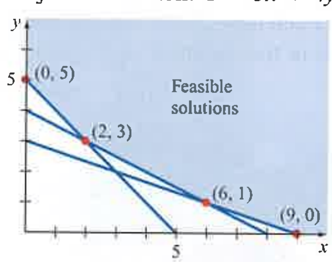
EXERCISE SET 6.6

Concept Check

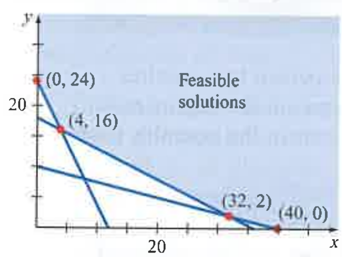
1. What is an optimization problem? It is a problem that seeks a solution that will maximize or minimize a situation.
2. What is the set of feasible solutions for a linear programming problem? The set of feasible solutions is the solution set of the constraints.
3. If a linear programming problem has an optimal solution, where in the set of feasible solutions must that solution occur? It must occur at a vertex of the set of feasible solutions.
4. A maximization problem has a maximum value of \$1225, at (300, 50). If a change is made to the objective function in this problem, might this change produce a different maximum solution, which occurs at a different ordered pair? Yes

In Exercises 5 and 6, find the minimum value of the given objective function for the given set of feasible solutions. Also state where the objective function takes on its minimum value.

5. Objective function: $C = 3x + 4y$ The minimum is 18 at (2, 3).

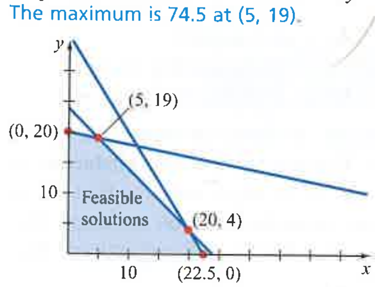


6. Objective function: $C = 12x + 2y + 48$
The minimum is 96 at (0, 24).

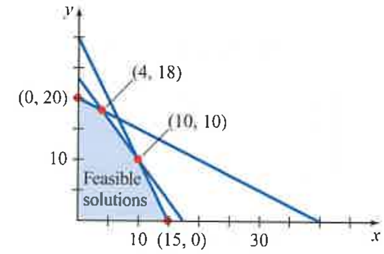


In Exercises 7 and 8, find the maximum value of the given objective function for the given set of feasible solutions. Also state where the objective function takes on its maximum value.

7. Objective function: $C = 2.5x + 3y + 5$
The maximum is 74.5 at (5, 19).



8. Objective function: $C = 6x + 4y + 15$
The maximum is 115 at (10, 10).



In Exercises 9 to 26, solve the linear programming problem. Assume $x \geq 0$ and $y \geq 0$.

9. Minimize $C = 4x + 2y$ with the constraints

The minimum is 16 at (0, 8).

$$\begin{cases} x + y \geq 7 \\ 4x + 3y \geq 24 \\ x \leq 10, y \leq 10 \end{cases}$$

10. Minimize $C = 5x + 4y$ with the constraints

The minimum is 32 at (0, 8).

$$\begin{cases} 3x + 4y \geq 32 \\ x + 4y \geq 24 \\ x \leq 12, y \leq 15 \end{cases}$$

11. Maximize $C = 6x + 7y$ with the constraints

The maximum is 71 at (6, 5).

$$\begin{cases} x + 2y \leq 16 \\ 5x + 3y \leq 45 \end{cases}$$

12. Maximize $C = 6x + 5y$ with the constraints

The maximum is 53 at (3, 7).

$$\begin{cases} 2x + 3y \leq 27 \\ 7x + 3y \leq 42 \end{cases}$$

13. Maximize $C = x + 6y$ with the constraints

The maximum is 72 at (0, 12).

$$\begin{cases} 5x + 8y \leq 120 \\ 7x + 16y \leq 192 \end{cases}$$

14. Minimize $C = 4x + 5y$ with the constraints

The minimum is 75 at (0, 15).

$$\begin{cases} x + 3y \geq 30 \\ 3x + 4y \geq 60 \end{cases}$$

15. Minimize $C = 4x + y$ with the constraints

The minimum is 32 at (0, 32).

