EXERCISE SET 6.2

Concept Check

1. Determine which of the following ordered triples are solutions of

$$3x - 2y + z = 6$$

a. (3, 4, 5) **b.** (5, 3, −3) **c.** (0, 3, 11) a. and b.

2. What is a homogeneous system of equations?
A homogeneous system is a system of linear equations in which the constant term is zero for all of the equations.

3. Consider the following nonsquare system of equations.

$$\begin{cases} x + y + z = 6 \\ 2x - y - 3z = 9 \end{cases}$$

Describe the graph of the solutions of this system.
The graph of the solutions is the line where the graphs of the equations, which are planes, intersect.

4. Is the following system an independent system, a dependent system, or an inconsistent system of equations?

$$\begin{cases} x + y + z = 9 \\ x - y + z = 1 \end{cases}$$

A dependent system

In Exercises 5 to 28, solve each system of equations.

5. $\begin{cases} 2x - y + z = 8 \\ 2y - 3z = -11 \\ 3y + 2z = 3 \end{cases}$
(2, −1, 3)

6. $\begin{cases} 3x + y + 2z = -4 \\ -3y - 2z = -5 \\ 2y + 5z = -4 \end{cases}$
(−1, 3, −2)

7. $\begin{cases} 2x + 2y - 3z = 18 \\ x - 3y + 4z = -6 \\ 3x - y + 2z = 10 \end{cases}$
(5, 1, −2)

8. $\begin{cases} x - 2y + 3z = 5 \\ 3x - 3y + z = 9 \\ 5x + y - 3z = 3 \end{cases}$
(1, −2, 0)

9. $\begin{cases} 3x + 4y - z = -7 \\ x - 5y + 2z = 19 \\ 5x + y - 2z = 5 \end{cases}$
(2, −3, 1)

10. $\begin{cases} 2x - 3y - 2z = 12 \\ x + 4y + z = -9 \\ 4x + 2y - 3z = 6 \end{cases}$
(1, −2, −2)

11. $\begin{cases} 3x - 2y + 3z = 8 \\ 2x + 3y - 4z = -28 \\ x - y + 2z = 9 \end{cases}$
(−3, 2, 7)

12. $\begin{cases} 4x - y + 2z = -1 \\ 2x + 3y - 3z = -13 \\ x + 5y + z = 7 \end{cases}$
(−2, 1, 4)

13. $\begin{cases} 3x - 2y + 4z = 16 \\ 2x + 3y + 2z = 16 \\ 5x + 2y + 4z = 24 \end{cases}$
(0, 2, 5)

14. $\begin{cases} x - 3y + 2z = -11 \\ 3x + y + 4z = 4 \\ 5x - 5y + 8z = -18 \end{cases}$
$\left(\frac{1 - 14c}{10}, \frac{37 + 2c}{10}, c\right)$

15. $\begin{cases} 2x - 5y + 2z = -4 \\ 3x + 2y + 3z = 13 \\ 5x - 3y - 4z = -18 \end{cases}$
(0, 2, 3)

16. $\begin{cases} 3x + 2y - 5z = 6 \\ 5x - 4y + 3z = -12 \\ 4x + 5y - 2z = 15 \end{cases}$
(0, 3, 0)

17. $\begin{cases} 2x + y - z = -2 \\ 3x + 2y + 3z = 21 \\ 7x + 4y + z = 17 \end{cases}$
(5c − 25, 48 − 9c, c)

18. $\begin{cases} 3x + y + 2z = 2 \\ 4x - 2y + z = -4 \\ 11x - 3y + 4z = -6 \end{cases}$
$\left(-\frac{c}{2}, \frac{4 - c}{2}, c\right)$

19. $\begin{cases} 6x - 7y + 12z = 81 \\ 4x + 3y + 2z = 7 \\ 3x - 5y + 11z = 65 \end{cases}$
(2, −3, 4)

20. $\begin{cases} x + 3y - 2z = 2 \\ -2x - 4y + z = 0 \\ -3x - 7y + 3z = -1 \end{cases}$
No solution

21. $\begin{cases} 2x - 3y + 6z = 3 \\ x + 2y - 4z = 5 \\ 3x + 4y - 8z = 7 \end{cases}$
No solution

