Thus we have a system of two linear equations in the variables r_1 and r_2.

$$\begin{cases} 9 = r_1 + r_2 \\ 5 = r_1 - r_2 \end{cases}$$

Solving the system by using the elimination method, we find that r_1 is 7 miles per hour and r_2 is 2 miles per hour. Thus the rate of the boat in calm water is 7 miles per hour and the rate of the current is 2 miles per hour. You should verify these solutions.

▶ Try Exercise 50, page 487

EXERCISE SET 6.1

Concept Check

1. Is (4, 7) a solution of this system? $\begin{cases} 2x + 3y = 29 \\ -5x + 2y = -6 \end{cases}$ Yes

2. Identify each of the following systems of equations as an independent system, a dependent system, or as an inconsistent system.

   a. $\begin{cases} 3x - 6y = 8 \\ -6x + 12y = 10 \end{cases}$ b. $\begin{cases} 5x - 11y = 4 \\ 2x + 3y = 7 \end{cases}$

   c. $\begin{cases} x - y = 3 \\ -2x + 2y = -6 \end{cases}$ a. Inconsistent b. Independent c. Dependent

3. A system of two linear equations in two variables has no solution. What does this indicate concerning the relationship between the graphs of the equations? The graphs are lines that are parallel to each other.

4. The solutions of a dependent system of equations consist of all ordered pairs of the form

$$\left(c, \frac{3}{5}c - \frac{4}{5}\right)$$

Is (8, 4) a solution of this system? Yes

In Exercises 5 to 24, solve each system of equations by using the substitution method.

5. $\begin{cases} 3x - 2y = 1 \\ y = 4 \end{cases}$ (3, 4)

6. $\begin{cases} 3x - 2y = -11 \\ y = 1 \end{cases}$ (−3, 1)

7. $\begin{cases} 3x + 4y = 18 \\ y = -2x + 3 \end{cases}$ $\left(-\frac{6}{5}, \frac{27}{5}\right)$

8. $\begin{cases} 5x - 4y = -22 \\ y = 5x - 2 \end{cases}$ (2, 8)

9. $\begin{cases} -2x + 3y = 6 \\ x = 2y - 5 \end{cases}$ (3, 4)

10. $\begin{cases} 8x + 3y = -7 \\ x = 3y + 15 \end{cases}$ $\left(\frac{8}{9}, -\frac{127}{27}\right)$

11. $\begin{cases} 6x + 5y = 1 \\ x - 3y = 4 \end{cases}$ (1, −1)

12. $\begin{cases} -3x + 7y = 14 \\ 2x - y = -13 \end{cases}$ (−7, −1)

13. $\begin{cases} 7x + 6y = -3 \\ y = \frac{2}{3}x - 6 \end{cases}$ (3, −4)

14. $\begin{cases} 9x - 4y = 3 \\ x = \frac{4}{3}y + 3 \end{cases}$ (−1, −3)

15. $\begin{cases} y = 3x - 5 \\ y = 5x - 7 \end{cases}$ (1, −2)

16. $\begin{cases} y = 3x - 2 \\ 2x = y + 5 \end{cases}$ (−3, −11)

17. $\begin{cases} 2y = 3x + 19 \\ x = -2y + 7 \end{cases}$ (−3, 5)

18. $\begin{cases} 18x - 12y = 4 \\ -6x + 4y = 5 \end{cases}$ No solution

19. $\begin{cases} 3x - 4y = 2 \\ 4x + 3y = 14 \end{cases}$ $\left(\frac{62}{25}, \frac{34}{25}\right)$

20. $\begin{cases} 6x + 7y = -4 \\ 2x + 5y = 4 \end{cases}$ (−3, 2)

21. $\begin{cases} 3x - 3y = 5 \\ 4x - 4y = 9 \end{cases}$ No solution

22. $\begin{cases} 3x - 4y = 8 \\ 6x - 8y = 9 \end{cases}$ No solution

23. $\begin{cases} 4x + 3y = 6 \\ y = -\frac{4}{3}x + 2 \end{cases}$ $\left(c, -\frac{4}{3}c + 2\right)$

24. $\begin{cases} 5x + 2y = 2 \\ y = -\frac{5}{2}x + 1 \end{cases}$ $\left(c, -\frac{5}{2}c + 1\right)$

In Exercises 25 to 42, solve each system of equations by using the elimination method.

