 30x + 30y = 720 \end{cases}$$ • The vertex is (8, 16).

$$\begin{cases} 9x + 5y = 180 \\ 30x + 30y = 720 \end{cases}$$ • The vertex is (15, 9).

The vertices on the x- and y-axes are the x- and y-intercepts $(20, 0)$ and $(0, 20)$.

Substitute the coordinates of the vertices into the objective function.

(x, y)	$P = 100x + 85y$	
$(0, 20)$	$P = 100(0) + 85(20) = 1700$	
$(8, 16)$	$P = 100(8) + 85(16) = 2160$	
$(15, 9)$	$P = 100(15) + 85(9) = 2265$	• Maximum
$(20, 0)$	$P = 100(20) + 85(0) = 2000$	

The maximum value of the objective function is \$2265 when the company produces 15 kiloliters of the all-purpose cleanser and 9 kiloliters of the industrial strength cleanser.

▶ Try Exercise 32, page 531

EXERCISE SET 6.6

Concept Check

1. What is an optimization problem?

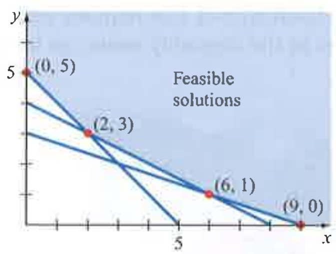
2. What is the set of feasible solutions for a linear programming problem?

3. If a linear programming problem has an optimal solution, where in the set of feasible solutions must that solution occur?

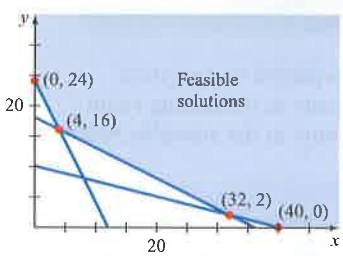
4. A maximization problem has a maximum value of $1225, at (300, 50). If a change is made to the objective function in this problem, might this change produce a different maximum solution, which occurs at a different ordered pair?

In Exercises 5 and 6, find the minimum value of the given objective function for the given set of feasible solutions. Also state where the objective function takes on its minimum value.

5. Objective function: $C = 3x + 4y$

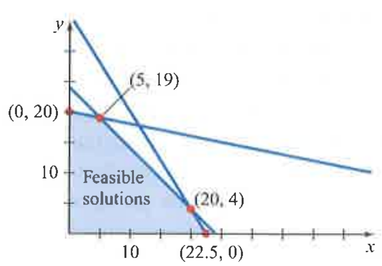


6. Objective function: $C = 12x + 2y + 48$



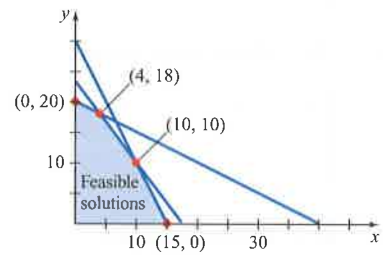
In Exercises 7 and 8, find the maximum value of the given objective function for the given set of feasible solutions. Also state where the objective function takes on its maximum value.

7. Objective function: $C = 2.5x + 3y + 5$



Indicates Try It Exercises

8. Objective function: $C = 6x + 4y + 15$



In Exercises 9 to 26, solve the linear programming problem. Assume $x \geq 0$ and $y \geq 0$.

9. Minimize $C = 4x + 2y$ with the constraints

$$\begin{cases} x + y \geq 7 \\ 4x + 3y \geq 24 \\ x \leq 10, y \leq 10 \end{cases}$$

10. Minimize $C = 5x + 4y$ with the constraints

$$\begin{cases} 3x + 4y \geq 32 \\ x + 4y \geq 24 \\ x \leq 12, y \leq 15 \end{cases}$$