22. $\begin{cases} 2x + 3y - 6z = 4 \\ 3x - 2y - 9z = -7 \\ 2x + 5y - 6z = 8 \end{cases}$
(3c − 1, 2, c)

23. $\begin{cases} 2x - 3y + 5z = 14 \\ x + 4y - 3z = -2 \end{cases}$
$\left(\frac{50 - 11c}{11}, \frac{11c - 18}{11}, c\right)$

24. $\begin{cases} x - 3y + 4z = 9 \\ 3x - 8y - 2z = 4 \end{cases}$
(38c − 60, 14c − 23, c)

25. $\begin{cases} 6x - 9y + 6z = 7 \\ 4x - 6y + 4z = 9 \end{cases}$
No solution

26. $\begin{cases} 4x - 2y + 6z = 5 \\ 2x - y + 3z = 2 \end{cases}$
No solution

27. $\begin{cases} 5x + 3y + 2z = 10 \\ 3x - 4y - 4z = -5 \end{cases}$
$\left(\frac{25 + 4c}{29}, \frac{55 - 26c}{29}, c\right)$

28. $\begin{cases} 3x - 4y - 7z = -5 \\ 2x + 3y - 5z = 2 \end{cases}$
$\left(\frac{41c - 7}{17}, \frac{16 + c}{17}, c\right)$

In Exercises 29 to 36, solve each homogeneous system of equations.

29. $\begin{cases} x + 3y - 4z = 0 \\ 2x + 7y + z = 0 \\ 3x - 5y - 2z = 0 \end{cases}$
(0, 0, 0)

30. $\begin{cases} x - 2y + 3z = 0 \\ 3x - 7y - 4z = 0 \\ 4x - 4y + z = 0 \end{cases}$
(0, 0, 0)

31. $\begin{cases} 2x - 3y + z = 0 \\ 2x + 4y - 3z = 0 \\ 6x - 2y - z = 0 \end{cases}$
$\left(\frac{5c}{14}, \frac{4c}{7}, c\right)$

32. $\begin{cases} 5x - 4y - 3z = 0 \\ 2x + y + 2z = 0 \\ x - 6y - 7z = 0 \end{cases}$
$\left(-\frac{5}{13}c, -\frac{16}{13}c, c\right)$

33. $\begin{cases} 3x - 5y + 3z = 0 \\ 2x - 3y + 4z = 0 \\ 7x - 11y + 11z = 0 \end{cases}$
(−11c, −6c, c)

34. $\begin{cases} 5x - 2y - 3z = 0 \\ 3x - y - 4z = 0 \\ 4x - y - 9z = 0 \end{cases}$
(5c, 11c, c)

35. $\begin{cases} 4x - 7y - 2z = 0 \\ 2x + 4y + 3z = 0 \\ 3x - 2y - 5z = 0 \end{cases}$
(0, 0, 0)

36. $\begin{cases} 5x + 2y + 3z = 0 \\ 3x + y - 2z = 0 \\ 4x - 7y + 5z = 0 \end{cases}$
(0, 0, 0)

In Exercises 37 to 50, solve each exercise by solving a system of equations.

37. Curve Fitting Find the quadratic equation of the form $y = ax^2 + bx + c$ whose graph passes through the points (2, 3), (−2, 7), and (1, −2). $y = 2x^2 - x - 3$

38. Curve Fitting Find the quadratic equation of the form $y = ax^2 + bx + c$ whose graph passes through the points (1, −2), (3, −4), and (2, −2). $y = -x^2 + 3x - 4$

39. Curve Fitting Find the equation of the circle whose graph passes through the points (5, 3), (−1, −5), and (−2, 2). (*Hint:* Use the equation $x^2 + y^2 + ax + by + c = 0$.)
$x^2 + y^2 - 4x + 2y - 20 = 0$

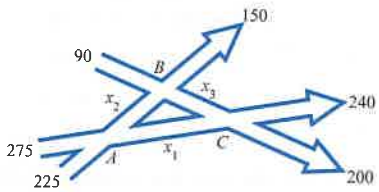
40. Curve Fitting Find the equation of the circle whose graph passes through the points (0, 6), (1, 5), and (−7, −1). (*Hint:* See Exercise 39.) $x^2 + y^2 + 6x - 4y - 12 = 0$

