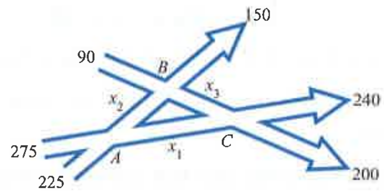
41. Curve Fitting Find the center and radius of the circle whose graph passes through the points $(-2, 10)$, $(-12, -14)$, and $(5, 3)$. (*Hint:* See Exercise 39.)

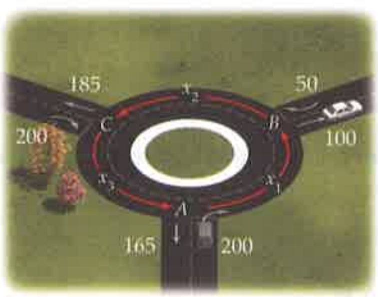
42. Curve Fitting Find the center and radius of the circle whose graph passes through the points $(2, 5)$, $(-4, -3)$, and $(3, 4)$. (*Hint:* See Exercise 39.)

43. Traffic Flow Suppose that the traffic flow for some one-way streets can be modeled by the diagram below, where each number or variable represents the number of cars entering or leaving an intersection per hour.



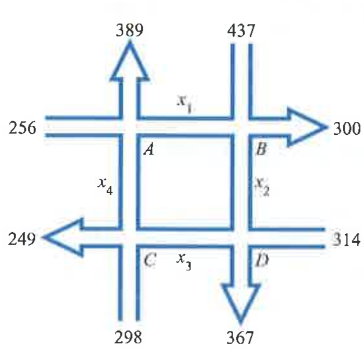
If the street connecting intersections A and B has a traffic flow of 150 to 250 cars per hour, what is the traffic flow between B and C?

44. Traffic Flow A roundabout is a type of intersection that accommodates traffic flow in one direction, around a circular island. The graphic model below shows the numbers of cars per hour that are entering or leaving a roundabout. The variables x_1, x_2, and x_3 represent the traffic flow per hour along the three portions of the roundabout.



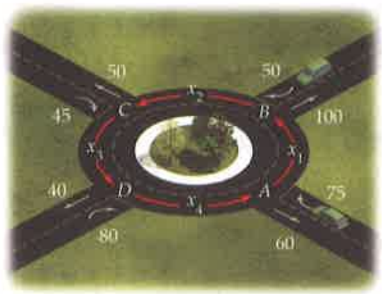
If the portion of the roundabout between A and B has a traffic flow of from 60 to 80 cars per hour, what is the traffic flow between C and A and between B and C?

45. Traffic Flow Suppose that the traffic flow for some one-way streets can be modeled by the accompanying diagram, where each number or variable represents the number of cars entering or leaving an intersection per hour.



If the street connecting intersections A and B has an estimated traffic flow of from 125 to 175 cars per hour, what is the estimated traffic flow between C and A, D and C, and B and D?

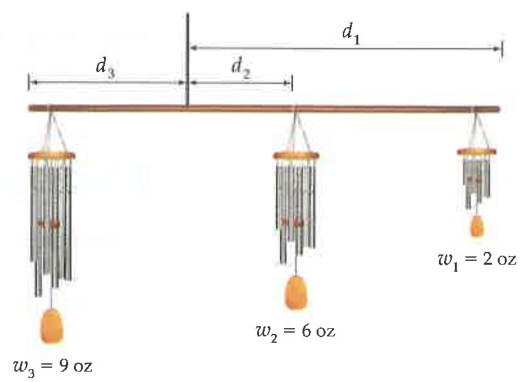
46. Traffic Flow The graphic model below shows the numbers of cars per hour that are entering and leaving a roundabout. What is the minimum number of cars per hour that can travel between B and C?



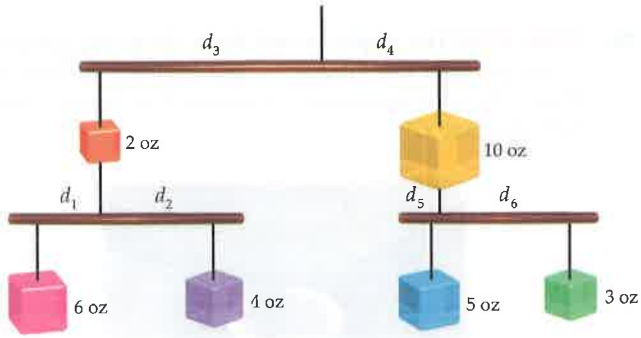
47. Coin Exchange Every day Emily puts her quarters, dimes, and nickels into a shoe box. After several weeks, she decides to take the coins to a machine that counts coins and provides a voucher that she can exchange for dollars. The coin machine charges a fee equal to 10% of the total value of the coins. After the machine has counted her coins, she receives a voucher for \$139.50. The voucher also shows that she had a total of 1025 coins and twice as many quarters as nickels. How many coins of each denomination did Emily have?