25. $\begin{cases} 3x - y = 10 \\ 4x + 3y = -4 \end{cases}$ (2, −4)

26. $\begin{cases} 3x + 4y = -5 \\ x - 5y = -8 \end{cases}$ (−3, 1)

27. $\begin{cases} 4x + 7y = 21 \\ 5x - 4y = -12 \end{cases}$ (0, 3)

28. $\begin{cases} 3x - 8y = -6 \\ -5x + 4y = 10 \end{cases}$ (−2, 0)

29. $\begin{cases} 5x - 3y = 0 \\ 10x - 6y = 0 \end{cases}$ $\left(\frac{3}{5}c, c\right)$

30. $\begin{cases} 3x + 2y = 0 \\ 2x + 3y = 0 \end{cases}$ (0, 0)

31. $\begin{cases} 3x - y = 4 \\ -6x + 2y = -8 \end{cases}$ (c, 3c − 4)

32. $\begin{cases} 4x + y = 2 \\ 8x + 2y = 4 \end{cases}$ (c, −4c + 2)

33. $\begin{cases} 3x + 6y = 11 \\ 2x + 4y = 9 \end{cases}$ No solution

34. $\begin{cases} 4x - 2y = 9 \\ 2x - y = 3 \end{cases}$ No solution

35. $\begin{cases} \frac{5}{6}x - \frac{1}{3}y = -6 \\ \frac{1}{6}x + \frac{2}{3}y = 1 \end{cases}$ (−6, 3)

36. $\begin{cases} \frac{3}{4}x + \frac{2}{5}y = 1 \\ \frac{1}{2}x - \frac{3}{5}y = -1 \end{cases}$ $\left(\frac{4}{13}, \frac{25}{13}\right)$

37. $\begin{cases} \frac{3}{4}x + \frac{1}{3}y = 1 \\ \frac{1}{2}x + \frac{2}{3}y = 0 \end{cases}$ $\left(2, -\frac{3}{2}\right)$

38. $\begin{cases} \frac{3}{5}x - \frac{2}{3}y = 7 \\ \frac{2}{5}x - \frac{5}{6}y = 7 \end{cases}$ (5, −6)

39. $\begin{cases} 2\sqrt{3}x - 3y = 3 \\ 3\sqrt{3}x + 2y = 24 \end{cases}$ $(2\sqrt{3}, 3)$

40. $\begin{cases} 4x - 3\sqrt{5}y = -19 \\ 3x + 4\sqrt{5}y = 17 \end{cases}$ $(-1, \sqrt{5})$

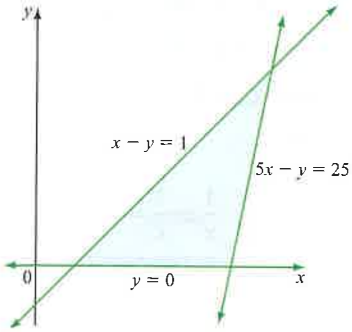
41. $\begin{cases} 3\sqrt{2}x - 4\sqrt{3}y = -6 \\ 2\sqrt{2}x + 3\sqrt{3}y = 13 \end{cases}$ $(\sqrt{2}, \sqrt{3})$

42. $\begin{cases} 2\sqrt{2}x + 3\sqrt{5}y = 7 \\ 3\sqrt{2}x - \sqrt{5}y = -17 \end{cases}$ $(-2\sqrt{2}, \sqrt{5})$

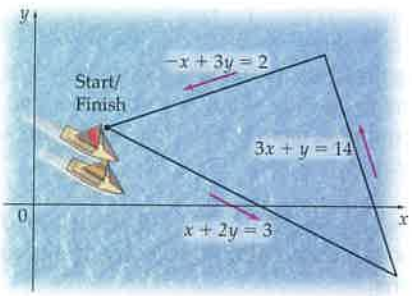
Indicates Try It Exercises

In Exercises 43 to 64, solve by using a system of equations.