Indicates Try It Exercises

41. Curve Fitting Find the center and radius of the circle whose graph passes through the points $(-2, 10)$, $(-12, -14)$, and $(5, 3)$. (*Hint:* See Exercise 39.) Center $(-7, -2)$, radius 13

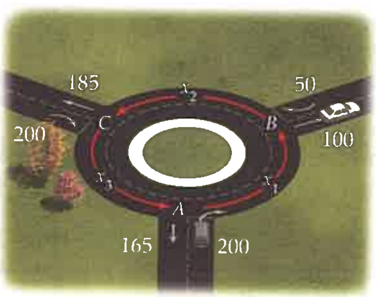
42. Curve Fitting Find the center and radius of the circle whose graph passes through the points $(2, 5)$, $(-4, -3)$, and $(3, 4)$. (*Hint:* See Exercise 39.) Center $(-1, 1)$, radius 5

43. Traffic Flow Suppose that the traffic flow for some one-way streets can be modeled by the diagram below, where each number or variable represents the number of cars entering or leaving an intersection per hour.



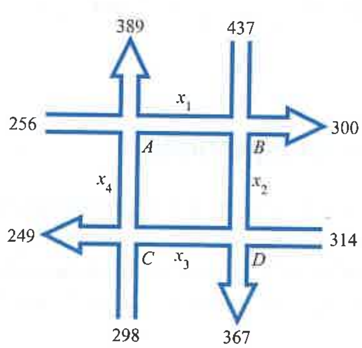
If the street connecting intersections A and B has a traffic flow of 150 to 250 cars per hour, what is the traffic flow between B and C? 90 to 190 cars/h

44. Traffic Flow A roundabout is a type of intersection that accommodates traffic flow in one direction, around a circular island. The graphic model below shows the numbers of cars per hour that are entering or leaving a roundabout. The variables x_1, x_2, and x_3 represent the traffic flow per hour along the three portions of the roundabout.



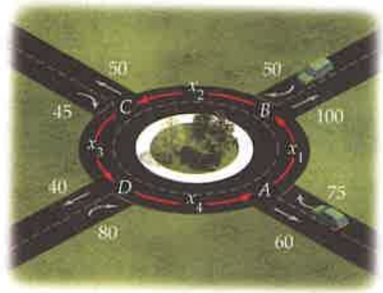
If the portion of the roundabout between A and B has a traffic flow of from 60 to 80 cars per hour, what is the traffic flow between C and A and between B and C? CA: 25 to 45 cars/h; BC: 10 to 30 cars/h

45. Traffic Flow Suppose that the traffic flow for some one-way streets can be modeled by the accompanying diagram, where each number or variable represents the number of cars entering or leaving an intersection per hour.



If the street connecting intersections A and B has an estimated traffic flow of from 125 to 175 cars per hour, what is the estimated traffic flow between C and A, D and C, and B and D? CA: 258 to 308 cars/h; DC: 209 to 259 cars/h; BD: 262 to 312 cars/h

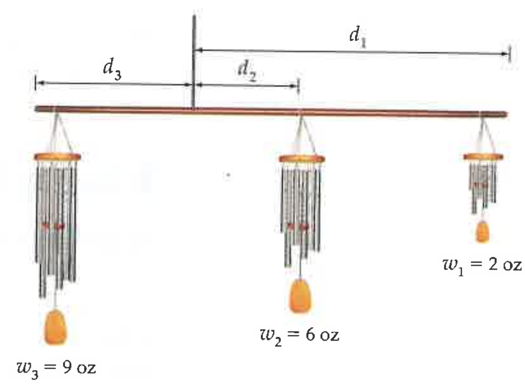
46. Traffic Flow The graphic model below shows the numbers of cars per hour that are entering and leaving a roundabout. What is the minimum number of cars per hour that can travel between B and C? 5 cars/h