48. Hotel Data A hotel resources report listed the total number of hotels in the Holiday Inn, Hilton, and Hyatt hotel chains as 2238. The number of Holiday Inn hotels was 400 more than the total number of Hilton and Hyatt hotels. There were 121 more Hilton hotels than Hyatt hotels. Determine the number of hotels in each of these hotel chains.

49. Art A sculptor is creating a wind chime consisting of three chimes that will be suspended from a rod 13 inches long. The weights, in ounces, of the chimes are shown in the diagram. For the rod to remain horizontal, the chimes must be positioned so that $w_1 d_1 + w_2 d_2 = w_3 d_3$. If the sculptor wants d_2 to be one-third of d_1, find the position of the middle chime that will make the wind chime balance.



50. **Art** A designer wants to create a mobile of colored blocks as shown in the diagram below. The weight, in ounces, of each of the blocks is shown next to the block.



Given that $d_3 + d_4 = 20$ inches, $d_1 + d_2 = 10$ inches, and $d_5 + d_6 = 8$ inches, find the values of d_1 through d_6 so that each bar is horizontal. (A bar is horizontal when the value of weight times distance on each side of a vertical support is equal. For instance, for the diagram above, $6d_1$ must equal $4d_2$. Because there are six variables, the resulting system of equations must contain six equations.)

In Exercises 51 and 52, find an equation of the plane that contains the given points. (*Hint:* The equation of a plane can be written as $z = ax + by + c$.)

51. $(1, -1, 5), (2, -2, 9), (-3, -1, -1)$

52. $(2, 1, 1), (-1, 2, 12), (3, 2, 0)$

Enrichment Exercises

In Exercises 53 and 54, use the system of equations

$$\begin{cases} x - 3y - 2z = A^2 \\ 2x - 5y + Az = 9 \\ 2x - 8y + z = 18 \end{cases}$$

53. Find all values of A for which the system has no solution.

54. Find all values of A for which the system has a unique solution.

In Exercises 55 and 56, use the system of equations

$$\begin{cases} x + 2y + z = A^2 \\ -2x - 3y + Az = 1 \\ 7x + 12y + A^2z = 4A^2 - 3 \end{cases}$$

55. Find all values of A for which the system has a unique solution.

56. Find all values of A for which the system has an infinite number of solutions.

SECTION 6.3

Solving Nonlinear Systems of Equations

Nonlinear Systems of Equations

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A39.

PS1. Solve $x^2 + 2x - 2 = 0$ for x. [1.3]

PS2. Solve: $\begin{cases} x + 4y = -11 \\ 3x - 2y = 9 \end{cases}$ [6.1]

PS3. Name the graph of $(y + 3)^2 = 8x$. [5.1]

PS4. Name the graph of $\dfrac{(x-2)^2}{4} - \dfrac{(y+3)^2}{9} = 1$. [5.3]

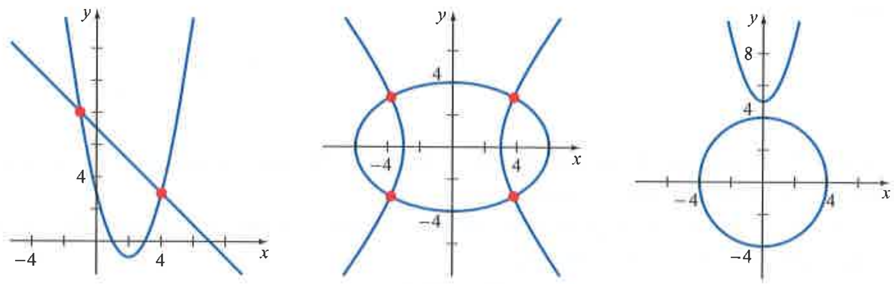
PS5. How many times do the graphs of $y = 2x - 1$ and $x^2 + y^2 = 4$ intersect? [2.1/2.2]

PS6. How many times do the graphs of $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ and $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ intersect? [5.2]

Solving Nonlinear Systems of Equations

A **nonlinear system of equations** is one in which one or more equations of the system are not linear equations. In this section, we will consider only solutions whose coordinates are real numbers. Therefore, if a system of equations does not have any solution in which both coordinates are real numbers, we will simply state that the system has no solution.

