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CHAPTER 6

Systems of Equations and Inequalities

Applications of Systems of Equations and Inequalities

Systems of equations and inequalities are often used to model applications that involve several variables. In this chapter, systems of equations and inequalities are used to solve applications from diverse fields, including business, transportation, operations research, marketing, chemistry, and mathematics.

In Exercise 31, page 531, linear programming concepts are used to maximize the profit of a skateboard company, and in Example 5, page 518, a system of inequalities is used to determine a person's targeted exercise heart rate range.

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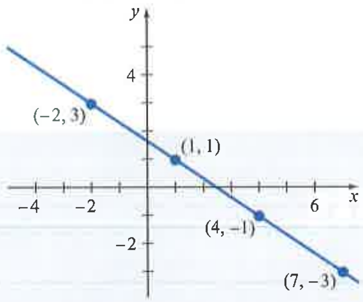
SECTION 6.1

Substitution Method for Solving a System of Equations
Elimination Method for Solving a System of Equations
Applications of Systems of Equations

Systems of Linear Equations in Two Variables

Recall that an equation of the form $Ax + By = C$ is a linear equation in two variables. A solution of a linear equation in two variables is an ordered pair (x, y) that makes the equation a true statement. For example, $(-2, 3)$ is a solution of the equation

$$2x + 3y = 5 \qquad \text{because} \qquad 2(-2) + 3(3) = 5$$



$2x + 3y = 5$

Figure 6.1

The graph of a linear equation in two variables, a straight line, is the set of points whose ordered pairs satisfy the equation. Figure 6.1 is the graph of $2x + 3y = 5$.

A **system of equations** is two or more equations considered together. The following system of equations is a **linear system of equations** in two variables.

$$\begin{cases} 2x + 3y = 4 \\ 3x - 2y = -7 \end{cases}$$

A **solution of a system of equations** in two variables is an ordered pair that is a solution of both equations.

In Figure 6.2, the graphs of the two equations in the system of equations above intersect at the point $(-1, 2)$. Because that point lies on both lines, $(-1, 2)$ is a solution of both equations and thus is a solution of the system of equations. The point $(5, -2)$ is a solution of the first equation but not a solution of the second equation. Therefore, $(5, -2)$ is not a solution of the system of equations.

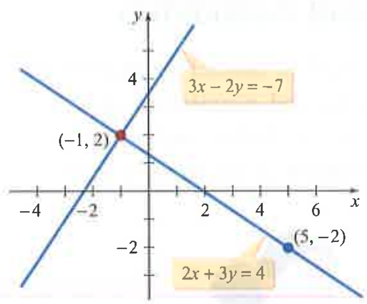


Figure 6.2

Question • Is $(3, -4)$ a solution of the system shown at the right? $\begin{cases} 2x - 3y = 18 \\ x + 4y = -13 \end{cases}$

- A system of equations is a **consistent system** if it has at least one solution.
- A system of equations with no solution is an **inconsistent system.**
- A system of linear equations with exactly one solution is an **independent system.**
- A system of linear equations with an infinite number of solutions is a **dependent system.**

The graphs of the two equations in a linear system of two variables can intersect at a single point, be the same line, or be parallel lines. See Figure 6.3.

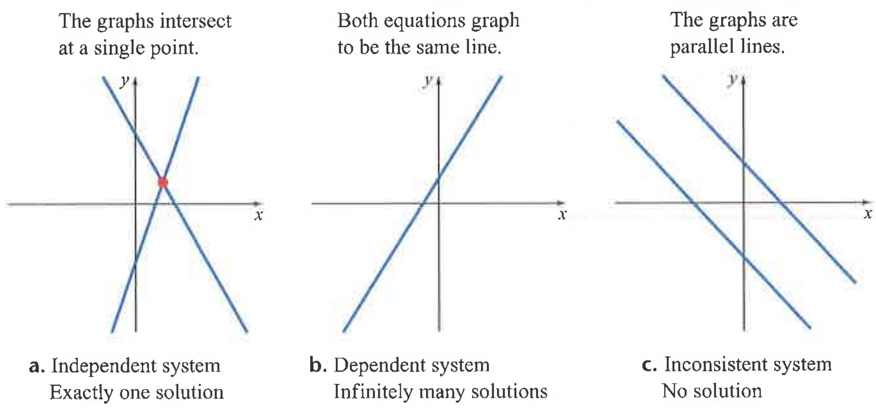


Figure 6.3

Answer • Yes. The ordered pair is a solution of both equations; thus it is a solution of the system of equations.

Substitution Method for Solving a System of Equations

The **substitution method** is one procedure for solving a system of equations. This method is illustrated in Example 1.

