5.5 Performing Function Operations

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Learning Target Perform arithmetic operations on two functions.

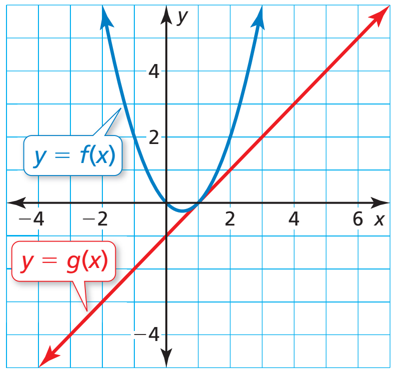
Success Criteria

- I can explain what it means to perform an arithmetic operation on two functions.
- I can find arithmetic combinations of two functions.
- I can state the domain of an arithmetic combination of two functions.
- I can evaluate an arithmetic combination of two functions for a given input.

Just as two real numbers can be combined by the operations of addition, subtraction, multiplication, and division to form other real numbers, two functions can be combined to form other functions.

EXPLORE IT! Graphing Arithmetic Combinations of Two Functions

Work with a partner. Consider the graphs of f and g.



a. Describe what it means to add two functions. Then describe what it means to subtract one function from another function.

b. Match each function with its graph. Explain your reasoning.

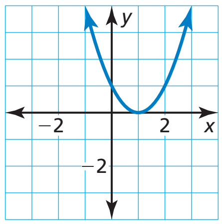
i. $m(x) = f(x) + g(x)$

ii. $n(x) = f(x) - g(x)$

iii. $p(x) = f(x) \cdot g(x)$

iv. $q(x) = f(x) \div g(x)$

A.



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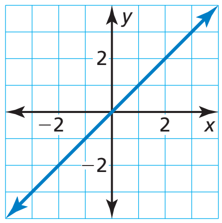

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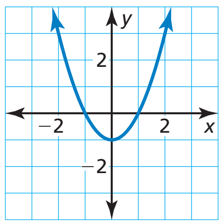

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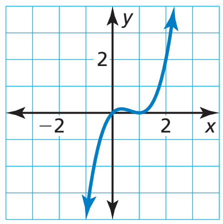

c. What is the domain of each function in part (b)? How do you know?

d. Check your answers in part (b) by writing function rules for f and g, performing each arithmetic combination, and graphing the results.

Math Practice

Use a Table

How can you use a table to organize your work in part (b)?

Operations on Functions

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You have learned how to add, subtract, multiply, and divide polynomial expressions. These operations are also defined for functions.

KEY IDEA

Operations on Functions

Let f and g be any two functions. A new function can be defined by performing any of the four basic operations on f and g.

Operation	Definition	Example: $f(x) = 5x$, $g(x) = x + 2$
Addition	$(f + g)(x) = f(x) + g(x)$	$(f + g)(x) = 5x + (x + 2) = 6x + 2$
Subtraction	$(f - g)(x) = f(x) - g(x)$	$(f - g)(x) = 5x - (x + 2) = 4x - 2$
Multiplication	$(fg)(x) = f(x) \bullet g(x)$	$(fg)(x) = 5x(x + 2) = 5x^2 + 10x$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	$\left(\frac{f}{g}\right)(x) = \frac{5x}{x + 2}$

The domains of the sum, difference, product, and quotient functions consist of the x-values that are in the domains of both f and g. Additionally, the domain of the quotient does not include x-values for which $g(x) = 0$.

EXAMPLE 1 Adding Two Functions

WATCH

Let $f(x) = 3\sqrt{x}$ and $g(x) = -10\sqrt{x}$. Find $(f + g)(x)$ and state the domain. Then evaluate $(f + g)(4)$.

SOLUTION

$$(f + g)(x) = f(x) + g(x) \qquad \text{Definition of function addition}$$
$$= 3\sqrt{x} + (-10\sqrt{x}) \qquad \text{Write sum of } f(x) \text{ and } g(x).$$
$$= (3 - 10)\sqrt{x} \qquad \text{Distributive Property}$$
$$= -7\sqrt{x} \qquad \text{Subtract.}$$

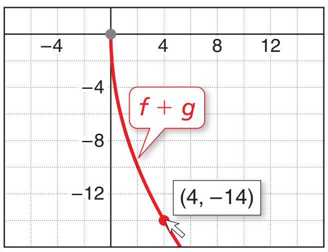
The functions f and g each have the same domain: all nonnegative real numbers. So, the domain of $f + g$ also consists of all nonnegative real numbers. To evaluate $f + g$ when $x = 4$, you can use several methods. Here are two:

Method 1 Use an algebraic approach.

$$(f + g)(4) = -7\sqrt{4} = -14$$

Method 2 Use a graphical approach.

Use technology to graph the sum of the functions. The graph shows that $(f + g)(4) = -14$.



EXAMPLE 2 Subtracting Two Functions

Let $f(x) = 3x^3 - 2x^2 + 5$ and $g(x) = x^3 - 3x^2 + 4x - 2$. Find $(f - g)(x)$ and state the domain. Then evaluate $(f - g)(-2)$.