Figure 6.17 shows examples of nonlinear systems of equations and the corresponding graphs of the equations. Each point of intersection of the graphs is a solution of the system of equations. In the third example, the graphs do not intersect; therefore, the system of equations has no solution.



$\begin{cases} y = x^2 - 4x + 3 \\ y = -x + 7 \end{cases}$

Two solutions

$\begin{cases} \dfrac{x^2}{36} + \dfrac{y^2}{16} = 1 \\ \dfrac{x^2}{9} - \dfrac{y^2}{16} = 1 \end{cases}$

Four solutions

$\begin{cases} y = x^2 + 5 \\ x^2 + y^2 = 16 \end{cases}$

No solution

Figure 6.17

Question • Is $\begin{cases} x + y = 1 \\ xy = 1 \end{cases}$ a nonlinear system of equations?

To solve a nonlinear system of equations, use the substitution method or the elimination method. The substitution method is usually easier for solving a nonlinear system that contains a linear equation.

EXAMPLE 1 Solve a Nonlinear System by the Substitution Method

Solve: $\begin{cases} y = x^2 - x - 1 & (1) \\ 3x - y = 4 & (2) \end{cases}$

Algebraic Solution

We will use the substitution method. Using the equation $y = x^2 - x - 1$, substitute the expression for y into $3x - y = 4$.

$$3x - y = 4$$

$$3x - (x^2 - x - 1) = 4 \quad \bullet\ y = x^2 - x - 1$$

$$-x^2 + 4x + 1 = 4 \quad \bullet\ \text{Simplify.}$$

$$x^2 - 4x + 3 = 0 \quad \bullet\ \text{Write the quadratic equation in standard form.}$$

$$(x - 3)(x - 1) = 0 \quad \bullet\ \text{Solve for } x.$$

$$x - 3 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = 3 \quad \text{or} \quad x = 1$$

Substitute these values into Equation (1) and solve for y.

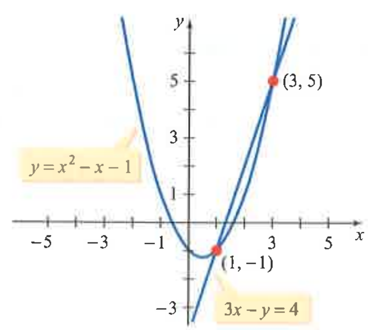
$$y = 3^2 - 3 - 1 = 5 \quad \text{or} \quad y = 1^2 - 1 - 1 = -1$$

The solutions are $(3, 5)$ and $(1, -1)$. Check by showing that $(3, 5)$ and $(1, -1)$ satisfy both equations in the original system.

▸ **Try Exercise 12, page 505**

Visualize the Solution

Graphing $y = x^2 - x - 1$ and $3x - y = 4$ shows that the points located at $(1, -1)$ and $(3, 5)$ belong to each graph. Therefore, these ordered pairs are the solutions of the system of equations.



Answer • Yes.

Integrating Technology

Use a Graphing Utility and WolframAlpha to Solve a Nonlinear System of Equations

You can use a graphing utility to solve some nonlinear systems of equations in two variables. For instance, to solve

$$\begin{cases} y = x^2 - 2x + 2 \\ y = x^3 + 2x^2 - 7x - 3 \end{cases}$$

enter X²–2X+2 into Y₁ and X^3+2X²–7X–3 into Y₂ and graph the two equations. Use a viewing window that shows all points of intersection.

Now repeat the procedure described in the Integrating Technology box on page 483, three times to find the intersection points shown below.

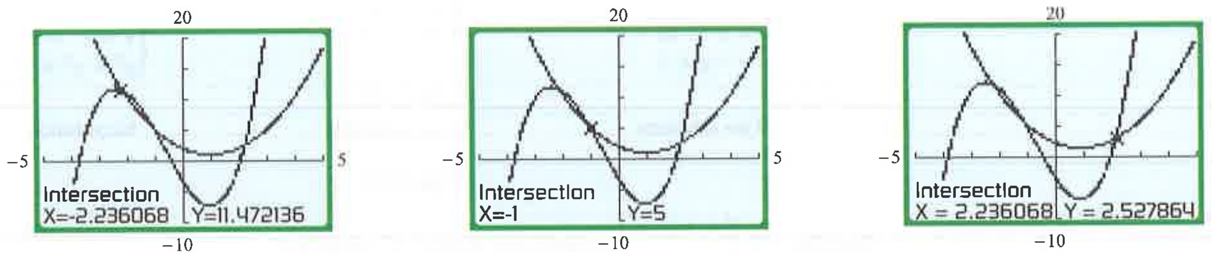


Figure 6.18

Approximate solutions of the system are (–2.24, 11.47), (–1, 5), and (2.24, 2.53).

