5.6 Composition of Functions

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Learning Target Evaluate and find compositions of functions.

Success Criteria
- I can evaluate a composition of functions.
- I can find a composition of functions.
- I can state the domain of a composition of functions.

EXPLORE IT! Finding a Composition of Functions

Work with a partner. The formulas below represent the temperature F (in degrees Fahrenheit) when the temperature is C degrees Celsius, and the temperature C when the temperature is K (Kelvin).

$$F = \frac{9}{5}C + 32 \qquad C = K - 273$$

Math Practice

Make Sense of Quantities

Does $g(f(x))$ make sense in this context? Explain.

a. Write an expression for F in terms of K.

b. Given that

$$f(x) = \frac{9}{5}x + 32$$

and

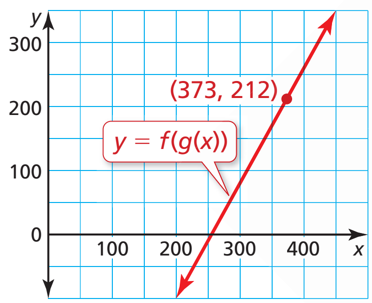
$$g(x) = x - 273$$

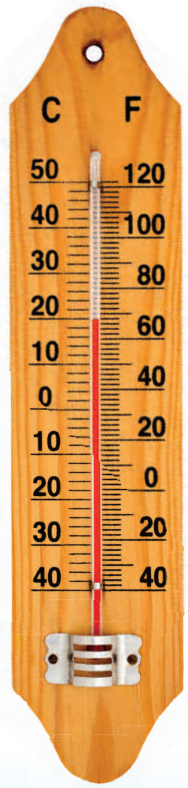
write an expression for $f(g(x))$. What does $f(g(x))$ represent in this situation?

c. Water freezes at about 273 Kelvin. Find $f(g(273))$. Does your answer make sense? Explain your reasoning.

d. Interpret the point shown on the graph.

Temperature Conversion





Evaluating Compositions of Functions

Vocabulary

composition, *p. 268*

You have combined functions by finding sums, differences, products, and quotients of functions. Another way of combining two functions is to form a *composition*.

READING

The composition $g(f(x))$ can be read as "g of f of x."

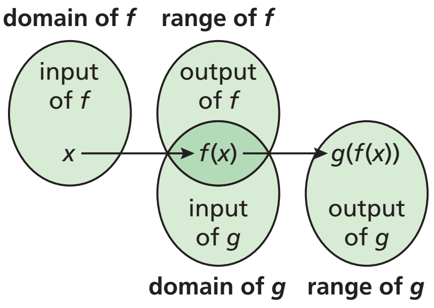
KEY IDEA

Composition of Functions

The **composition** of a function g with a function f is

$$h(x) = g(f(x)).$$

The domain of h is the set of all x-values such that x is in the domain of f and $f(x)$ is in the domain of g.



EXAMPLE 1 Evaluating Compositions of Functions

Let $f(x) = \sqrt{2x + 1}$ and $g(x) = x^2 - 4$. Find the indicated value.

a. $g(f(4))$ **b.** $f(g(2))$ **c.** $g(g(-2))$

SOLUTION

a. To evaluate $g(f(4))$, first find $f(4)$.

$$f(4) = \sqrt{2(4) + 1} = \sqrt{8 + 1} = \sqrt{9} = 3$$

Then $g(f(4)) = g(3) = 3^2 - 4 = 9 - 4 = 5$.

▶ So, $g(f(4))$ is 5.

READING

As with subtraction and division of functions, you need to be aware of the order of functions when they are composed. In general, $f(g(x)) \neq g(f(x))$.

b. To evaluate $f(g(2))$, first find $g(2)$.

$$g(2) = 2^2 - 4 = 4 - 4 = 0$$

Then $f(g(2)) = f(0) = \sqrt{2(0) + 1} = \sqrt{0 + 1} = \sqrt{1} = 1$.

▶ So, $f(g(2))$ is 1.

c. To evaluate $g(g(-2))$, first find $g(-2)$.

$$g(-2) = (-2)^2 - 4 = 4 - 4 = 0$$

Then $g(g(-2)) = g(0) = 0^2 - 4 = 0 - 4 = -4$.

▶ So, $g(g(-2))$ is -4.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Let $f(x) = x - 2$, $g(x) = x^2$, and $h(x) = \dfrac{x + 5}{2}$. Find the indicated value.