SOLUTION

$$\begin{aligned}(f - g)(x) &= f(x) - g(x)\\ &= 3x^3 - 2x^2 + 5 - (x^3 - 3x^2 + 4x - 2)\\ &= 2x^3 + x^2 - 4x + 7\end{aligned}$$

The functions f and g each have the same domain: all real numbers. So, the domain of $f - g$ also consists of all real numbers.

$$(f - g)(-2) = 2(-2)^3 + (-2)^2 - 4(-2) + 7 = 3$$

EXAMPLE 3 Multiplying Two Functions

Let $f(x) = x^2$ and $g(x) = \sqrt{x}$. Find $(fg)(x)$ and state the domain. Then evaluate $(fg)(9)$.

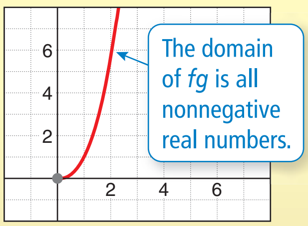
SOLUTION

$$(fg)(x) = f(x) \cdot g(x) = x^2(\sqrt{x}) = x^2(x^{1/2}) = x^{(2 + 1/2)} = x^{5/2}$$

The domain of f consists of all real numbers, and the domain of g consists of all nonnegative real numbers. So, the domain of fg consists of all nonnegative real numbers.

$$(fg)(9) = 9^{5/2} = (9^{1/2})^5 = 3^5 = 243$$

Check



EXAMPLE 4 Dividing Two Functions

Let $f(x) = 6x$ and $g(x) = x^{3/4}$. Find $\left(\frac{f}{g}\right)(x)$ and state the domain. Then evaluate $\left(\frac{f}{g}\right)(16)$.

SOLUTION

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{6x}{x^{3/4}} = 6x^{(1 - 3/4)} = 6x^{1/4}$$

The domain of f consists of all real numbers, the domain of g consists of all nonnegative real numbers, and $g(0) = 0$. So, the domain of $\frac{f}{g}$ is restricted to all *positive* real numbers.

$$\left(\frac{f}{g}\right)(16) = 6(16)^{1/4} = 6(2^4)^{1/4} = 12$$

ANOTHER WAY

In Example 4, you can also evaluate $\left(\frac{f}{g}\right)(16)$ as

$$\begin{aligned}\left(\frac{f}{g}\right)(16) &= \frac{f(16)}{g(16)}\\ &= \frac{6(16)}{(16)^{3/4}}\\ &= \frac{96}{8}\\ &= 12.\end{aligned}$$

SELF-ASSESSMENT

1 I do not understand. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

1. Let $f(x) = -2x^{2/3}$ and $g(x) = 7x^{2/3}$. Find $(f + g)(x)$ and $(f - g)(x)$ and state the domain of each. Then evaluate $(f + g)(8)$ and $(f - g)(8)$.

2. Let $f(x) = 3x$ and $g(x) = x^{1/5}$. Find $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$ and state the domain of each. Then evaluate $(fg)(32)$ and $\left(\frac{f}{g}\right)(32)$.

$f(x) = \sqrt{x}$

$g(x) = \sqrt{9 - x^2}$

$f(2) + g(2)$
$= 3.65028153987$

$f(2) - g(2)$
$= -0.821854415127$

$f(2) \cdot g(2)$
$= 3.16227766017$

$\frac{f(2)}{g(2)}$
$= 0.632455532034$

EXAMPLE 5 Performing Function Operations Using Technology

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Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{9 - x^2}$. Use technology to evaluate $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $\left(\frac{f}{g}\right)(x)$ when $x = 2$. Round your answers to two decimal places.

SOLUTION

Enter f and g. From the screen, you can see that $f(2) + g(2) \approx 3.65$, so $(f + g)(2) \approx 3.65$. Similarly,

$$(f - g)(2) \approx -0.82,\ (fg)(2) \approx 3.16, \text{ and } \left(\frac{f}{g}\right)(2) \approx 0.63.$$

EXAMPLE 6 Modeling Real Life

WATCH STEM INFO

For a white rhino, heart rate (in beats per minute) and life span (in minutes) are related to body mass m (in kilograms) by the following functions.

Heart rate: $r(m) = 241m^{-0.25}$

Life span: $s(m) = (6 \times 10^6)m^{0.2}$

Find $(rs)(m)$ and explain what it represents.

SOLUTION

$(rs)(m) = r(m) \cdot s(m)$	Definition of function multiplication
$= 241m^{-0.25}[(6 \times 10^6)m^{0.2}]$	Write product of $r(m)$ and $s(m)$.
$= 241(6 \times 10^6)m^{-0.25 + 0.2}$	Product of Powers Property
$= (1446 \times 10^6)m^{-0.05}$	Simplify.
$= (1.446 \times 10^9)m^{-0.05}$	Use scientific notation.