43. **Area of a Triangle** Determine the area of the triangle shown in blue. Assume that x and y are measured in miles. 10 mi^2



44. **A Sailboat Racecourse** Determine the distance around the racecourse shown by the triangle in the following diagram. Assume that x and y are measured in kilometers. Round your answer to the nearest tenth of a kilometer. 10.8 km



45. **Supply–Demand** The number x of smartphones a manufacturer is willing to sell is given by $x = 25p - 3400$, where p is the price, in dollars, per smartphone. The number x of smartphones a wholesale distributor is willing to purchase is given by $x = -5p + 2000$, where p is the price per smartphone. Find the equilibrium price. $180

46. **Supply–Demand** The number x of digital cameras a manufacturer is willing to sell is given by $x = 25p - 500$, where p is the price, in dollars, per digital camera. The number x of digital cameras a store is willing to purchase is given by $x = -7p + 1100$, where p is the price per digital camera. Find the equilibrium price. $50

47. **Rate of Wind** Flying with the wind, a plane traveled 450 miles in 3 hours. Flying against the wind, the plane traveled the same distance in 5 hours. Find the rate of the plane in calm air and the rate of the wind. Plane: 120 mph; wind: 30 mph

48. **Rate of Wind** A plane flew 800 miles in 4 hours while flying with the wind. Against the wind, it took the plane 5 hours to travel 800 miles. Find the rate of the plane in calm air and the rate of the wind. Plane: 180 mph; wind: 20 mph

49. **Rate of Current** A motorboat traveled a distance of 120 miles in 4 hours while traveling with the current. Against the current, the same trip took 6 hours. Find the rate of the boat in calm water and the rate of the current. Boat: 25 mph; current: 5 mph

50. **Rate of Current** A canoeist can row 12 miles with the current in 2 hours. Rowing against the current, it takes the canoeist 4 hours to travel the same distance. Find the rate of the canoeist in calm water and the rate of the current. Canoeist: 4.5 mph; current: 1.5 mph

51. **Metallurgy** A metallurgist made two purchases. The first purchase, which cost $1080, included 30 kilograms of an iron alloy and 45 kilograms of a lead alloy. The second purchase, at the same prices, cost $372 and included 15 kilograms of the iron alloy and 12 kilograms of the lead alloy. Find the cost per kilogram of the iron and lead alloys. Iron: $12/kg; lead: $16/kg

52. **Chemistry** For $14.10, a chemist purchased 10 liters of hydrochloric acid and 15 liters of silver nitrate. A second purchase, at the same prices, cost $18.16 and included 12 liters of hydrochloric acid and 20 liters of silver nitrate. Find the cost per liter of each of the two chemicals. Hydrochloric acid: $0.48/L: silver nitrate: $0.62/L

53. **Chemistry** A goldsmith has two gold alloys. The first alloy is 40% gold; the second alloy is 60% gold. How many grams of each should be mixed to produce 20 grams of an alloy that is 52% gold? 40% gold: 8 g; 60% gold: 12 g

54. **Chemistry** One acetic acid solution is 70% water, and another is 30% water. How many liters of each solution should be mixed to produce 20 liters of a solution that is 40% water? 70% solution: 5 L; 30% solution: 15 L

55. **Geometry** A right triangle in the first quadrant is bounded by the lines $y = 0$, $y = \frac{1}{2}x$, and $y = -2x + 6$. Find its area. $\frac{9}{5}$ square units

56. **Geometry** The lines whose equations are $2x + 3y = 1$, $3x - 4y = 10$, and $4x + ky = 5$ all intersect at the same point. What is the value of k? 3

57. **Number Theory** Adding a three-digit number $5Z7$ to 256 gives $XY3$. If $XY3$ is divisible by 3, then what is the largest possible value of Z? 8

58. **Number Theory** Find the value of k if $2x + 5 = 6x + k = 4x - 7$. −19

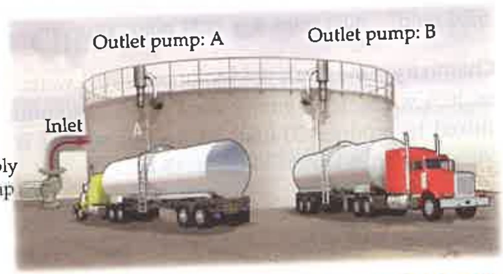
59. **Number Theory** A Pythagorean triple is three positive integers—a, b, and c—for which $a^2 + b^2 = c^2$. Given $a = 42$, find all the values of b and c such that a, b, and c form a Pythagorean triple. *Suggestion:* If $a = 42$, then $1764 + b^2 = c^2$ or $1764 = c^2 - b^2 = (c - b)(c + b)$. Because the product $(c - b)(c + b) = 1764$, $c - b$ and $c + b$ must be factors of 1764. For instance, one possibility is $2 = c - b$ and $882 = c + b$. Solving this system of equations yields one set of Pythagorean triples. Now repeat the process for other possible factors of 1764. Remember that answers must be positive integers. 42, 56, 70; 42, 40, 58; 42, 144, 150; 42, 440, 442