EXAMPLE 1 Solve a System of Equations by the Substitution Method

Solve: $\begin{cases} 3x - 5y = 7 & (1) \\ y = 2x & (2) \end{cases}$

Algebraic Solution

The solutions of $y = 2x$ are the ordered pairs $(x, 2x)$. For the system of equations to have a solution, ordered pairs of the form $(x, 2x)$ also must be solutions of $3x - 5y = 7$. To determine whether the ordered pairs $(x, 2x)$ are solutions of Equation (1), substitute $(x, 2x)$ into Equation (1) and solve for x. Think of this as *substituting* $2x$ for y.

$$3x - 5y = 7 \quad \text{• Equation (1)}$$
$$3x - 5(2x) = 7 \quad \text{• Substitute } 2x \text{ for } y.$$
$$3x - 10x = 7 \quad \text{• Solve for } x.$$
$$-7x = 7$$
$$x = -1$$

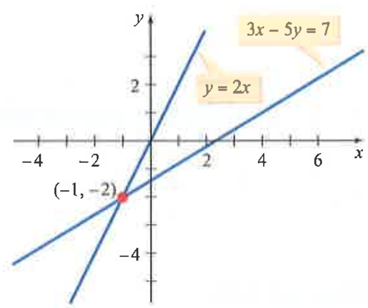
$$y = 2x \quad \text{• Equation (2)}$$
$$= 2(-1) = -2 \quad \text{• Substitute } -1 \text{ for } x \text{ in Equation (2).}$$

The only solution of the system of equations is $(-1, -2)$. You can check your work by showing that $(-1, -2)$ satisfies both equations in the original system.

▶ Try Exercise 10, page 486

Visualize the Solution

Graphing $3x - 5y = 7$ and $y = 2x$ shows that the point $(-1, -2)$ belongs to both lines. Therefore, $(-1, -2)$ is a solution of the system of equations.



An independent system of equations

EXAMPLE 2 Identify an Inconsistent System of Equations

Solve: $\begin{cases} x + 3y = 6 & (1) \\ 2x + 6y = -18 & (2) \end{cases}$

Algebraic Solution

Solve Equation (1) for y:

$$x + 3y = 6$$
$$y = -\frac{1}{3}x + 2$$

The solutions of $y = -\frac{1}{3}x + 2$ are the ordered pairs $\left(x, -\frac{1}{3}x + 2\right)$. For the system of equations to have a solution, ordered pairs of this form must also be solutions of $2x + 6y = -18$. To determine whether the ordered pairs

Visualize the Solution

Solving Equations (1) and (2) for y gives $y = -\frac{1}{3}x + 2$ and $y = -\frac{1}{3}x - 3$. The graphs of these two equations have the same slope, $-\frac{1}{3}$, and different y-intercepts.

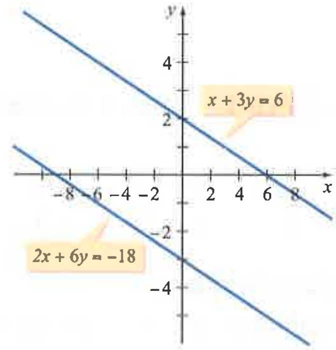
(continued)

$\left(x, -\frac{1}{3}x + 2\right)$ are solutions of Equation (2), substitute $\left(x, -\frac{1}{3}x + 2\right)$ into Equation (2) and solve for x.

$$2x + 6y = -18 \quad \text{• Equation (2)}$$
$$2x + 6\left(-\frac{1}{3}x + 2\right) = -18 \quad \text{• Substitute } -\frac{1}{3}x + 2 \text{ for } y.$$
$$2x - 2x + 12 = -18$$
$$12 = -18 \quad \text{• A false statement}$$

The false statement $12 = -18$ means that no ordered pair that is a solution of Equation (1) is also a solution of Equation (2). The equations have no ordered-pair solutions in common; thus the system of equations has no solution. This is an inconsistent system of equations.

The graphs of the two lines are parallel and never intersect.



An inconsistent system of equations

▶ Try Exercise 22, page 486

EXAMPLE 3 Identify a Dependent System of Equations

Solve: $\begin{cases} 8x - 4y = 16 & (1) \\ 2x - y = 4 & (2) \end{cases}$

Algebraic Solution

Solve Equation (2) for y:

$$2x - y = 4$$
$$y = 2x - 4$$

The solutions of $y = 2x - 4$ are the ordered pairs $(x, 2x - 4)$. For the system of equations to have a solution, ordered pairs of the form $(x, 2x - 4)$ also must be solutions of $8x - 4y = 16$. To determine whether the ordered pairs $(x, 2x - 4)$ are solutions of Equation (1), substitute $(x, 2x - 4)$ into Equation (1) and solve for x.

$$8x - 4y = 16 \quad \text{• Equation (1)}$$
$$8x - 4(2x - 4) = 16 \quad \text{• Substitute } 2x - 4 \text{ for } y.$$
$$8x - 8x + 16 = 16$$
$$16 = 16 \quad \text{• A true statement}$$

The true statement $16 = 16$ means that the ordered pairs $(x, 2x - 4)$ that are solutions of Equation (2) are also solutions of Equation (1). Because x can be replaced by any real number c, the solutions of the system of equations are all ordered pairs of the form $(c, 2c - 4)$. This is a dependent system of equations.

Visualize the Solution

Solving Equations (1) and (2) for y gives $y = 2x - 4$ and $y = 2x - 4$. The graphs of these two equations have the same slope, 2, and the same y-intercept, $(0, -4)$. Therefore, the graphs of the two equations are exactly the same. See Figure 6.4.