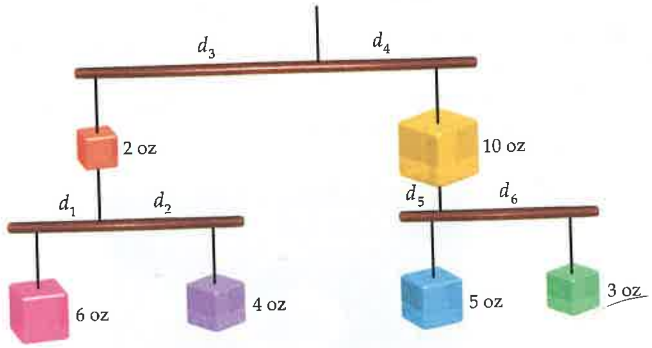
47. Coin Exchange Every day Emily puts her quarters, dimes, and nickels into a shoe box. After several weeks, she decides to take the coins to a machine that counts coins and provides a voucher that she can exchange for dollars. The coin machine charges a fee equal to 10% of the total value of the coins. After the machine has counted her coins, she receives a voucher for \$139.50. The voucher also shows that she had a total of 1025 coins and twice as many quarters as nickels. How many coins of each denomination did Emily have? 420 quarters, 395 dimes, 210 nickels

48. Hotel Data A hotel resources report listed the total number of hotels in the Holiday Inn, Hilton, and Hyatt hotel chains as 2238. The number of Holiday Inn hotels was 400 more than the total number of Hilton and Hyatt hotels. There were 121 more Hilton hotels than Hyatt hotels. Determine the number of hotels in each of these hotel chains. Holiday Inn: 1319 hotels; Hilton: 520 hotels; Hyatt: 399 hotels

49. Art A sculptor is creating a wind chime consisting of three chimes that will be suspended from a rod 13 inches long. The weights, in ounces, of the chimes are shown in the diagram. For the rod to remain horizontal, the chimes must be positioned so that $w_1d_1 + w_2d_2 = w_3d_3$. If the sculptor wants d_2 to be one-third of d_1, find the position of the middle chime that will make the wind chime balance. 7 in. from the 9-oz chime and 6 in. from the 2-oz chime



50. **Art** A designer wants to create a mobile of colored blocks as shown in the diagram below. The weight, in ounces, of each of the blocks is shown next to the block.



Given that $d_3 + d_4 = 20$ inches, $d_1 + d_2 = 10$ inches, and $d_5 + d_6 = 8$ inches, find the values of d_1 through d_6 so that each bar is horizontal. (A bar is horizontal when the value of weight times distance on each side of a vertical support is equal. For instance, for the diagram above, $6d_1$ must equal $4d_2$. Because there are six variables, the resulting system of equations must contain six equations.)
$d_1 = 4$ in., $d_2 = 6$ in., $d_3 = 12$ in., $d_4 = 8$ in., $d_5 = 3$ in., $d_6 = 5$ in.

In Exercises 51 and 52, find an equation of the plane that contains the given points. (*Hint:* The equation of a plane can be written as $z = ax + by + c$.)

51. $(1, -1, 5), (2, -2, 9), (-3, -1, -1)$ $3x - 5y - 2z = -2$

52. $(2, 1, 1), (-1, 2, 12), (3, 2, 0)$ $z = -3x + 2y + 5$

Enrichment Exercises

In Exercises 53 and 54, use the system of equations

$$\begin{cases} x - 3y - 2z = A^2 \\ 2x - 5y + Az = 9 \\ 2x - 8y + z = 18 \end{cases}$$

53. Find all values of A for which the system has no solution. $A = -\frac{13}{2}$

54. Find all values of A for which the system has a unique solution. $A \neq -\frac{13}{2}$

In Exercises 55 and 56, use the system of equations

$$\begin{cases} x + 2y + z = A^2 \\ -2x - 3y + Az = 1 \\ 7x + 12y + A^2z = 4A^2 - 3 \end{cases}$$

55. Find all values of A for which the system has a unique solution. $A \neq -3, A \neq 1$

56. Find all values of A for which the system has an infinite number of solutions. $A = 1$

SECTION 6.3

Solving Nonlinear Systems of Equations

Nonlinear Systems of Equations

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A39.