1. $f(g(-1))$

2. $g(h(-7))$

3. $h(g(5))$

4. $f(f(0))$

5. MP **STRUCTURE** For functions f and g, $f(2) = -3$ and $g(-3) = 10$. Find $g(f(2))$.

Finding Compositions of Functions

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EXAMPLE 2 Finding Compositions of Functions

WATCH

Let $f(x) = 5x^{-1}$ and $g(x) = 3x - 3$. Perform the indicated composition and state the domain.

a. $f(g(x))$

b. $g(f(x))$

c. $f(f(x))$

SOLUTION

STUDY TIP

$g(1) = 0$ is not in the domain of f because $f(0) = \frac{5}{0}$, which is undefined.

a. $f(g(x)) = f(3x - 3)$ — Substitute $3x - 3$ for $g(x)$.

$= 5(3x - 3)^{-1}$ — Replace x with $3x - 3$ in $f(x)$.

$= \dfrac{5}{3x - 3}$ — Definition of negative exponents

▶ The domain of $y = f(g(x))$ is all real numbers except $x = 1$, because $g(1) = 0$ is not in the domain of f.

b. $g(f(x)) = g(5x^{-1})$ — Substitute $5x^{-1}$ for $f(x)$.

$= 3(5x^{-1}) - 3$ — Replace x with $5x^{-1}$ in $g(x)$.

$= 15x^{-1} - 3$ — Multiply.

$= \dfrac{15}{x} - 3$ — Definition of negative exponents

▶ The domain of $y = g(f(x))$ is all real numbers except $x = 0$, because 0 is not in the domain of f.

c. $f(f(x)) = f(5x^{-1})$ — Substitute $5x^{-1}$ for $f(x)$.

$= 5(5x^{-1})^{-1}$ — Replace x with $5x^{-1}$ in $f(x)$.

$= 5(5^{-1}x^{1})$ — Use properties of exponents.

$= 5\left(\frac{1}{5}x\right)$ — Definition of negative exponents

$= x$ — Multiply.

▶ The domain of $y = f(f(x))$ is all real numbers except $x = 0$, because 0 is not in the domain of f.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Let $f(x) = 2x^{-1}$, $g(x) = 4x - 3$, and $h(x) = 0.5x + 2$. Perform the indicated composition and state the domain.

6. $f(g(x))$

7. $g(f(x))$

8. $f(f(x))$

9. $h(h(x))$

10. **MP REASONING** Let f and g be linear functions. Is $y = f(g(x))$ a linear function? Explain your reasoning.

Solving Real-Life Problems

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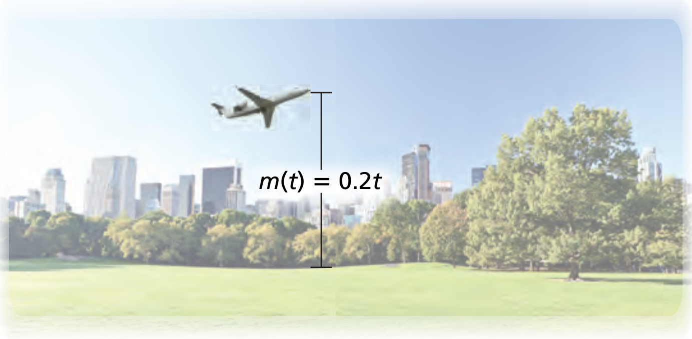
EXAMPLE 3 **Modeling Real Life** WATCH INFO

The function $C(m) = 15 - 10.5m$ approximates the temperature (in degrees Celsius) at an altitude of m miles. The diagram shows the altitude (in miles) of an airplane t minutes after taking off, where $0 \le t \le 30$. Find $C(m(t))$. Evaluate $C(m(30))$ and explain what it represents.



SOLUTION

The composition $C(m(t))$ represents the temperature at the airplane's altitude t minutes after taking off. Find $C(m(t))$.

$C(m(t)) = C(0.2t)$ — Substitute $0.2t$ for $m(t)$.

$= 15 - 10.5(0.2t)$ — Replace m with $0.2t$ in $C(m)$.

$= 15 - 2.1t$ — Multiply.

Evaluate $C(m(30))$.

$C(m(30)) = 15 - 2.1(30)$ — Substitute 30 for t.

$= 15 - 63$ — Multiply.