60. **Number Theory** Given $a = 30$, find all the values of b and c such that a, b, and c form a Pythagorean triple. (See the preceding exercise.) 30, 16, 34; 30, 40, 50; 30, 72, 78; 30, 224, 226

61. **Marketing** A marketing company asked 100 people whether they liked a new skin cream and lip balm. The company found that 80% of the people who liked the new skin cream also liked the new lip balm and that 50% of the people who did not like the new skin cream liked the new lip balm. If 77 people liked the lip balm, how many people liked the skin cream? 90 people

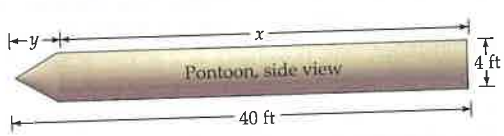
62. **Fire Science** An analysis of 200 scores on a firefighter qualifying exam found that 75% of those who passed the basic fire science exam also passed the exam on containing chemical fires. Of those who did not pass the basic fire science exam, 25% passed the exam on containing chemical fires. If 120 people passed the exam on containing chemical fires, how many people passed the basic fire science exam? 140 people

63. **Inlet and Outlet Pump Rates** A fuel storage tank has one supply pump and two identical outlet pumps. With one outlet pump running, the supply pump can increase the fuel level in the storage tank by 8750 gallons in 30 minutes. With both outlet pumps running, the supply pump can increase the fuel level in the storage tank by 11,250 gallons in 45 minutes. Find the pumping rate, in gallons per hour, for each of the pumps.



Supply pump: 20,000 gal/h; each outlet pump: 2500 gal/h

64. **Dimensions of a Pontoon** The pontoons on a boat are cylinders with conical tips. The length of a pontoon is 40 feet, and its diameter is 4 feet. The volume of each pontoon is 477.5 cubic feet.



a. Write a system of equations that describes the relationships between x and y. See the accompanying figure.

b. Find x and y. Round to the nearest tenth of a foot.

a. $\begin{cases} x + y = 40 \\ \pi(2)^2x + \frac{1}{3}\pi(2)^2y = 477.5 \end{cases}$ b. $x = 37.0$ ft; $y = 3.0$ ft

Enrichment Exercises

The system of equations

$$\begin{cases} \frac{3}{x} + \frac{2}{y} = 1 \\ -\frac{7}{x} + \frac{6}{y} = -1 \end{cases}$$

is not a linear system of equations. However, the system can be written in the form of a linear system involving the variables u and v, by using the substitutions $u = 1/x$ and $v = 1/y$ to produce

$$\begin{cases} 3u + 2v = 1 \\ -7u + 6v = -1 \end{cases}$$

Solving this system yields $u = 1/4$ and $v = 1/8$, which allows us to determine that $x = 4$ and $y = 8$, since x and u are reciprocals, and y and v are reciprocals. Thus $(4, 8)$ is the solution of the original system.

In Exercises 65 and 66, use the substitutions $u = 1/x$ and $v = 1/y$, as explained above, to solve the given systems of equations.

65. $\begin{cases} \frac{2}{x} - \frac{1}{y} = \frac{2}{3} \\ \frac{5}{x} + \frac{3}{y} = -\frac{1}{6} \end{cases}$ $(6, -3)$

66. $\begin{cases} \frac{1}{x} + \frac{2}{y} = 1 \\ \frac{2}{x} - \frac{1}{y} = 0 \end{cases}$ $\left(5, \frac{5}{2}\right)$

67. Use a graphing utility to graph each of the equations given in Exercise 65.

a. Is the point $(6, -3)$ an intersection point of the graphs? Yes

b. The point $(0, 0)$ appears to be an intersection point for the graphs. Explain how you know that the graphs do not intersect at $(0, 0)$. The variable terms in the system are undefined for $x = 0$ and for $y = 0$.

SECTION 6.2

Systems of Linear Equations in Three Variables
Triangular Form
Nonsquare Systems of Equations
Homogeneous Systems of Equations
Applications of Systems of Equations

Systems of Linear Equations in Three Variables

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A38.