PS1. Solve $x^2 + 2x - 2 = 0$ for x. [1.3] $-1 \pm \sqrt{3}$

PS2. Solve: $\begin{cases} x + 4y = -11 \\ 3x - 2y = 9 \end{cases}$ [6.1] $(1, -3)$

PS3. Name the graph of $(y + 3)^2 = 8x$. [5.1] Parabola

PS4. Name the graph of $\frac{(x - 2)^2}{4} - \frac{(y + 3)^2}{9} = 1$. [5.3] Hyperbola

PS5. How many times do the graphs of $y = 2x - 1$ and $x^2 + y^2 = 4$ intersect? [2.1/2.2] Two

PS6. How many times do the graphs of $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and $\frac{x^2}{9} + \frac{y^2}{4} = 1$ intersect? [5.2] Four

Solving Nonlinear Systems of Equations

A **nonlinear system of equations** is one in which one or more equations of the system are not linear equations. In this section, we will consider only solutions whose coordinates are real numbers. Therefore, if a system of equations does not have any solution in which both coordinates are real numbers, we will simply state that the system has no solution.

$$\frac{337}{81}y^2 = 63^2$$ • Collect like terms.

$$y^2 = \frac{81}{337} \cdot 63^2$$ • Solve for y.

$$y = \sqrt{\frac{81}{337} \cdot 63^2}$$ • Because $y > 0$, we find only the positive square root.

$$y \approx 30.886$$

Now use Equation (2) to find x.

$$x = \frac{16}{9}y \approx \frac{16}{9} \cdot 30.886 \approx 54.908$$

To the nearest tenth of an inch, the width is 54.9 inches and the height is 30.9 inches.

Try Exercise 40, page 506

EXERCISE SET 6.3

Concept Check

1. Is $\begin{cases} 3x + 2y = 6 \\ y = 2x^2 \end{cases}$ a nonlinear system of equations? Yes

2. Consider the following nonlinear system of equations, where c is a real number.

$$\begin{cases} \dfrac{x^2}{16} + \dfrac{y^2}{9} = 1 \\ y = x^2 + c \end{cases}$$

Determine the number of solutions of this system for each of the following values of c. Recall that in this section we consider only solutions whose coordinates are real numbers.

a. $c = -5$ 4 **b.** $c = -3$ 3

c. $c = 0$ 2 **d.** $c = 5$ 0

3. The graph of the following nonlinear system consists of a hyperbola and its asymptotes. How many solutions does this system have? 0

$$\begin{cases} \dfrac{x^2}{25} - \dfrac{y^2}{16} = 1 \\ y = \dfrac{4}{5}x \\ y = -\dfrac{4}{5}x \end{cases}$$

4. The graph of the following nonlinear system consists of an ellipse and a hyperbola. How many solutions does this system have? 2

$$\begin{cases} \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1 \\ \dfrac{x^2}{9} - \dfrac{y^2}{4} = 1 \end{cases}$$

In Exercises 5 to 36, solve the system of equations.

5. $\begin{cases} y = x^2 + 3x \\ y = 4x + 6 \end{cases}$ $(-2, -2), (3, 18)$

6. $\begin{cases} y = x^2 + 2x - 3 \\ y = x - 1 \end{cases}$ $(-2, -3), (1, 0)$

7. $\begin{cases} y = 2x^2 - 3x - 3 \\ y = x - 4 \end{cases}$ $\left(\frac{2+\sqrt{2}}{2}, \frac{-6+\sqrt{2}}{2}\right), \left(\frac{2-\sqrt{2}}{2}, \frac{-6-\sqrt{2}}{2}\right)$

8. $\begin{cases} y = -x^2 + 2x - 4 \\ y = \frac{1}{2}x + 1 \end{cases}$ No solution

9. $\begin{cases} x^2 - 3x + y - 5 = 0 \\ x^2 - 2x - y - 7 = 0 \end{cases}$ $\left(-\frac{3}{2}, -\frac{7}{4}\right), (4, 1)$

10. $\begin{cases} y = 2x^2 - x + 1 \\ y = x^2 + 2x + 5 \end{cases}$ $(4, 29), (-1, 4)$

11. $\begin{cases} 2x + 3y = 16 \\ xy = 10 \end{cases}$ $\left(3, \frac{10}{3}\right), (5, 2)$

12. $\begin{cases} x - 2y = 3 \\ xy = -1 \end{cases}$ $\left(2, -\frac{1}{2}\right), (1, -1)$

13. $\begin{cases} 2x - y = 1 \\ xy = 6 \end{cases}$ $\left(-\frac{3}{2}, -4\right), (2, 3)$