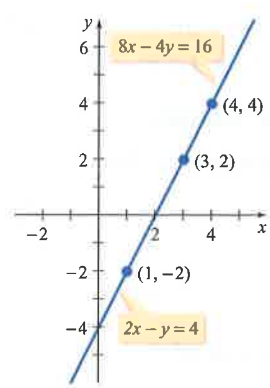


Figure 6.4
A dependent system of equations

▶ Try Exercise 24, page 486

Some of the specific ordered-pair solutions in Example 3 can be found by choosing various values for c. Table 6.1 shows the ordered pairs that result from choosing 1, 3, and 4 for c. The ordered pairs $(1, -2)$, $(3, 2)$, and $(4, 4)$ are specific solutions of the system of equations. These points are on the graphs of Equation (1) and Equation (2), as shown in Figure 6.4.

Table 6.1

c	$(c, 2c - 4)$	(x, y)
1	$(1, 2(1) - 4)$	$(1, -2)$
3	$(3, 2(3) - 4)$	$(3, 2)$
4	$(4, 2(4) - 4)$	$(4, 4)$

Note

When a system of equations is dependent, there is more than one way to write the solution set. The solution to Example 3 is the set of ordered pairs

$$(c, 2c - 4) \text{ or } \left(\frac{1}{2}b + 2, b\right)$$

However, there are infinitely more ways in which the ordered pairs could be expressed. For instance, let $b = 2w$. Then

$$\frac{1}{2}b + 2 = \frac{1}{2}(2w) + 2 = w + 2$$

The ordered-pair solutions, written in terms of w, are $(w + 2, 2w)$.

Before leaving Example 3, note that there is more than one way to represent the ordered-pair solutions. To illustrate this point, solve Equation (2) for x.

$$2x - y = 4 \qquad \text{• Equation (2)}$$

$$x = \frac{1}{2}y + 2 \qquad \text{• Solve for } x.$$

Because y can be replaced by any real number b, there are an infinite number of ordered pairs $\left(\frac{1}{2}b + 2, b\right)$ that are solutions of the system of equations. Choosing -2, 2, and 4 for b produces the same ordered pairs—$(1, -2)$, $(3, 2)$, and $(4, 4)$—that we found in Table 6.1. There is always more than one way to describe the ordered pairs when writing the solution of a dependent system of equations. For Example 3, either of the ordered pairs $(c, 2c - 4)$ or $\left(\frac{1}{2}b + 2, b\right)$ would generate all the solutions of the system of equations.

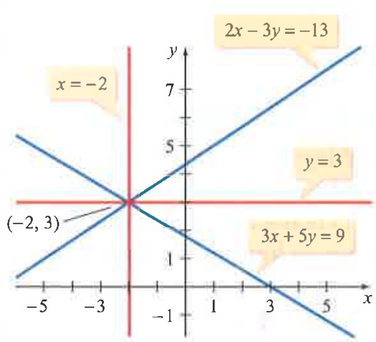


Figure 6.5

Elimination Method for Solving a System of Equations

Two systems of equations are **equivalent** if the systems have exactly the same solutions. The systems

$$\begin{cases} 3x + 5y = 9 \\ 2x - 3y = -13 \end{cases} \quad \text{and} \quad \begin{cases} x = -2 \\ y = 3 \end{cases}$$

are equivalent systems of equations. Each system has the solution $(-2, 3)$, as shown in Figure 6.5.

A second technique for solving a system of equations is similar to the strategy for solving first-degree equations in one variable. The system of equations is replaced by a series of equivalent systems until the solution is apparent.

Operations That Produce Equivalent Systems of Equations

1. Interchange any two equations.
2. Replace an equation with a nonzero constant multiple of that equation.
3. Replace an equation with the sum of that equation and a nonzero constant multiple of another equation in the system.

Because the order in which the equations are written does not affect the system of equations, interchanging the equations does not affect its solution. The second operation restates the property that multiplying each side of an equation by the same nonzero constant does not change the solutions of the equation.

The third operation can be illustrated as follows. Consider the system of equations

$$\begin{cases} 3x + 2y = 10 & (1) \\ 2x - 3y = -2 & (2) \end{cases}$$

Multiply each side of Equation (2) by 2. (Any nonzero number would work.) Add the resulting equation to Equation (1).

$$\begin{aligned} 3x + 2y &= 10 && \text{• Equation (1)} \\ 4x - 6y &= -4 && \text{• 2 times Equation (2)} \\ \hline 7x - 4y &= 6 \quad (3) && \text{• Add the equations.} \end{aligned}$$

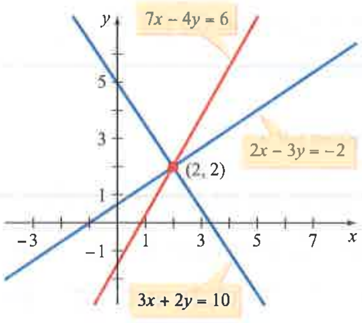


Figure 6.6

Replace Equation (1) with the new Equation (3) to produce the following equivalent system of equations.

$$\begin{cases} 7x - 4y = 6 & (3) \\ 2x - 3y = -2 & (2) \end{cases}$$

The third property states that the resulting system of equations has the same solutions as the original system and is therefore equivalent to the original system of equations. Figure 6.6 shows the graph of $7x - 4y = 6$. Note that this line passes through the same point at which the lines of the original system of equations intersect, the point $(2, 2)$.