PS1. Solve $2x - 5y = 15$ for y. [1.1] $y = \frac{2}{5}x - 3$

PS2. If $x = 2c + 1$, $y = -c + 3$, and $z = 2x + 5y - 4$, write z in terms of c. [...] $z = -c$

PS3. Solve: $\begin{cases} 5x - 2y = 10 \\ 2y = 8 \end{cases}$ [6.1] $\left(\frac{18}{5}, 4\right)$

PS4. Solve: $\begin{cases} 3x - y = 11 \\ 2x + 3y = -11 \end{cases}$ [6.1] (2, −5)

PS5. Solve: $\begin{cases} y = 3x - 4 \\ y = 4x - 2 \end{cases}$ [6.1] (−2, −10)

PS6. Solve: $\begin{cases} 4x + y = 9 \\ -8x - 2y = -18 \end{cases}$ [6.1] (c, −4c + 9)

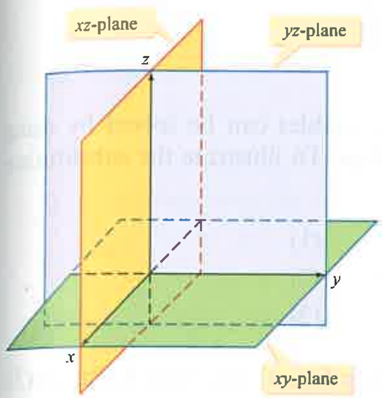


Figure 6.9

Systems of Linear Equations in Three Variables

An equation of the form $Ax + By + Cz = D$, with A, B, and C not all zero, is a linear equation in three variables. A solution of an equation in three variables is an **ordered triple** (x, y, z).

The ordered triple $(2, -1, -3)$ is one of the solutions of the equation $2x - 3y + z = 4$. The ordered triple $(3, 1, 1)$ is another solution. In fact, an infinite number of ordered triples are solutions of the equation.

Graphing an equation in three variables requires a third coordinate axis perpendicular to the xy-plane. This third axis is commonly called the **z-axis.** The result is a three-dimensional coordinate system called the xyz-coordinate system (see Figure 6.9). To visualize a three-dimensional coordinate system, think of a corner of a room: the floor is the xy-plane, one wall is the yz-plane, and the other wall is the xz-plane.

Graphing an ordered triple requires three moves: the first along the x-axis, the second along the y-axis, and the third along the z-axis. Figure 6.10 is the graph of the points $(-5, -4, 3)$ and $(4, 5, -2)$.

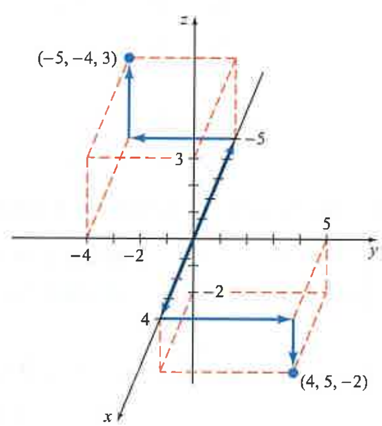


Figure 6.10

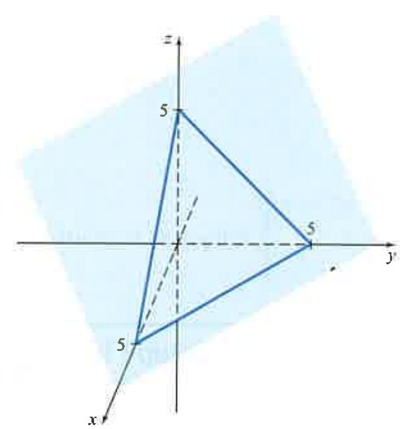


Figure 6.11

The graph of a linear equation in three variables is a plane. That is, if all the solutions of a linear equation in three variables were plotted in an xyz-coordinate system, the graph would look like a large, flat piece of paper with infinite extent. Figure 6.11 is a portion of the graph of $x + y + z = 5$.

There are different ways in which three planes can be oriented in an xyz-coordinate system. Figure 6.12 on the next page illustrates several ways.

For a system of linear equations in three variables to have a solution, the graphs of the equations must be three planes that intersect at a point, be three planes that intersect along a common line, or all be the same plane. In Figure 6.12, the graphs in **a.**, **b.**, and **c.** represent systems of equations that have a solution. The system of equations represented in Figure 6.12a is a consistent system of equations. Figure 6.12b and Figure 6.12c are graphs of dependent systems of equations. The remaining graphs are the graphs of inconsistent systems of equations.