11. Maximize $C = 6x + 7y$ with the constraints

$$\begin{cases} x + 2y \leq 16 \\ 5x + 3y \leq 45 \end{cases}$$

12. Maximize $C = 6x + 5y$ with the constraints

$$\begin{cases} 2x + 3y \leq 27 \\ 7x + 3y \leq 42 \end{cases}$$

13. Maximize $C = x + 6y$ with the constraints

$$\begin{cases} 5x + 8y \leq 120 \\ 7x + 16y \leq 192 \end{cases}$$

14. Minimize $C = 4x + 5y$ with the constraints

$$\begin{cases} x + 3y \geq 30 \\ 3x + 4y \geq 60 \end{cases}$$

15. Minimize $C = 4x + y$ with the constraints

$$\begin{cases} 3x + 5y \geq 120 \\ x + y \geq 32 \end{cases}$$

16. Maximize $C = 7x + 2y$ with the constraints

$$\begin{cases} x + 3y \leq 108 \\ 7x + 4y \leq 280 \end{cases}$$

17. Maximize $C = 2x + 7y$ with the constraints

$$\begin{cases} x + y \leq 10 \\ x + 2y \leq 16 \\ 2x + y \leq 16 \end{cases}$$

18. Minimize $C = 4x + 3y$ with the constraints

$$\begin{cases} 2x + y \geq 8 \\ 2x + 3y \geq 16 \\ x + 3y \geq 11 \\ x \leq 20, y \leq 20 \end{cases}$$

19. Minimize $C = 5x + 2.5y$ with the constraints

$$\begin{cases} 3x + y \geq 12 \\ 2x + 7y \geq 21 \\ x + y \geq 8 \end{cases}$$

20. Maximize $C = 7x + 6y$ with the constraints

$$\begin{cases} x + y \leq 12 \\ 3x + 4y \leq 40 \\ x + 2y \leq 18 \end{cases}$$

21. Maximize $C = x + 4y$ with the constraints

$$\begin{cases} 2x + y \leq 10 \\ 2x + 3y \leq 18 \\ x - y \leq 2 \end{cases}$$

22. Minimize $C = 4x + 2y$ with the constraints

$$\begin{cases} x + y \geq 9 \\ 3x + 4y \geq 32 \\ x + 2y \geq 12 \end{cases}$$

23. Minimize $C = 3x + 2y$ with the constraints

$$\begin{cases} x + 2y \geq 8 \\ 3x + y \geq 9 \\ x + 4y \geq 12 \end{cases}$$

24. Maximize $C = 4x + 5y$ with the constraints

$$\begin{cases} 3x + 4y \leq 250 \\ x + y \leq 75 \\ 2x + 3y \leq 180 \end{cases}$$

25. Maximize $C = 6x + 7y$ with the constraints

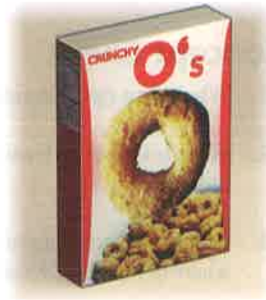
$$\begin{cases} x + 2y \leq 900 \\ x + y \leq 500 \\ 3x + 2y \leq 1200 \end{cases}$$

26. Minimize $C = 11x + 16y$ with the constraints

$$\begin{cases} x + 2y \geq 45 \\ x + y \geq 40 \\ 2x + y \geq 45 \end{cases}$$

27. **Minimize Cost** A dietician formulates a special breakfast cereal by mixing Oat Flakes and Crunchy O's. The cereals each provide protein and carbohydrates in the amounts shown below.

1 cup: 6 grams of protein;
30 grams of carbohydrates



1 cup: 3 grams of protein;
40 grams of carbohydrates

The dietician wants to produce a mixture that contains at least 210 grams of protein and at least 1200 grams of carbohydrates. The cost is 38 cents for 1 cup of Oat Flakes and 32 cents for 1 cup of Crunchy O's. How many cups of each cereal will satisfy the constraints and minimize the cost? What is the minimum cost?

28. **Maximize Profit** A tent manufacturer makes a two-person tent and a family tent. Each type of tent requires time in the cutting room and time in the assembly room, as indicated below.