14. $\begin{cases} x - 3y = 7 \\ xy = -4 \end{cases}$ $\left(3, -\frac{4}{3}\right), (4, -1)$

15. $\begin{cases} 5x^2 - 3y^2 = -7 \\ -3x + y = -3 \end{cases}$ $\left(\frac{5}{11}, -\frac{18}{11}\right), (2, 3)$

16. $\begin{cases} x^2 + 3y^2 = 7 \\ x + 4y = 6 \end{cases}$ $\left(-\frac{2}{19}, \frac{29}{19}\right), (2, 1)$

17. $\begin{cases} y = x^3 + 4x^2 - 3x - 5 \\ y = 2x^2 - 2x - 3 \end{cases}$ $(-2, 9), (1, -3), (-1, 1)$

18. $\begin{cases} y = x^3 - 2x^2 + 5x + 1 \\ y = x^2 + 7x - 5 \end{cases}$ $(3, 25), (\sqrt{2}, -3 + 7\sqrt{2}), (-\sqrt{2}, -3 - 7\sqrt{2})$

Indicates Try It Exercises

19. $\begin{cases} 2x^2 + y^2 = 9 \\ x^2 - y^2 = 3 \end{cases}$ $(-2, 1), (-2, -1), (2, 1), (2, -1)$

20. $\begin{cases} 3x^2 - 2y^2 = 19 \\ x^2 - y^2 = 5 \end{cases}$ $(3, -2), (3, 2), (-3, 2), (-3, -2)$

21. $\begin{cases} x^2 - 2y^2 = 8 \\ x^2 + 3y^2 = 28 \end{cases}$ $(4, 2), (-4, 2), (4, -2), (-4, -2)$

22. $\begin{cases} 2x^2 + 3y^2 = 5 \\ x^2 - 3y^2 = 4 \end{cases}$ No solution

23. $\begin{cases} 2x^2 + 4y^2 = 5 \\ 3x^2 + 8y^2 = 14 \end{cases}$ No solution

24. $\begin{cases} 2x^2 + 3y^2 = 11 \\ 3x^2 + 2y^2 = 19 \end{cases}$ No solution

25. $\begin{cases} x^2 - 2x + y^2 = 1 \\ 2x + y = 5 \end{cases}$ $\left(\frac{12}{5}, \frac{1}{5}\right), (2, 1)$

26. $\begin{cases} x^2 + y^2 + 5y = 66 \\ 3x + 2y = 22 \end{cases}$ $(4, 5), \left(\frac{110}{13}, -\frac{22}{13}\right)$

27. $\begin{cases} (x - 3)^2 + (y + 1)^2 = 5 \\ x - 3y = 7 \end{cases}$ $\left(\frac{26}{5}, -\frac{3}{5}\right), (1, -2)$

28. $\begin{cases} (x + 2)^2 + (y - 2)^2 = 13 \\ 2x + y = 6 \end{cases}$ $\left(\frac{7}{5}, \frac{16}{5}\right), (1, 4)$

29. $\begin{cases} x^2 - 3x + y^2 = 4 \\ 3x + y = 11 \end{cases}$ $\left(\frac{39}{10}, -\frac{7}{10}\right), (3, 2)$

30. $\begin{cases} x^2 + y^2 - 4y = 4 \\ 5x - 2y = 2 \end{cases}$ $\left(\frac{2}{29}, -\frac{24}{29}\right), (2, 4)$

31. $\begin{cases} (x - 2)^2 + (y + 2)^2 = 160 \\ (x + 3)^2 + (y - 1)^2 = 162 \end{cases}$ $(6, 10), \left(-\frac{114}{17}, -\frac{190}{17}\right)$

32. $\begin{cases} (x + 2)^2 + (y - 3)^2 = 10 \\ (x - 3)^2 + (y + 1)^2 = 13 \end{cases}$ $\left(-\frac{15}{41}, \frac{12}{41}\right), (1, 2)$

33. $\begin{cases} (x + 3)^2 + (y - 2)^2 = 20 \\ (x - 2)^2 + (y - 3)^2 = 2 \end{cases}$ $\left(\frac{19}{13}, \frac{22}{13}\right), (1, 4)$

34. $\begin{cases} (x - 4)^2 + (y - 5)^2 = 8 \\ (x + 1)^2 + (y + 2)^2 = 34 \end{cases}$ $\left(\frac{102}{37}, \frac{91}{37}\right), (2, 3)$