WolframAlpha can also be used to solve the above nonlinear system of equations. Just enter the equations separated by a comma, as shown below.

y = x^2 - 2x + 2, y = x^3 + 2x^2 - 7x - 3

Click on the equal sign icon to display the exact solutions:

$$(-\sqrt{5}, 7 + 2\sqrt{5}), (-1, 5), (\sqrt{5}, 7 - 2\sqrt{5})$$

EXAMPLE 2 Solve a Nonlinear System by the Elimination Method

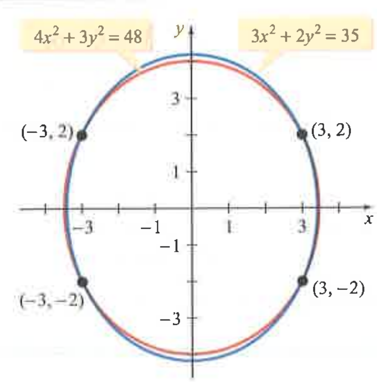
Solve: $\begin{cases} 4x^2 + 3y^2 = 48 & (1) \\ 3x^2 + 2y^2 = 35 & (2) \end{cases}$

Algebraic Solution

We will eliminate the x^2 term. Multiply Equation (1) by -3 and Equation (2) by 4. Then add the two equations.

$$\begin{aligned} -12x^2 - 9y^2 &= -144 \\ 12x^2 + 8y^2 &= 140 \\ \hline -y^2 &= -4 \\ y^2 &= 4 \\ y &= \pm 2 \end{aligned}$$

Visualize the Solution



Substitute 2 for y into Equation (1) and solve for x.

$$\begin{aligned} 4x^2 + 3(2)^2 &= 48 \\ 4x^2 &= 36 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

Because $(-2)^2 = 2^2$, replacing y with -2 yields the same values of x: $x = 3$ or $x = -3$. The solutions are $(3, 2)$, $(3, -2)$, $(-3, 2)$, and $(-3, -2)$.

▶ **Try Exercise 20, page 506**

EXAMPLE 3 Identify an Inconsistent System of Equations

Solve: $\begin{cases} 4x^2 + 9y^2 = 36 & (1) \\ x^2 - y^2 = 25 & (2) \end{cases}$

Algebraic Solution

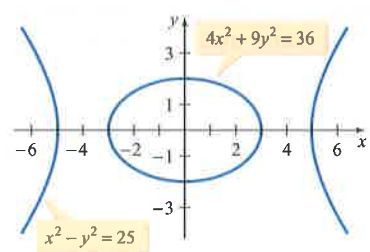
Using the elimination method, we will eliminate the x^2 term from each equation. Multiplying Equation (2) by -4 and then adding, we have

$$\begin{aligned} 4x^2 + 9y^2 &= 36 \\ -4x^2 + 4y^2 &= -100 \\ \hline 13y^2 &= -64 \end{aligned}$$

Because the equation $13y^2 = -64$ has no real solutions, the system of equations has no solutions. The graphs of the equations do not intersect.

▶ **Try Exercise 24, page 506**

Visualize the Solution



EXAMPLE 4 Solve a Nonlinear System of Equations

Solve: $\begin{cases} (x + 3)^2 + (y - 4)^2 = 20 \\ (x + 4)^2 + (y - 3)^2 = 26 \end{cases}$

Algebraic Solution

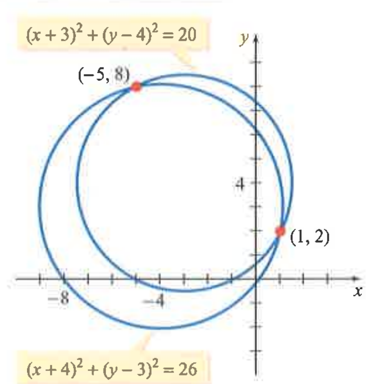
Expand the binomials in each equation. Then subtract the two equations and simplify.