$= -48$ — Subtract.

▶ So, $C(m(30)) = -48$ indicates that after 30 minutes, the airplane is at an altitude that has a temperature of about -48°C.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

11. The function $C(x) = 50x + 100$ represents the cost (in dollars) of producing x bee hive boxes. The number of bee hive boxes produced in t hours is represented by $x(t) = 6t$.

a. Find $C(x(t))$.

b. Evaluate $C(x(8))$ and explain what it represents.

12. A worker earning an hourly wage changes positions within a company. The new position comes with a 20% raise in hourly wage. The worker also receives a $2 increase in hourly wage. Use composition of functions to write a function that represents the worker's new hourly wage when the 20% raise is applied before the $2 raise.

5.6 Practice WITH CalcChat® AND CalcView®

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In Exercises 1–8, let $f(x) = \sqrt{x+1}$, $g(x) = 2x - 5$, and $h(x) = 3x^2 - 3$. Find the indicated value. *Example 1*

1. $f(g(4))$

2. $g(f(0))$

3. $g(h(-2))$

4. $h(g(-1))$

5. $h(f(10))$

6. $f(h(-3))$

7. $g(g(-2.5))$

8. $h\left(h\left(\frac{2}{3}\right)\right)$

In Exercises 9–20, find (a) $f(g(x))$, (b) $g(f(x))$, and (c) $f(f(x))$. State the domain of each composition. *Example 2*

9. $f(x) = -5x$, $g(x) = x + 6$

10. $f(x) = x - 9$, $g(x) = |x + 2|$

11. $f(x) = 2x^2$, $g(x) = x - 1$

12. $f(x) = x^2 + 7$, $g(x) = 2x + 5$

13. $f(x) = 3x^{-1}$, $g(x) = 4x + 8$

14. $f(x) = 10x^{-1}$, $g(x) = x^2 - 9$

15. $f(x) = 3x - 7$, $g(x) = \sqrt{x + 7}$

16. $f(x) = 4x + 2$, $g(x) = \sqrt{x - 6}$

17. $f(x) = -x + 11$, $g(x) = \sqrt[3]{x - 3}$

18. $f(x) = -6x - 5$, $g(x) = \sqrt[3]{x + 4}$

19. $f(x) = 2x + 1$, $g(x) = x^2 + 6x - 10$

20. $f(x) = 3x - 1$, $g(x) = x^3 - 2x + 4$

ERROR ANALYSIS **In Exercises 21 and 22, let $f(x) = x^2 - 3$ and $g(x) = 4x$. Describe and correct the error in performing the composition.**

21.

$$f(g(x)) = (x^2 - 3)(4x)$$
$$= 4x^3 - 12x$$

22.

$$g(f(x)) = g(x^2 - 3)$$
$$= 4x^2 - 3$$

23. MODELING REAL LIFE The function $C(g) = 2.75g$ represents the cost (in dollars) of g gallons of gasoline at a gas station. The function $g(m) = 0.04m$ approximates the number of gallons of gasoline a vehicle uses to travel m miles. *Example 3*

a. Find $C(g(m))$. Interpret the coefficient.

b. Evaluate $C(g(100))$ and explain what it represents.

24. MODELING REAL LIFE The function $p(d) = 0.03d + 1$ approximates the pressure (in atmospheres) at a depth of d feet below sea level. The function $d(t) = 60t$ represents the depth (in feet) of a diver t minutes after beginning a descent from sea level, where $0 \le t \le 2$.

a. Find $p(d(t))$. Interpret the terms and coefficient.

b. Evaluate $p(d(1.5))$ and explain what it represents.

25. MP REASONING The table shows the inputs and outputs of two functions f and g. Use the table to find each value.

x	-2	-1	0	1	2	3
$f(x)$	7	5	3	2	0	-2
$g(x)$	1	-2	-1	5	2	0

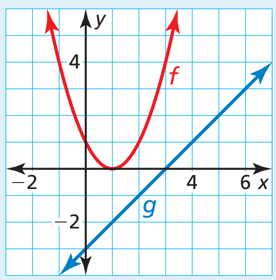
a. $f(g(-1))$

b. $g(f(2))$

c. $f(f(0))$

d. $g(g(-2))$

26. HOW DO YOU SEE IT? Use the graphs of f and g to find each value.



a. $f(g(6))$

b. $g(f(-1))$

c. $f(f(2))$

d. $g(g(3))$

27. MP REASONING Functions f and g consist only of the ordered pairs shown. Find the ordered pairs for $y = f(g(x))$.

f: $(-12, 11), (-4, 9), (1, 3), (2, -4), (6, -5)$

g: $(-10, 6), (-3, 1), (0, -4), (5, 2), (8, -12)$

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28. **COLLEGE PREP** Let $f(x) = x^2 + 1$ and $g(x) = 3x + 1$. What is $f(f(x)) - g(f(x))$?