EXAMPLE 4 Solve a System of Equations by the Elimination Method

Solve: $\begin{cases} 3x - 4y = 10 & (1) \\ 2x + 5y = -1 & (2) \end{cases}$

Algebraic Solution

Use the operations that produce equivalent equations to eliminate a variable from one of the equations. We will eliminate x from Equation (2) by multiplying each equation by a different constant so as to create a new system of equations in which the coefficients of x are additive inverses.

$$\begin{aligned} 6x - 8y &= 20 && \text{• 2 times Equation (1)} \\ -6x - 15y &= 3 && \text{• } -3 \text{ times Equation (2)} \\ \hline -23y &= 23 && \text{• Add the equations.} \\ y &= -1 && \text{• Solve for } y. \end{aligned}$$

Solve Equation (1) for x by substituting -1 for y.

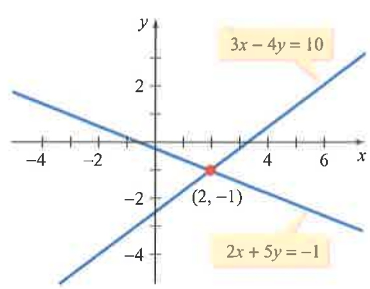
$$\begin{aligned} 3x - 4(-1) &= 10 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

The solution of the system of equations is $(2, -1)$.

Try Exercise 28, page 486

Visualize the Solution

Graphing $3x - 4y = 10$ and $2x + 5y = -1$ shows that $(2, -1)$ is the only point that belongs to both lines. Therefore, $(2, -1)$ is the solution of the system of equations.



The method just described is called the **elimination method** for solving a system of equations, because it involves *eliminating* a variable from one of the equations.

Integrating Technology

Use a Graphing Utility and WolframAlpha to Solve a System of Equations

You can use a TI-83/84 graphing calculator to solve a system of equations in two variables. Start by solving each equation for y.

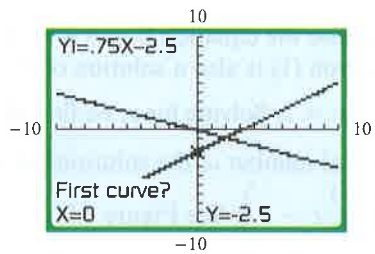
Solve for y.

$$3x - 4y = 10 \quad \rightarrow \quad y = 0.75x - 2.5$$
$$2x + 5y = -1 \quad \rightarrow \quad y = -0.4x - 0.2$$

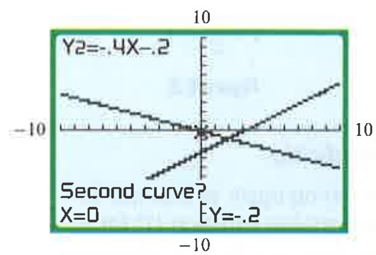
Enter 0.75X–2.5 into Y_1 and -0.4X–0.2 into Y_2 and graph the two equations in the standard viewing window. Continue by following the steps shown in Figure 6.7.

Press [2nd] [CALC]. Select 5: intersect. Press [ENTER].

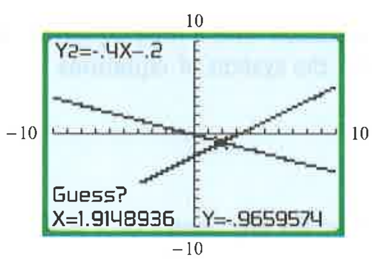
The question First curve? shown at the bottom of the screen means to select the first of the two graphs that intersect. Just press [ENTER].



The question Second curve? shown at the bottom of the screen means to select the second of the two graphs that intersect. Just press [ENTER].



Guess? is shown at the bottom of the screen. Move the cursor until it is approximately on the point of intersection. Press [ENTER].



The coordinates of the point of intersection, $(2, -1)$, are shown at the bottom of the screen.

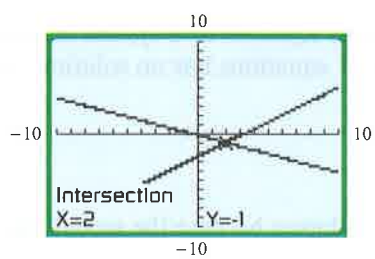


Figure 6.7

In this example, the intersection of the two graphs occurs at a point in the standard viewing window. If the point of intersection does not appear on the screen, you must adjust the viewing window so that the point of intersection is visible.

WolframAlpha can be used to solve a system of equations. For instance, to solve

$$\begin{cases} 3x - 4y = 10 \\ 2x + 5y = -1 \end{cases}$$

enter the equations separated by a comma as shown below.

3x-4y=10, 2x+5y=-1

Click on the equal sign icon to display a graph of the system and its solution $(2, -1)$.

Scan the following QR code to access WolframAlpha on a mobile device.

www.wolframalpha.com

EXAMPLE 5 Solve a Dependent System of Equations

Solve: $\begin{cases} x - 2y = 2 & (1) \\ 3x - 6y = 6 & (2) \end{cases}$

Solution

Eliminate x by multiplying Equation (2) by $-\frac{1}{3}$ and then adding the result to Equation (1).