$$\begin{aligned} x^2 + 6x + 9 + y^2 - 8y + 16 &= 20 \quad (1) \\ x^2 + 8x + 16 + y^2 - 6y + 9 &= 26 \quad (2) \\ \hline -2x - 7 \quad - 2y + 7 &= -6 \\ x + y &= 3 \end{aligned}$$

Now solve the resulting equation for y.

$$y = -x + 3$$

Visualize the Solution



(continued)

Substitute $-x + 3$ for y in Equation (1) and solve for x.

$$x^2 + 6x + 9 + (-x + 3)^2 - 8(-x + 3) + 16 = 20$$
$$2(x^2 + 4x - 5) = 0$$
$$2(x + 5)(x - 1) = 0$$
$$x = -5 \quad \text{or} \quad x = 1$$

Substitute -5 and 1 for x in the equation $y = -x + 3$ and solve for y. This yields $y = 8$ or $y = 2$. The solutions of the system of equations are $(-5, 8)$ and $(1, 2)$.

▶ Try Exercise 32, page 506

In Example 5, a nonlinear system of equations is used to solve an application.

EXAMPLE 5 Television Dimensions

A television screen has a diagonal of 63 inches. The ratio of the width of the screen, x, to the height of the screen, y, is 16 to 9. Find the width and the height of the screen. Round to the nearest tenth of an inch.

Algebraic Solution

A diagonal of the screen measures 63 inches. Therefore, by the Pythagorean Theorem,

$$x^2 + y^2 = 63^2$$

The ratio of the width x to the height y is 16 to 9. Thus $\frac{x}{y} = \frac{16}{9}$, which can be written as $x = \frac{16}{9}y$. We need to solve the following system.

$$\begin{cases} x^2 + y^2 = 63^2 & (1) \\ x = \frac{16}{9}y & (2) \end{cases}$$

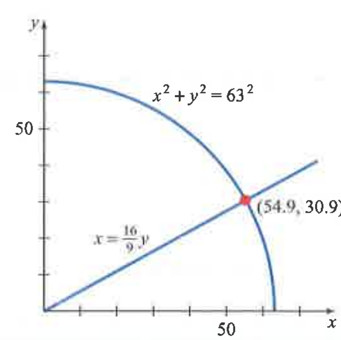
Substitute $\frac{16}{9}y$ for x in Equation (1) and solve for y.

$$\left(\frac{16}{9}y\right)^2 + y^2 = 63^2 \qquad \text{• Substitute.}$$

$$\frac{256}{81}y^2 + \frac{81}{81}y^2 = 63^2 \qquad \text{• Simplify.}$$

Visualize the Solution

The graph of Equation (1) is a circle, and the graph of Equation (2) is a line. Because the width x and the height y are both positive, we know that the dimensions of the television screen will be given by the coordinates of the point in Quadrant I at which the graphs intersect.



$$\frac{337}{81}y^2 = 63^2$$ • Collect like terms.

$$y^2 = \frac{81}{337} \cdot 63^2$$ • Solve for y.

$$y = \sqrt{\frac{81}{337} \cdot 63^2}$$ • Because $y > 0$, we find only the positive square root.

$$y \approx 30.886$$

Now use Equation (2) to find x.

$$x = \frac{16}{9}y \approx \frac{16}{9} \cdot 30.886 \approx 54.908$$

To the nearest tenth of an inch, the width is 54.9 inches and the height is 30.9 inches.

Try Exercise 40, page 506

EXERCISE SET 6.3

Concept Check

1. Is $\begin{cases} 3x + 2y = 6 \\ y = 2x^2 \end{cases}$ a nonlinear system of equations?

2. Consider the following nonlinear system of equations, where c is a real number.

$$\begin{cases} \dfrac{x^2}{16} + \dfrac{y^2}{9} = 1 \\ y = x^2 + c \end{cases}$$

Determine the number of solutions of this system for each of the following values of c. Recall that in this section we consider only solutions whose coordinates are real numbers.

a. $c = -5$ **b.** $c = -3$

c. $c = 0$ **d.** $c = 5$

3. The graph of the following nonlinear system consists of a hyperbola and its asymptotes. How many solutions does this system have?

$$\begin{cases} \dfrac{x^2}{25} - \dfrac{y^2}{16} = 1 \\ y = \dfrac{4}{5}x \\ y = -\dfrac{4}{5}x \end{cases}$$

4. The graph of the following nonlinear system consists of an ellipse and a hyperbola. How many solutions does this system have?

$$\begin{cases} \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1 \\ \dfrac{x^2}{9} - \dfrac{y^2}{4} = 1 \end{cases}$$

Indicates Try It Exercises

In Exercises 5 to 36, solve the system of equations.