Ⓐ $x^2 - 3x$

Ⓑ $x^4 - x^2 - 2$

Ⓒ $x^4 - 7x^2 - 6x$

Ⓓ $x^4 - 3x^3 + x^2 - 3x$

29. **CONNECTING CONCEPTS** The radius of a circular region increases at a rate of 2 inches per minute. Use composition of functions to write a function that represents the area A (in square inches) of the region after t minutes.

30. **MP PROBLEM SOLVING** You have two coupons for a store, one for \$10 off your entire purchase and another for 20% off your entire purchase. Both coupons can be used on the same purchase. Which order of discounts results in a lesser total? Use composition of functions to justify your answer.

MP STRUCTURE In Exercises 31–34, find functions f and g such that $f(g(x)) = h(x)$, $f(x) \neq x$, and $g(x) \neq x$.

31. $h(x) = \frac{1}{2}x + 6$

32. $h(x) = |2x + 9|$

33. $h(x) = \sqrt[3]{x + 2}$

34. $h(x) = \dfrac{4}{3x^2 + 7}$

35. **MP LOGIC** Complete the table using the following information.

- f and g are linear functions.
- $f(g(1)) = 6.5$
- $g(f(2)) = -5$

x	$f(x)$	$g(x)$
1		
2		
4		−5
7	6.5	

36. **THOUGHT PROVOKING**
Write two different nonlinear functions f and g such that $f(g(x)) = x$ and $g(f(x)) = x$.

In Exercises 37–40, let $f(x) = 3x + 5$, $g(x) = x^2$, and $h(x) = -2x - 1$. Perform the indicated composition.

37. $f(g(h(x)))$

38. $h(g(f(x)))$

39. $f(f(f(x)))$

40. $g(h(g(x)))$

41. **DIG DEEPER** Show that the function $f(x) = \frac{1}{3}\sqrt{x - 2} + 3$ is a composition, in some order, of functions g, h, p, and q.

$g(x) = \frac{1}{3}x$ $\qquad h(x) = x - 2$

$p(x) = x + 9$ $\qquad q(x) = \sqrt{x}$

REVIEW & REFRESH

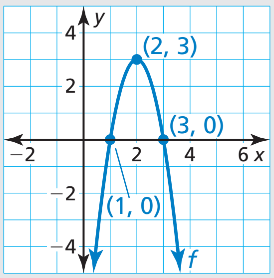
WATCH

In Exercises 42 and 43, solve the inequality.

42. $5\sqrt{x} - 3 < 17$

43. $\sqrt[3]{x + 1} + 4 \geq -2$

44. Describe the x-values for which (a) f is increasing or decreasing, (b) $f(x) > 0$, and (c) $f(x) < 0$.



In Exercises 45 and 46, let $f(x) = -x + 4$ and $g(x) = \dfrac{2x - 1}{3}$. Find the indicated value.

45. $f(g(5))$

46. $g(f(-4))$

47. Let g be a horizontal stretch by a factor of 2, followed by a translation 3 units up of the graph of $f(x) = \sqrt{4x}$. Write a rule for g.

In Exercises 48 and 49, solve the system using any method. Explain your choice of method.

48. $2x^2 + 4x - y = -5$
$2x + y = 1$

49. $x^2 - 3x - y = 4$
$-x^2 + 7x + y = 10$

50. **MODELING REAL LIFE** From 2012 to 2017, the United States population (in millions) ages 5 and over can be modeled by

$$P(t) = 0.0208t^4 - 0.203t^3 + 0.56t^2 + 2.1t + 289$$

and the number of people in that group that speak a language other than English at home can be modeled by

$$S(t) = 0.0037t^3 - 0.042t^2 + 1.08t + 59.4$$

where t is the number of years since 2012. Find $(P - S)(t)$. Explain what $(P - S)(t)$ represents.

51. Find the volume of the cone. Round your answer to the nearest tenth.