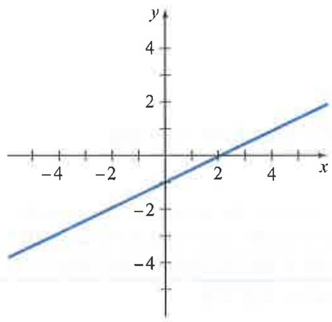
$$\begin{aligned} x - 2y &= 2 && \text{• Equation (1)} \\ -x + 2y &= -2 && \text{• } -\tfrac{1}{3} \text{ times Equation (2)} \\ 0 &= 0 && \text{• Add the two equations.} \end{aligned}$$

Replace Equation (2) with $0 = 0$.

$$\begin{cases} x - 2y = 2 \\ 0 = 0 \end{cases} \quad \text{• This is an equivalent system of equations.}$$

Because the equation $0 = 0$ is an identity, an ordered pair that is a solution of Equation (1) is also a solution of $0 = 0$. Thus the solutions are the solutions of $x - 2y = 2$. Solving for y, we find that $y = \frac{1}{2}x - 1$. Because x can be replaced by any real number c, the solutions of the system of equations are the ordered pairs $\left(c, \frac{1}{2}c - 1\right)$. See Figure 6.8.

▶ Try Exercise 32, page 486



$y = \frac{1}{2}x - 1$

Figure 6.8

Study tip

Referring again to Example 5 and solving Equation (1) for x, we have $x = 2y + 2$. Because y can be any real number b, the ordered-pair solutions of the system of equations can also be written as $(2b + 2, b)$.

If one equation of a system of equations is replaced with a false equation, the system of equations has no solution. For example, the system of equations

$$\begin{cases} x + y = 4 \\ 0 = 5 \end{cases}$$

has no solution because the second equation is false for any choice of x and y.

▶ Applications of Systems of Equations

Consider the situation of a Corvette car dealership. If the dealership were willing to sell a Corvette for \$10, there would be many consumers willing to buy a Corvette. The problem with this plan is that the dealership would soon be out of business. On the other hand, if the dealership tried to sell each Corvette for \$1 million, the dealership would not sell any cars and would still go out of business. Between \$10 and \$1 million, there is a price at which a dealership can sell Corvettes (and stay in business) and at which consumers are willing to pay that price. This price is referred to as the **equilibrium price.**

Economists refer to these types of problems as **supply–demand problems.** Businesses are willing to *supply* a product at a certain price, and there is consumer *demand* for the product at that price. To find the equilibrium point, a system of equations is created. One equation of the system is the supply model of the business. The second equation is the demand model of the consumer.

EXAMPLE 6 Solve a Supply–Demand Problem

Suppose that the number x of bushels of apples a farmer is willing to sell is given by $x = 100p - 25$, where p is the price, in dollars, per bushel of apples. The number x of bushels of apples a grocer is willing to purchase is given by $x = -150p + 655$, where p is the price per bushel of apples. Find the equilibrium price.

Solution

Using the supply and demand equations, we have the system of equations

$$\begin{cases} x = 100p - 25 \\ x = -150p + 655 \end{cases}$$

Solve the system of equations by using substitution.

$$-150p + 655 = 100p - 25$$

$$-250p + 655 = -25 \qquad \text{• Subtract } 100p \text{ from each side.}$$

$$-250p = -680 \qquad \text{• Subtract 655 from each side.}$$

$$p = 2.72 \qquad \text{• Divide each side by } -250.$$

The equilibrium price is \$2.72 per bushel.

▶ Try Exercise 46, page 487

TO REVIEW

Uniform Motion Problems
See page 88.

As the types of application problems we studied earlier in the text become more complicated, a system of equations may be the best method for solving these problems. The next example involves the distance–rate–time equation $d = rt$.

EXAMPLE 7 Solve an Application Involving Distance, Rate, and Time

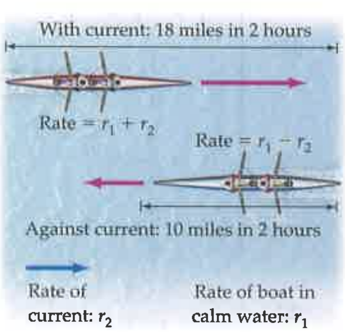
A rowing team rowing with the current traveled 18 miles in 2 hours. Against the current, the team rowed 10 miles in 2 hours. Find the rate of the boat in calm water and the rate of the current.

Solution

Let r_1 represent the rate of the boat in calm water, and let r_2 represent the rate of the current.

The rate of the boat *with the current* is $r_1 + r_2$.

The rate of the boat *against the current* is $r_1 - r_2$.