A two-person tent requires
2 hours in the cutting room
2 hours in the assembly room

A family tent requires
2 hours in the cutting room
4 hours in the assembly room

The total number of hours available per week in the cutting room is 50. There are 80 hours available per week in the assembly room. The manager requires that the number of two-person tents manufactured be no more than four times the number of family tents manufactured. The profit for the two-person tent is $34, and the profit for the family tent is $49. Assuming that all the tents produced can be sold, how many of each should be manufactured per week to maximize the profit? What is the maximum profit?

29. **Maximize Profit** A farmer is planning to raise wheat and barley. Each acre of wheat yields a profit of $50, and each acre of barley yields a profit of $70. To sow the crop, two machines, a tractor and a tiller, are rented. The tractor is available for 200 hours, and the tiller is available for 100 hours. Sowing an acre of barley requires 3 hours of tractor time and 2 hours of tilling. Sowing an acre of wheat requires 4 hours of tractor time and 1 hour of tilling. How many acres of each crop should be planted to maximize the farmer's profit?

30. **Minimize Cost** An ice cream supplier has two machines that produce vanilla and chocolate ice cream. The production rates of each machine are shown below.

Machine 1 produces
4 gallons of vanilla per hour
5 gallons of chocolate per hour

Machine 2 produces
3 gallons of vanilla per hour
10 gallons of chocolate per hour

To meet one of its contractual obligations, the company must produce at least 60 gallons of vanilla ice cream and 100 gallons of chocolate ice cream per day. It costs $28 per hour to run machine 1 and $25 per hour to run machine 2. How many hours should each machine be operated to fulfill the contract at the least expense?

31. **Maximize Profit** A small skateboard company manufactures two types of skateboards. Each type of skateboard has labor requirements as indicated below.

An economy board requires
2 hours for cutting and laminating
2 hours for finishing

A superior board requires
2.5 hours for cutting and laminating
4 hours for finishing

The cutting and laminating employees are available for 240 hours per week, and the finishing room employees are available for 312 hours per week. The profit from each economy board is $26, and the profit from each superior board is $42. Determine how many of each model should be manufactured, per week, to maximize profit. Assume the company can sell all the skateboards it produces. What is the maximum weekly profit?

32. **Maximize Profit** A company makes two types of telephone answering machines: the standard model and the deluxe model. Each machine passes through three processes: P_1, P_2, and P_3. One standard answering machine requires 1 hour in P_1, 1 hour in P_2, and 2 hours in P_3. One deluxe answering machine requires 3 hours in P_1, 1 hour in P_2, and 1 hour in P_3. Because of employee work schedules, P_1 is available for 24 hours, P_2 is available for 10 hours, and P_3 is available for 16 hours. If the profit is $25 for each standard model and $35 for each deluxe model, how many units of each type should the company produce to maximize profit?

33. **Minimize Cost** A dietician formulates a special diet from two food groups: A and B. Each ounce of food group A contains 3 units of vitamin A, 1 unit of vitamin C, and 1 unit of vitamin D. Each ounce of food group B contains 1 unit of vitamin A, 1 unit of vitamin C, and 3 units of vitamin D. Each ounce of food group A costs 40 cents, and each ounce of food group B costs 10 cents. The dietary constraints are such that at least 24 units of vitamin A, 16 units of vitamin C, and 30 units of vitamin D are required. Find the amount of each food group that should be used to minimize the cost. What is the minimum cost?

34. **Maximize Profit** Among the many products it produces, an oil refinery makes two specialized petroleum distillates: Pymex A and Pymex B. Each distillate passes through three stages: S_1, S_2, and S_3. Each liter of Pymex A requires 1 hour in S_1, 3 hours in S_2, and 3 hours in S_3. Each liter of Pymex B requires 1 hour in S_1, 4 hours in S_2, and 2 hours in S_3. There are 10 hours available for S_1, 36 hours available for S_2, and 27 hours available for S_3. The profit per liter of Pymex A is $12, and the profit per liter of Pymex B is $9. How many liters of each distillate should be produced to maximize profit? What is the maximum profit?