35. $\begin{cases} (x - 1)^2 + (y + 1)^2 = 2 \\ (x + 2)^2 + (y - 3)^2 = 3 \end{cases}$ No solution

36. $\begin{cases} (x + 1)^2 + (y - 3)^2 = 4 \\ (x - 3)^2 + (y + 2)^2 = 2 \end{cases}$ No solution

37. **Dimensions of a Brochure** A rectangular brochure is designed so that it has an area of 37.5 square inches and a perimeter of 25 inches. Find the width and the height of the brochure. Assume the height is greater than the width.
Width: 5 in.; height: 7.5 in.

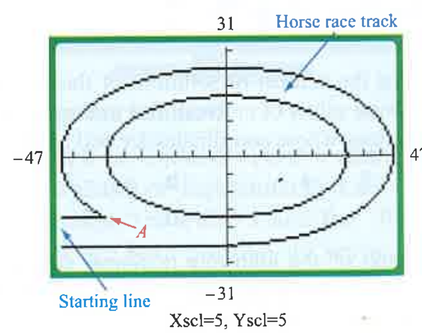
38. **Dimensions of a Container** With the lid closed, a takeout box used by a restaurant has a volume of 121 cubic inches. Its length l equals its width w. A strip of tape is wrapped around the box to keep it closed. The length of the tape measures 20 inches, which is 1 inch more than the shortest distance around the box. Find the dimensions of the box.
Height: 4 in.; length = width: 5.5 in.

39. **Dimensions of Carpets** Two square carpets are used in the reception area of a hotel. The sum of the areas of the carpets is 865 square feet. The difference of the areas of the carpets is 703 square feet. Find the dimensions of each carpet.
Small carpet: 9 by 9 ft; large carpet: 28 by 28 ft

40. **Dimensions of a Sign** A large, rectangular electronic advertising sign for a hotel has a diagonal of 25.0 feet. The height of the sign is 1.6 times its width. Find the width and the height of the sign. Round to the nearest tenth of a foot.
Width: 13.2 ft; height: 21.2 ft

41. **Dimensions of Globes** A company sells a large globe and a small globe. The volume of the large globe is eight times the volume of the small globe. The difference between the volumes is approximately 15,012.62 cubic inches. Find the radius of each globe. Round to the nearest tenth of an inch.
Large radius: 16.0 in.; small radius: 8.0 in.

42. **Horse Race Simulation** A student is writing a horse race simulation for a TI-83/TI-83 Plus/TI-84 Plus graphing calculator. The figure below shows the layout of the track.

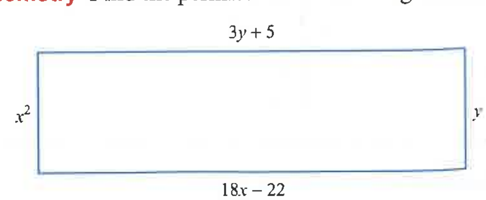


To produce the graph on a graphing calculator, the student needs to know the coordinates of point A in Quadrant III, the point at which the large ellipse

$$\frac{x^2}{47^2} + \frac{y^2}{25^2} = 1$$

intersects the horizontal line segment defined by $y = -16$. Find the coordinates of point A. Round the x value to the nearest tenth. $(-36.1, -16)$

43. **Geometry** Find the perimeter of the rectangle below. 82 units



44. **Construction** A painter leans a ladder against a vertical wall. The top of the ladder is 7 meters above the ground. When the bottom of the ladder is moved 1 meter farther away from the wall, the top of the ladder is 5 meters above the ground. What is the length of the ladder? Round to the nearest hundredth of a meter. 13.46 m

45. **Analytic Geometry** For what values of the radius r does the line $y = 2x + 1$ intersect (at one or more points) the circle whose equation is $x^2 + y^2 = r^2$? $r \geq \sqrt{\frac{1}{5}}$ or $\frac{\sqrt{5}}{5}$

46. **Geometry** Three rectangles have exactly the same area. The dimensions of the rectangles (as length and width) are a and b; $a - 3$ and $b + 2$; and $a + 3$ and $b - 1$. Find the area of the rectangles. 36 square units

47. **Find Numbers** Find two real numbers that have a sum of 5 and a product of 1.
$\frac{5 - \sqrt{21}}{2} \approx 0.208712$ and $\frac{5 + \sqrt{21}}{2} \approx 4.79129$

48. **Find Numbers** Find two positive real numbers that have a difference of 12 and a product of 5.
$6 + \sqrt{41} \approx 12.4031$ and $\sqrt{41} - 6 \approx 0.403124$

In Exercises 49 to 54, use a graphing utility or WolframAlpha to solve each system of equations. Round approximate values to the nearest ten-thousandth.