5. $\begin{cases} y = x^2 + 3x \\ y = 4x + 6 \end{cases}$

6. $\begin{cases} y = x^2 + 2x - 3 \\ y = x - 1 \end{cases}$

7. $\begin{cases} y = 2x^2 - 3x - 3 \\ y = x - 4 \end{cases}$

8. $\begin{cases} y = -x^2 + 2x - 4 \\ y = \dfrac{1}{2}x + 1 \end{cases}$

9. $\begin{cases} x^2 - 3x + y - 5 = 0 \\ x^2 - 2x - y - 7 = 0 \end{cases}$

10. $\begin{cases} y = 2x^2 - x + 1 \\ y = x^2 + 2x + 5 \end{cases}$

11. $\begin{cases} 2x + 3y = 16 \\ xy = 10 \end{cases}$

12. $\begin{cases} x - 2y = 3 \\ xy = -1 \end{cases}$

13. $\begin{cases} 2x - y = 1 \\ xy = 6 \end{cases}$

14. $\begin{cases} x - 3y = 7 \\ xy = -4 \end{cases}$

15. $\begin{cases} 5x^2 - 3y^2 = -7 \\ -3x + y = -3 \end{cases}$

16. $\begin{cases} x^2 + 3y^2 = 7 \\ x + 4y = 6 \end{cases}$

17. $\begin{cases} y = x^3 + 4x^2 - 3x - 5 \\ y = 2x^2 - 2x - 3 \end{cases}$

18. $\begin{cases} y = x^3 - 2x^2 + 5x + 1 \\ y = x^2 + 7x - 5 \end{cases}$

19. $\begin{cases} 2x^2 + y^2 = 9 \\ x^2 - y^2 = 3 \end{cases}$

20. $\begin{cases} 3x^2 - 2y^2 = 19 \\ x^2 - y^2 = 5 \end{cases}$

21. $\begin{cases} x^2 - 2y^2 = 8 \\ x^2 + 3y^2 = 28 \end{cases}$

22. $\begin{cases} 2x^2 + 3y^2 = 5 \\ x^2 - 3y^2 = 4 \end{cases}$

23. $\begin{cases} 2x^2 + 4y^2 = 5 \\ 3x^2 + 8y^2 = 14 \end{cases}$

24. $\begin{cases} 2x^2 + 3y^2 = 11 \\ 3x^2 + 2y^2 = 19 \end{cases}$

25. $\begin{cases} x^2 - 2x + y^2 = 1 \\ 2x + y = 5 \end{cases}$

26. $\begin{cases} x^2 + y^2 + 5y = 66 \\ 3x + 2y = 22 \end{cases}$

27. $\begin{cases} (x - 3)^2 + (y + 1)^2 = 5 \\ x - 3y = 7 \end{cases}$

28. $\begin{cases} (x + 2)^2 + (y - 2)^2 = 13 \\ 2x + y = 6 \end{cases}$

29. $\begin{cases} x^2 - 3x + y^2 = 4 \\ 3x + y = 11 \end{cases}$

30. $\begin{cases} x^2 + y^2 - 4y = 4 \\ 5x - 2y = 2 \end{cases}$

31. $\begin{cases} (x - 2)^2 + (y + 2)^2 = 160 \\ (x + 3)^2 + (y - 1)^2 = 162 \end{cases}$

32. $\begin{cases} (x + 2)^2 + (y - 3)^2 = 10 \\ (x - 3)^2 + (y + 1)^2 = 13 \end{cases}$

33. $\begin{cases} (x + 3)^2 + (y - 2)^2 = 20 \\ (x - 2)^2 + (y - 3)^2 = 2 \end{cases}$

34. $\begin{cases} (x - 4)^2 + (y - 5)^2 = 8 \\ (x + 1)^2 + (y + 2)^2 = 34 \end{cases}$

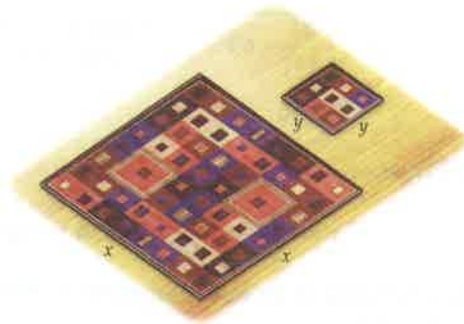
35. $\begin{cases} (x - 1)^2 + (y + 1)^2 = 2 \\ (x + 2)^2 + (y - 3)^2 = 3 \end{cases}$

36. $\begin{cases} (x + 1)^2 + (y - 3)^2 = 4 \\ (x - 3)^2 + (y + 2)^2 = 2 \end{cases}$

37. **Dimensions of a Brochure** A rectangular brochure is designed so that it has an area of 37.5 square inches and a perimeter of 25 inches. Find the width and the height of the brochure. Assume the height is greater than the width.