35. **Maximize Profit** A company sells a chocolate-covered strawberries platter and a crème brûlée. Each of these desserts requires preparation time, assembly time, and packing time as indicated below.

Darren K. Fisher/Shutterstock.com

Lestertair/Shutterstock.com

Each strawberry platter requires
- 4 minutes to prepare ingredients
- 3 minutes to assemble
- 1 minute to pack.

Each crème brûlée requires
- 6 minutes to prepare ingredients
- 6 minutes to assemble
- 0.75 minute to pack.

The number of employee minutes available per day in the:

- preparation area is 1200 minutes.
- assembly kitchen is 990 minutes.
- packing department is 300 minutes

The profit for each strawberry platter is \$3.25 and the profit for each crème brûlée is \$5.00. How many of each dessert should be produced per day to maximize the company's profit? Assume that all of the desserts that are produced each day can be sold. What is the maximum daily profit?

36. **Minimize Cost** A producer of animal feed makes two food products: F_1 and F_2. The products contain three major ingredients: M_1, M_2, and M_3. Each ton of F_1 requires 200 pounds of M_1, 100 pounds of M_2, and 100 pounds of M_3. Each ton of F_2 requires 100 pounds of M_1, 200 pounds of M_2, and 400 pounds of M_3. There are at least 5000 pounds of M_1 available, at least 7000 pounds of M_2 available, and at least 10,000 pounds of M_3 available. Each ton of F_1 costs \$450 to make, and each ton of F_2 costs \$300 to make. How many tons of each food product should the feed producer make to minimize cost? What is the minimum cost?

Enrichment Exercise

37. Find both the minimum value and the maximum value of $C = 5x + 3y$ subject to the following constraints.

$$\begin{cases} x - y \geq -2 \\ x + 4y \geq 18 \\ 4x + y \leq 42 \end{cases}$$

Exploring Concepts with Technology

Use WolframAlpha to Solve a Linear Programming Problem

Consider the following linear programming problem.

Maximize $C = x + y$ with the following constraints:

$$\begin{cases} 2x + y \leq 12 \\ 0.3x + y \leq 6 \\ 8x + 7y \leq 56 \\ x \geq 0, y \geq 0 \end{cases}$$

Scan the following QR code to access WolframAlpha on a mobile device.

www.wolframalpha.com

You can use WolframAlpha to solve this maximization problem by entering the objective function and the constraints as shown below.

maximize [{x + y, 2x + y ≤ 12 && 0.3x + y ≤ 6 && 8x + 7y ≤ 56 && x ≥ 0 && y ≥ 0}, {x, y}]

Objective function followed by a comma

5 constraints separated by && symbol

Click on the equal sign icon to display the approximate solution 7.66102, which occurs at $(x, y) \approx (2.37288, 5.28814)$.

If you receive a "Wolfram|Alpha doesn't understand your query" message, then something has not been entered properly. Edit your entry so that it appears exactly as shown above. If you have difficulty entering the "≤" symbols and the "≥" symbols, then just use a "<" symbol in place of each "≤" symbol and a ">" symbol in place of each "≥" symbol.

After you have found the solution, you can edit one of the constraints, or the objective function, to see what effect this has on the solution and tell whether the new solution still occurs at the same ordered pair. For instance, edit the objective function so that it becomes $3x + y$. Then you will find that the new solution is 18 and it occurs at $(x, y) = (6, 0)$.

Use the following text to produce a graph of the set of feasible solutions.

```
plot {2x + y ≤ 12 && 0.3x + y ≤ 6 && 8x + 7y ≤ 56 && x ≥ 0 && y ≥ 0}
```

CHAPTER 6 TEST PREP

The following test prep table summarizes essential concepts in this chapter. The references given in the right-hand column list Examples and Exercises that can be used to test your understanding of a concept.