Because the rowing team traveled 18 miles in 2 hours with the current, we use the equation $d = rt$.

$$d = r \cdot t$$

$$18 = (r_1 + r_2) \cdot 2 \qquad \text{• } r = r_1 + r_2, d = 18, t = 2$$

$$9 = r_1 + r_2 \qquad \text{• Divide each side by 2.}$$

Because the team rowed 10 miles in 2 hours against the current, we write

$$10 = (r_1 - r_2) \cdot 2 \qquad \text{• } r = r_1 - r_2, d = 10, t = 2$$

$$5 = r_1 - r_2 \qquad \text{• Divide each side by 2.}$$

(continued)

Thus we have a system of two linear equations in the variables r_1 and r_2.

$$\begin{cases} 9 = r_1 + r_2 \\ 5 = r_1 - r_2 \end{cases}$$

Solving the system by using the elimination method, we find that r_1 is 7 miles per hour and r_2 is 2 miles per hour. Thus the rate of the boat in calm water is 7 miles per hour and the rate of the current is 2 miles per hour. You should verify these solutions.

▸ Try Exercise 50, page 487

EXERCISE SET 6.1

Concept Check

1. Is (4, 7) a solution of this system? $\begin{cases} 2x + 3y = 29 \\ -5x + 2y = -6 \end{cases}$

2. Identify each of the following systems of equations as an independent system, a dependent system, or as an inconsistent system.

   a. $\begin{cases} 3x - 6y = 8 \\ -6x + 12y = 10 \end{cases}$ b. $\begin{cases} 5x - 11y = 4 \\ 2x + 3y = 7 \end{cases}$

   c. $\begin{cases} x - y = 3 \\ -2x + 2y = -6 \end{cases}$

3. A system of two linear equations in two variables has no solution. What does this indicate concerning the relationship between the graphs of the equations?

4. The solutions of a dependent system of equations consist of all ordered pairs of the form
$$\left(c, \frac{3}{5}c - \frac{4}{5}\right)$$
Is (8, 4) a solution of this system?

In Exercises 5 to 24, solve each system of equations by using the substitution method.

5. $\begin{cases} 3x - 2y = 1 \\ y = 4 \end{cases}$

6. $\begin{cases} 3x - 2y = -11 \\ y = 1 \end{cases}$

7. $\begin{cases} 3x + 4y = 18 \\ y = -2x + 3 \end{cases}$

8. $\begin{cases} 5x - 4y = -22 \\ y = 5x - 2 \end{cases}$

9. $\begin{cases} -2x + 3y = 6 \\ x = 2y - 5 \end{cases}$

10. $\begin{cases} 8x + 3y = -7 \\ x = 3y + 15 \end{cases}$

11. $\begin{cases} 6x + 5y = 1 \\ x - 3y = 4 \end{cases}$

12. $\begin{cases} -3x + 7y = 14 \\ 2x - y = -13 \end{cases}$

13. $\begin{cases} 7x + 6y = -3 \\ y = \frac{2}{3}x - 6 \end{cases}$

14. $\begin{cases} 9x - 4y = 3 \\ x = \frac{4}{3}y + 3 \end{cases}$

15. $\begin{cases} y = 3x - 5 \\ y = 5x - 7 \end{cases}$

16. $\begin{cases} y = 3x - 2 \\ 2x = y + 5 \end{cases}$

17. $\begin{cases} 2y = 3x + 19 \\ x = -2y + 7 \end{cases}$

18. $\begin{cases} 18x - 12y = 4 \\ -6x + 4y = 5 \end{cases}$

19. $\begin{cases} 3x - 4y = 2 \\ 4x + 3y = 14 \end{cases}$

20. $\begin{cases} 6x + 7y = -4 \\ 2x + 5y = 4 \end{cases}$

21. $\begin{cases} 3x - 3y = 5 \\ 4x - 4y = 9 \end{cases}$

22. $\begin{cases} 3x - 4y = 8 \\ 6x - 8y = 9 \end{cases}$

23. $\begin{cases} 4x + 3y = 6 \\ y = -\frac{4}{3}x + 2 \end{cases}$

24. $\begin{cases} 5x + 2y = 2 \\ y = -\frac{5}{2}x + 1 \end{cases}$

In Exercises 25 to 42, solve each system of equations by using the elimination method.

25. $\begin{cases} 3x - y = 10 \\ 4x + 3y = -4 \end{cases}$

26. $\begin{cases} 3x + 4y = -5 \\ x - 5y = -8 \end{cases}$

27. $\begin{cases} 4x + 7y = 21 \\ 5x - 4y = -12 \end{cases}$

28. $\begin{cases} 3x - 8y = -6 \\ -5x + 4y = 10 \end{cases}$

29. $\begin{cases} 5x - 3y = 0 \\ 10x - 6y = 0 \end{cases}$

30. $\begin{cases} 3x + 2y = 0 \\ 2x + 3y = 0 \end{cases}$

31. $\begin{cases} 3x - y = 4 \\ -6x + 2y = -8 \end{cases}$

32. $\begin{cases} 4x + y = 2 \\ 8x + 2y = 4 \end{cases}$

33. $\begin{cases} 3x + 6y = 11 \\ 2x + 4y = 9 \end{cases}$

34. $\begin{cases} 4x - 2y = 9 \\ 2x - y = 3 \end{cases}$

35. $\begin{cases} \frac{5}{6}x - \frac{1}{3}y = -6 \\ \frac{1}{6}x + \frac{2}{3}y = 1 \end{cases}$