$$\begin{cases} 3x + 5y \geq 120 \\ x + y \geq 32 \end{cases}$$

16. Maximize $C = 7x + 2y$ with the constraints

The maximum is 280 at (40, 0).

$$\begin{cases} x + 3y \leq 108 \\ 7x + 4y \leq 280 \end{cases}$$

17. Maximize $C = 2x + 7y$ with the constraints

The maximum is 56 at (0, 8).

$$\begin{cases} x + y \leq 10 \\ x + 2y \leq 16 \\ 2x + y \leq 16 \end{cases}$$

Indicates Try It Exercises

18. Minimize $C = 4x + 3y$ with the constraints

The minimum is 20 at (2, 4).

$$\begin{cases} 2x + y \geq 8 \\ 2x + 3y \geq 16 \\ x + 3y \geq 11 \\ x \leq 20, y \leq 20 \end{cases}$$

19. Minimize $C = 5x + 2.5y$ with the constraints

The minimum is 25 at (2, 6).

$$\begin{cases} 3x + y \geq 12 \\ 2x + 7y \geq 21 \\ x + y \geq 8 \end{cases}$$

20. Maximize $C = 7x + 6y$ with the constraints

The maximum is 84 at (12, 0).

$$\begin{cases} x + y \leq 12 \\ 3x + 4y \leq 40 \\ x + 2y \leq 18 \end{cases}$$

21. Maximize $C = x + 4y$ with the constraints

The maximum is 24 at (0, 6).

$$\begin{cases} 2x + y \leq 10 \\ 2x + 3y \leq 18 \\ x - y \leq 2 \end{cases}$$

22. Minimize $C = 4x + 2y$ with the constraints

The minimum is 18 at (0, 9).

$$\begin{cases} x + y \geq 9 \\ 3x + 4y \geq 32 \\ x + 2y \geq 12 \end{cases}$$

23. Minimize $C = 3x + 2y$ with the constraints

The minimum is 12 at (2, 3).

$$\begin{cases} x + 2y \geq 8 \\ 3x + y \geq 9 \\ x + 4y \geq 12 \end{cases}$$

24. Maximize $C = 4x + 5y$ with the constraints

The maximum is 325 at (50, 25).

$$\begin{cases} 3x + 4y \leq 250 \\ x + y \leq 75 \\ 2x + 3y \leq 180 \end{cases}$$

25. Maximize $C = 6x + 7y$ with the constraints

The maximum is 3400 at (100, 400).

$$\begin{cases} x + 2y \leq 900 \\ x + y \leq 500 \\ 3x + 2y \leq 1200 \end{cases}$$

26. Minimize $C = 11x + 16y$ with the constraints

The minimum is 465 at (35, 5).

$$\begin{cases} x + 2y \geq 45 \\ x + y \geq 40 \\ 2x + y \geq 45 \end{cases}$$

27. **Minimize Cost** A dietician formulates a special breakfast cereal by mixing Oat Flakes and Crunchy O's. The cereals each provide protein and carbohydrates in the amounts shown below.

1 cup: 6 grams of protein; 30 grams of carbohydrates

1 cup: 3 grams of protein; 40 grams of carbohydrates

The dietician wants to produce a mixture that contains at least 210 grams of protein and at least 1200 grams of carbohydrates. The cost is 38 cents for 1 cup of Oat Flakes and 32 cents for 1 cup of Crunchy O's. How many cups of each cereal will satisfy the constraints and minimize the cost? What is the minimum cost? 32 c of Oat Flakes and 6 c of Crunchy O's are needed. The minimum cost is $14.08.

28. **Maximize Profit** A tent manufacturer makes a two-person tent and a family tent. Each type of tent requires time in the cutting room and time in the assembly room, as indicated below.

A two-person tent requires
2 hours in the cutting room
2 hours in the assembly room

A family tent requires
2 hours in the cutting room
4 hours in the assembly room

The total number of hours available per week in the cutting room is 50. There are 80 hours available per week in the assembly room. The manager requires that the number of two-person tents manufactured be no more than four times the number of family tents manufactured. The profit