6.1 Systems of Linear Equations in Two Variables

Systems of Linear Equations in Two Variables A solution of a system of two linear equations in two variables is an ordered pair that satisfies each equation of the system. Systems of equations are equivalent if the systems have exactly the same solutions. The substitution method and the elimination method are often used to solve these systems.

- ***Substitution Method*** Solve one of the equations to find an expression for one variable in terms of the other variable. Substitute this expression into the other equation to produce an equation that involves only one variable.
- ***Elimination Method*** Multiply one or both equations by appropriate nonzero constants so that the sum of the resulting equations is an equation in one variable.

  The elimination method uses the following operations to produce equivalent systems until the solution or solutions of the original system are apparent.

  1. Interchange any two equations.
  2. Replace an equation with a nonzero constant multiple of that equation.
  3. Replace an equation with the sum of that equation and a nonzero constant multiple of another equation.

See Examples 1 and 4, pages 479 and 482, and then try Exercises 2 and 3, page 536.

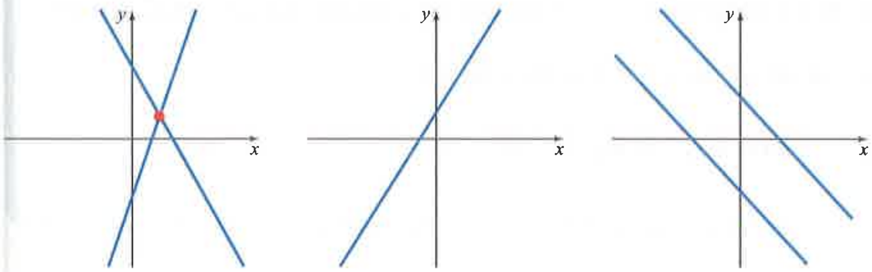
Classification of Systems of Equations A system of equations is a consistent system if it has at least one solution. A system of equations with no solution is an inconsistent system.

- A system of linear equations with exactly one solution is an independent system. A system of linear equations with an infinite number of solutions is a dependent system.

  The graphs of the two equations in a linear system of two variables can intersect at a single point, be the same line, or be parallel lines. See the graphs on page 534.

See Examples 2 and 3, pages 479 and 480, and then try Exercises 7 and 8, page 536.

The graphs intersect at a single point. | Both equations graph to be the same line. | The graphs are parallel lines.



a. Independent system Exactly one solution | **b.** Dependent system Infinitely many solutions | **c.** Inconsistent system No solution

6.2 Systems of Linear Equations in Three Variables

Systems of Linear Equations in Three Variables An equation of the form $ax + by + cz = d$, with constants a, b, and c not all zero, is a linear equation in three variables. A solution of a linear system of equations in three variables is an ordered triple whose coordinates satisfy each of the equations in the system. The elimination method is often used to solve systems of linear equations in three variables by rewriting the system in an equivalent triangular form.	See Examples 1 and 2, pages 491 and 492, and then try Exercises 9 and 11, page 536.
Nonsquare Systems of Equations A system of linear equations with fewer equations than variables forms a nonsquare system of equations. These systems of equations have either no solution or an infinite number of solutions. The elimination method can often be used to solve these systems.	See Example 4, page 495, and then try Exercises 17 and 18, page 536.
Homogeneous Systems of Equations A system of linear equations in which the constant term is 0 for all equations is called a homogeneous system of equations. The ordered triple (0, 0, 0) is always a solution of a homogeneous system of linear equations in three variables. This solution is called the trivial solution. A homogeneous system of linear equations will have exactly one solution (the trivial solution) or an infinite number of solutions.	See Example 5, page 495, and then try Exercises 15 and 16, page 536.

6.3 Nonlinear Systems of Equations

Solutions of Nonlinear Systems of Equations A nonlinear system of equations is a system in which one or more equations of the system are nonlinear. The substitution method and the elimination method are often used to find the solutions to these systems. A graph of the equations in the system can be used to visualize how many solutions to expect and the approximate coordinates of the solutions.	See Examples 1 and 2, pages 501 and 502, and then try Exercises 23 and 28, page 536.