38. **Dimensions of a Container** With the lid closed, a takeout box used by a restaurant has a volume of 121 cubic inches. Its length l equals its width w. A strip of tape is wrapped around the box to keep it closed. The length of the tape measures 20 inches, which is 1 inch more than the shortest distance around the box. Find the dimensions of the box.

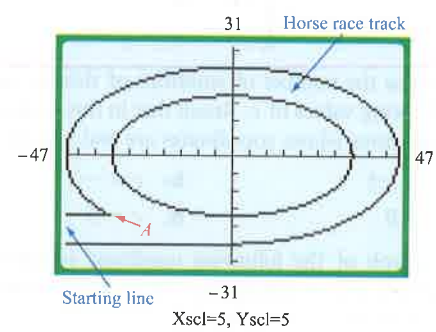
39. **Dimensions of Carpets** Two square carpets are used in the reception area of a hotel. The sum of the areas of the carpets is 865 square feet. The difference of the areas of the carpets is 703 square feet. Find the dimensions of each carpet.



40. **Dimensions of a Sign** A large, rectangular electronic advertising sign for a hotel has a diagonal of 25.0 feet. The height of the sign is 1.6 times its width. Find the width and the height of the sign. Round to the nearest tenth of a foot.

41. **Dimensions of Globes** A company sells a large globe and a small globe. The volume of the large globe is eight times the volume of the small globe. The difference between the volumes is approximately 15,012.62 cubic inches. Find the radius of each globe. Round to the nearest tenth of an inch.

42. **Horse Race Simulation** A student is writing a horse race simulation for a TI-83/TI-83 Plus/TI-84 Plus graphing calculator. The figure below shows the layout of the track.

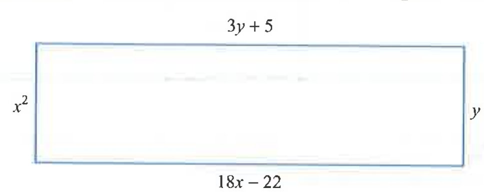


To produce the graph on a graphing calculator, the student needs to know the coordinates of point A in Quadrant III, the point at which the large ellipse

$$\frac{x^2}{47^2} + \frac{y^2}{25^2} = 1$$

intersects the horizontal line segment defined by $y = -16$. Find the coordinates of point A. Round the x value to the nearest tenth.

43. **Geometry** Find the perimeter of the rectangle below.



44. Construction A painter leans a ladder against a vertical wall. The top of the ladder is 7 meters above the ground. When the bottom of the ladder is moved 1 meter farther away from the wall, the top of the ladder is 5 meters above the ground. What is the length of the ladder? Round to the nearest hundredth of a meter.

45. Analytic Geometry For what values of the radius r does the line $y = 2x + 1$ intersect (at one or more points) the circle whose equation is $x^2 + y^2 = r^2$?

46. Geometry Three rectangles have exactly the same area. The dimensions of the rectangles (as length and width) are a and b; $a - 3$ and $b + 2$; and $a + 3$ and $b - 1$. Find the area of the rectangles.

47. Find Numbers Find two real numbers that have a sum of 5 and a product of 1.

48. Find Numbers Find two positive real numbers that have a difference of 12 and a product of 5.

In Exercises 49 to 54, use a graphing utility or WolframAlpha to solve each system of equations. Round approximate values to the nearest ten-thousandth.

49. $\begin{cases} y = 2^x \\ y = x + 1 \end{cases}$

50. $\begin{cases} y = \log_2 x \\ y = x - 3 \end{cases}$

51. $\begin{cases} y = e^{-x} \\ y = x^2 \end{cases}$

52. $\begin{cases} y = \ln x \\ y = -x + 4 \end{cases}$

53. $\begin{cases} y = \sqrt{x} \\ y = \dfrac{1}{x - 1} \end{cases}$

54. $\begin{cases} y = \dfrac{6}{x + 1} \\ y = \dfrac{x}{x - 1} \end{cases}$

In Exercises 55 to 60, solve the system of equations for rational-number ordered pairs.