49. $\begin{cases} y = 2^x \\ y = x + 1 \end{cases}$ (0, 1), (1, 2)

50. $\begin{cases} y = \log_2 x \\ y = x - 3 \end{cases}$ (0.1375, −2.8625), (5.4449, 2.4449)

51. $\begin{cases} y = e^{-x} \\ y = x^2 \end{cases}$ (0.7035, 0.4949)

52. $\begin{cases} y = \ln x \\ y = -x + 4 \end{cases}$ (2.9263, 1.0737)

53. $\begin{cases} y = \sqrt{x} \\ y = \dfrac{1}{x - 1} \end{cases}$ (1.7549, 1.3247)

54. $\begin{cases} y = \dfrac{6}{x + 1} \\ y = \dfrac{x}{x - 1} \end{cases}$ (2, 2), $\left(3, \frac{3}{2}\right)$

In Exercises 55 to 60, solve the system of equations for rational-number ordered pairs.

55. $\begin{cases} y = x^2 + 4 \\ x = y^2 - 24 \end{cases}$ (1, 5)

56. $\begin{cases} y = x^2 - 5 \\ x = y^2 - 13 \end{cases}$ (3, 4)

57. $\begin{cases} x^2 - 3xy + y^2 = 5 \\ x^2 - xy - 2y^2 = 0 \end{cases}$ (−1, 1), (1, −1)

(*Hint:* Factor the second equation. Now use the zero product principle and the substitution principle.)

58. $\begin{cases} x^2 + 2xy - y^2 = 1 \\ x^2 + 3xy + 2y^2 = 0 \end{cases}$ No rational-number solution

(*Hint:* See Exercise 57.)

59. $\begin{cases} 2x^2 - 4xy - y^2 = 6 \\ 4x^2 - 3xy - y^2 = 6 \end{cases}$ (1, −2), (−1, 2)

(*Hint:* Subtract the two equations.)

60. $\begin{cases} 3x^2 + 2xy - 5y^2 = 11 \\ x^2 + 3xy + y^2 = 11 \end{cases}$ (2, 1), (−2, −1)

(*Hint:* Subtract the two equations.)

Enrichment Exercises

61. **The Parade Problem** A parade that is 2 miles in length moves forward 2 miles at a constant rate of 4 miles per hour. During this time, a security guard rides a bicycle, at a constant rate, from the front of the parade to the back of the parade and then returns to the front.

a. How far did the security guard ride?
$2 + 2\sqrt{2}$ mi

b. What was the security guard's rate, in miles per hour, during this time period?
$4 + 4\sqrt{2}$ mph

62. **Number of Solutions** Determine the number of solutions of the following nonlinear system of equations. Explain how you determined your answer.

$$\begin{cases} 100x = 10^y \\ y = 2 + \log x \end{cases}$$

There are an infinite number of solutions, because the equations are equivalent.

MID-CHAPTER 6 QUIZ

1. Solve: $\begin{cases} 2x - 3y = -15 \\ -3x + 4y = 19 \end{cases}$
(3, 7) [6.1]

2. Solve: $\begin{cases} 6x - 3y = -9 \\ -2x + y = 3 \end{cases}$
(c, 2c + 3) [6.1]

3. Give an example of an inconsistent system of equations in two variables. Answers will vary. [6.1]

4. Find an equation of the form $y = ax^2 + bx + c$ whose graph passes through the points (−1, 6), (2, 3), and (3, 10).
$y = 2x^2 - 3x + 1$ [6.2]

5. Solve: $\begin{cases} 3x^2 + y^2 = 28 \\ x^2 - y^2 = 8 \end{cases}$ (3, 1), (3, −1), (−3, 1), (−3, −1) [6.3]