36. $\begin{cases} \frac{3}{4}x + \frac{2}{5}y = 1 \\ \frac{1}{2}x - \frac{3}{5}y = -1 \end{cases}$

37. $\begin{cases} \frac{3}{4}x + \frac{1}{3}y = 1 \\ \frac{1}{2}x + \frac{2}{3}y = 0 \end{cases}$

38. $\begin{cases} \frac{3}{5}x - \frac{2}{3}y = 7 \\ \frac{2}{5}x - \frac{5}{6}y = 7 \end{cases}$

39. $\begin{cases} 2\sqrt{3}x - 3y = 3 \\ 3\sqrt{3}x + 2y = 24 \end{cases}$

40. $\begin{cases} 4x - 3\sqrt{5}y = -19 \\ 3x + 4\sqrt{5}y = 17 \end{cases}$

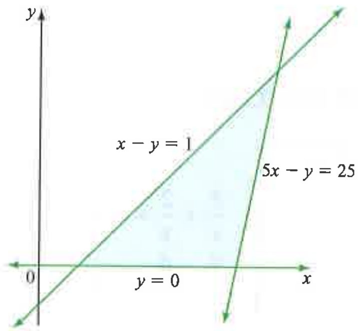
41. $\begin{cases} 3\sqrt{2}x - 4\sqrt{3}y = -6 \\ 2\sqrt{2}x + 3\sqrt{3}y = 13 \end{cases}$

42. $\begin{cases} 2\sqrt{2}x + 3\sqrt{5}y = 7 \\ 3\sqrt{2}x - \sqrt{5}y = -17 \end{cases}$

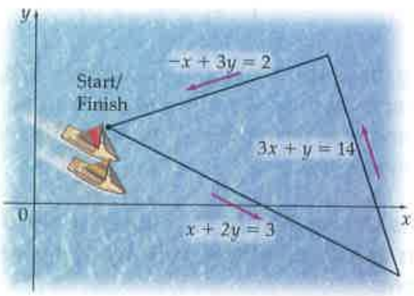
Indicates Try It Exercises

In Exercises 43 to 64, solve by using a system of equations.

43. Area of a Triangle Determine the area of the triangle shown in blue. Assume that x and y are measured in miles.



44. A Sailboat Racecourse Determine the distance around the racecourse shown by the triangle in the following diagram. Assume that x and y are measured in kilometers. Round your answer to the nearest tenth of a kilometer.



45. Supply–Demand The number x of smartphones a manufacturer is willing to sell is given by $x = 25p - 3400$, where p is the price, in dollars, per smartphone. The number x of smartphones a wholesale distributor is willing to purchase is given by $x = -5p + 2000$, where p is the price per smartphone. Find the equilibrium price.

46. Supply–Demand The number x of digital cameras a manufacturer is willing to sell is given by $x = 25p - 500$, where p is the price, in dollars, per digital camera. The number x of digital cameras a store is willing to purchase is given by $x = -7p + 1100$, where p is the price per digital camera. Find the equilibrium price.

47. Rate of Wind Flying with the wind, a plane traveled 450 miles in 3 hours. Flying against the wind, the plane traveled the same distance in 5 hours. Find the rate of the plane in calm air and the rate of the wind.

48. Rate of Wind A plane flew 800 miles in 4 hours while flying with the wind. Against the wind, it took the plane 5 hours to travel 800 miles. Find the rate of the plane in calm air and the rate of the wind.

49. Rate of Current A motorboat traveled a distance of 120 miles in 4 hours while traveling with the current. Against the current, the same trip took 6 hours. Find the rate of the boat in calm water and the rate of the current.

50. Rate of Current A canoeist can row 12 miles with the current in 2 hours. Rowing against the current, it takes the canoeist 4 hours to travel the same distance. Find the rate of the canoeist in calm water and the rate of the current.

51. Metallurgy A metallurgist made two purchases. The first purchase, which cost \$1080, included 30 kilograms of an iron alloy and 45 kilograms of a lead alloy. The second purchase, at the same prices, cost \$372 and included 15 kilograms of the iron alloy and 12 kilograms of the lead alloy. Find the cost per kilogram of the iron and lead alloys.

52. Chemistry For \$14.10, a chemist purchased 10 liters of hydrochloric acid and 15 liters of silver nitrate. A second purchase, at the same prices, cost \$18.16 and included 12 liters of hydrochloric acid and 20 liters of silver nitrate. Find the cost per liter of each of the two chemicals.

53. Chemistry A goldsmith has two gold alloys. The first alloy is 40% gold; the second alloy is 60% gold. How many grams of each should be mixed to produce 20 grams of an alloy that is 52% gold?

54. Chemistry One acetic acid solution is 70% water, and another is 30% water. How many liters of each solution should be mixed to produce 20 liters of a solution that is 40% water?

55. Geometry A right triangle in the first quadrant is bounded by the lines $y = 0$, $y = \frac{1}{2}x$, and $y = -2x + 6$. Find its area.

56. Geometry The lines whose equations are $2x + 3y = 1$, $3x - 4y = 10$, and $4x + ky = 5$ all intersect at the same point. What is the value of k?

57. Number Theory Adding a three-digit number $5Z7$ to 256 gives $XY3$. If $XY3$ is divisible by 3, then what is the largest possible value of Z?