55. $\begin{cases} y = x^2 + 4 \\ x = y^2 - 24 \end{cases}$

56. $\begin{cases} y = x^2 - 5 \\ x = y^2 - 13 \end{cases}$

57. $\begin{cases} x^2 - 3xy + y^2 = 5 \\ x^2 - xy - 2y^2 = 0 \end{cases}$

(*Hint:* Factor the second equation. Now use the zero product principle and the substitution principle.)

58. $\begin{cases} x^2 + 2xy - y^2 = 1 \\ x^2 + 3xy + 2y^2 = 0 \end{cases}$

(*Hint:* See Exercise 57.)

59. $\begin{cases} 2x^2 - 4xy - y^2 = 6 \\ 4x^2 - 3xy - y^2 = 6 \end{cases}$

(*Hint:* Subtract the two equations.)

60. $\begin{cases} 3x^2 + 2xy - 5y^2 = 11 \\ x^2 + 3xy + y^2 = 11 \end{cases}$

(*Hint:* Subtract the two equations.)

Enrichment Exercises

61. The Parade Problem A parade that is 2 miles in length moves forward 2 miles at a constant rate of 4 miles per hour. During this time, a security guard rides a bicycle, at a constant rate, from the front of the parade to the back of the parade and then returns to the front.

a. How far did the security guard ride?

b. What was the security guard's rate, in miles per hour, during this time period?

62. Number of Solutions Determine the number of solutions of the following nonlinear system of equations. Explain how you determined your answer.

$$\begin{cases} 100x = 10^y \\ y = 2 + \log x \end{cases}$$

MID-CHAPTER 6 QUIZ

1. Solve: $\begin{cases} 2x - 3y = -15 \\ -3x + 4y = 19 \end{cases}$

2. Solve: $\begin{cases} 6x - 3y = -9 \\ -2x + y = 3 \end{cases}$

3. Give an example of an inconsistent system of equations in two variables.

4. Find an equation of the form $y = ax^2 + bx + c$ whose graph passes through the points (–1, 6), (2, 3), and (3, 10).

5. Solve: $\begin{cases} 3x^2 + y^2 = 28 \\ x^2 - y^2 = 8 \end{cases}$

SECTION 6.4

Partial Fraction Decomposition

Partial Fractions

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A39.

PS1. Factor $x^4 + 14x^2 + 49$ over the real numbers. [P.4]

PS2. Add: $\dfrac{5}{x-1} + \dfrac{1}{x+2}$ [P.5]

PS3. Simplify: $\dfrac{7}{x} - \dfrac{6}{x-1} + \dfrac{10}{(x-1)^2}$ [P.5]

PS4. Solve: $\begin{cases} 1 = A + B \\ 11 = -5A + 3B \end{cases}$ [6.1]

PS5. Solve: $\begin{cases} 0 = A + B \\ 3 = -2B + C \\ 16 = 7A - 2C \end{cases}$ [6.2]

PS6. Divide: $\dfrac{x^3 - 4x^2 - 19x - 35}{x^2 - 7x}$ [3.1]

Partial Fraction Decomposition

TO REVIEW

Rational Expressions
See pages 50 to 53.

An algebraic application of systems of equations is a technique known as *partial fractions.* In Chapter P, we reviewed the problem of adding two rational expressions. For example,

$$\frac{5}{x-1} + \frac{1}{x+2} = \frac{6x+9}{(x-1)(x+2)}$$

Now we will take the opposite approach. That is, given a rational expression, we will find simpler rational expressions whose sum is the given expression. The method by which a more complicated rational expression is written as a sum of simpler rational expressions is called **partial fraction decomposition.** This technique is based on the following theorem.

CALCULUS CONNECTION

Partial Fraction Decomposition Theorem

If
$$f(x) = \frac{p(x)}{q(x)}$$

is a rational expression in which the degree of the numerator is less than the degree of the denominator, and $p(x)$ and $q(x)$ have no common factors, then $f(x)$ can be written as a partial fraction decomposition in the form

$$f(x) = f_1(x) + f_2(x) + \cdots + f_n(x)$$

where each $f_i(x)$ has one of the following forms.

$$\frac{A}{(ax+b)^m} \quad \text{or} \quad \frac{Ax+B}{(ax^2+bx+c)^m}$$