58. Number Theory Find the value of k if
$2x + 5 = 6x + k = 4x - 7$.

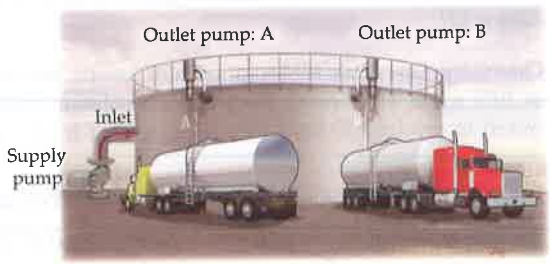
59. Number Theory A Pythagorean triple is three positive integers—a, b, and c—for which $a^2 + b^2 = c^2$. Given $a = 42$, find all the values of b and c such that a, b, and c form a Pythagorean triple. *Suggestion:* If $a = 42$, then $1764 + b^2 = c^2$ or $1764 = c^2 - b^2 = (c - b)(c + b)$. Because the product $(c - b)(c + b) = 1764$, $c - b$ and $c + b$ must be factors of 1764. For instance, one possibility is $2 = c - b$ and $882 = c + b$. Solving this system of equations yields one set of Pythagorean triples. Now repeat the process for other possible factors of 1764. Remember that answers must be positive integers.

60. Number Theory Given $a = 30$, find all the values of b and c such that a, b, and c form a Pythagorean triple. (See the preceding exercise.)

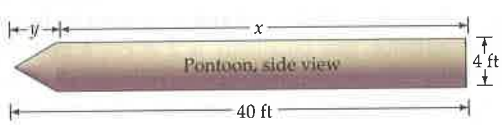
61. Marketing A marketing company asked 100 people whether they liked a new skin cream and lip balm. The company found that 80% of the people who liked the new skin cream also liked the new lip balm and that 50% of the people who did not like the new skin cream liked the new lip balm. If 77 people liked the lip balm, how many people liked the skin cream?

62. **Fire Science** An analysis of 200 scores on a firefighter qualifying exam found that 75% of those who passed the basic fire science exam also passed the exam on containing chemical fires. Of those who did not pass the basic fire science exam, 25% passed the exam on containing chemical fires. If 120 people passed the exam on containing chemical fires, how many people passed the basic fire science exam?

63. **Inlet and Outlet Pump Rates** A fuel storage tank has one supply pump and two identical outlet pumps. With one outlet pump running, the supply pump can increase the fuel level in the storage tank by 8750 gallons in 30 minutes. With both outlet pumps running, the supply pump can increase the fuel level in the storage tank by 11,250 gallons in 45 minutes. Find the pumping rate, in gallons per hour, for each of the pumps.



64. **Dimensions of a Pontoon** The pontoons on a boat are cylinders with conical tips. The length of a pontoon is 40 feet, and its diameter is 4 feet. The volume of each pontoon is 477.5 cubic feet.



a. Write a system of equations that describes the relationships between x and y. See the accompanying figure.

b. Find x and y. Round to the nearest tenth of a foot.

Enrichment Exercises

The system of equations

$$\begin{cases} \dfrac{3}{x} + \dfrac{2}{y} = 1 \\ -\dfrac{7}{x} + \dfrac{6}{y} = -1 \end{cases}$$

is not a linear system of equations. However, the system can be written in the form of a linear system involving the variables u and v, by using the substitutions $u = 1/x$ and $v = 1/y$ to produce

$$\begin{cases} 3u + 2v = 1 \\ -7u + 6v = -1 \end{cases}$$

Solving this system yields $u = 1/4$ and $v = 1/8$, which allows us to determine that $x = 4$ and $y = 8$, since x and u are reciprocals, and y and v are reciprocals. Thus (4, 8) is the solution of the original system.

In Exercises 65 and 66, use the substitutions $u = 1/x$ and $v = 1/y$, as explained above, to solve the given systems of equations.

65. $\begin{cases} \dfrac{2}{x} - \dfrac{1}{y} = \dfrac{2}{3} \\ \dfrac{5}{x} + \dfrac{3}{y} = -\dfrac{1}{6} \end{cases}$

66. $\begin{cases} \dfrac{1}{x} + \dfrac{2}{y} = 1 \\ \dfrac{2}{x} - \dfrac{1}{y} = 0 \end{cases}$

67. Use a graphing utility to graph each of the equations given in Exercise 65.

a. Is the point $(6, -3)$ an intersection point of the graphs?

b. The point (0, 0) appears to be an intersection point for the graphs. Explain how you know that the graphs do not intersect at (0, 0).

SECTION 6.2 Systems of Linear Equations in Three Variables

Systems of Linear Equations in Three Variables
Triangular Form
Nonsquare Systems of Equations
Homogeneous Systems of Equations
Applications of Systems of Equations

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A38.

PS1. Solve $2x - 5y = 15$ for y. [1.1]

PS2. If $x = 2c + 1$, $y = -c + 3$, and $z = 2x + 5y - 4$, write z in terms of c. [P.1]

PS3. Solve: $\begin{cases} 5x - 2y = 10 \\ \quad\;\; 2y = 8 \end{cases}$ [6.1]