▶ So, $(rs)(m) = (1.446 \times 10^9)m^{-0.05}$. Multiplying heart rate by life span gives the total number of heartbeats over the lifetime of a white rhino with body mass m.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

3. Let $f(x) = 8x$ and $g(x) = 2x^{5/6}$. Use technology to evaluate $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $\left(\frac{f}{g}\right)(x)$ when $x = 5$. Round your answers to two decimal places.

4. **WRITING** In Example 5, explain why you can evaluate $(f + g)(3)$, $(f - g)(3)$, and $(fg)(3)$ but not $\left(\frac{f}{g}\right)(3)$.

5. Use the answer in Example 6 to find the total number of heartbeats over the lifetime of a white rhino when its body mass is 1.7×10^5 kilograms.

6. The cost (in dollars) to rent a scooter for x minutes in City A is represented by $A(x) = 0.15x + 1$. The cost (in dollars) in City B is represented by $B(x) = 0.29x + 1$. Find $(B - A)(x)$ and explain what it represents.

5.5 Practice WITH CalcChat® AND CalcView®

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In Exercises 1–4, find $(f+g)(x)$ and $(f-g)(x)$ and state the domain of each. Then evaluate $f+g$ and $f-g$ for the given value of x. *Examples 1 and 2*

1. $f(x) = -5\sqrt[4]{x}$, $g(x) = 19\sqrt[4]{x}$; $x = 16$

2. $f(x) = \sqrt[3]{2x}$, $g(x) = -11\sqrt[3]{2x}$; $x = -4$

3. $f(x) = 6x - 4x^2 - 7x^3$, $g(x) = 9x^2 - 5x$; $x = -1$

4. $f(x) = 11x + 2x^2$, $g(x) = -7x - 3x^2 + 4$; $x = 2$

In Exercises 5–10, find $(fg)(x)$ and $\left(\dfrac{f}{g}\right)(x)$ and state the domain of each. Then evaluate fg and $\dfrac{f}{g}$ for the given value of x. *Examples 3 and 4*

5. $f(x) = 2x^3$, $g(x) = \sqrt[3]{x}$; $x = -27$

6. $f(x) = x^4$, $g(x) = 3\sqrt{x}$; $x = 4$

7. $f(x) = 4x$, $g(x) = 9x^{1/2}$; $x = 9$

8. $f(x) = 11x^3$, $g(x) = 7x^{7/3}$; $x = -8$

9. $f(x) = 7x^{3/2}$, $g(x) = -14x^{1/3}$; $x = 64$

10. $f(x) = 4x^{5/4}$, $g(x) = 2x^{1/2}$; $x = 16$

MP USING TOOLS In Exercises 11–14, use technology to evaluate $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$, and $\left(\dfrac{f}{g}\right)(x)$ when $x = 5$. Round your answers to two decimal places. *Example 5*

11. $f(x) = 4x^4$; $g(x) = 24x^{1/3}$

12. $f(x) = 7x^{5/3}$; $g(x) = 49x^{2/3}$

13. $f(x) = -2x^{1/3}$; $g(x) = 5x^{1/2}$

14. $f(x) = 4x^{1/2}$; $g(x) = 6x^{3/4}$

ERROR ANALYSIS In Exercises 15 and 16, describe and correct the error in stating the domain.

15. $f(x) = x^3$ and $g(x) = x^2 - 4$

The domain of $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^3}{x^2 - 4}$ is all real numbers except $x = 2$.

16. $f(x) = x^{1/2}$ and $g(x) = x^{3/2}$

The domain of $(fg)(x) = x^2$ is all real numbers.

17. **MODELING REAL LIFE** Over a period of 8 years, the numbers (in millions) of female and male employees in the United States over the age of 16 can be modeled by $F(t) = 0.0134t^3 - 0.160t^2 + 0.98t + 72.9$ and $M(t) = 0.0093t^3 - 0.078t^2 + 0.58t + 82.3$, where t is the number of years since 2010. *Example 6*

   a. Find $(F + M)(t)$.

   b. Explain what $(F + M)(t)$ represents.

18. **MODELING REAL LIFE** For a mammal that weighs w grams, the volume b (in milliliters) of air breathed in and the volume d (in milliliters) of "dead space" (the portion of the lungs not filled with air) can be modeled by

$$b(w) = 0.007w \text{ and } d(w) = 0.002w.$$

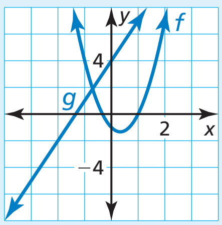
The breathing rate r (in breaths per minute) of a mammal that weighs w grams can be modeled by

$$r(w) = \frac{1.1w^{0.734}}{b(w) - d(w)}.$$

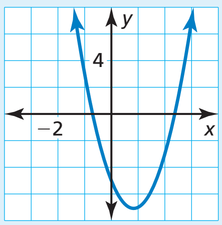
Simplify $r(w)$ and calculate the breathing rate for body weights of 6.5 grams, 300 grams, and 70,000 grams.

19. **MAKING AN ARGUMENT** Is the addition of functions commutative? the multiplication of functions? Explain your reasoning.

20. **HOW DO YOU SEE IT?** The graphs of the functions $f(x) = 3x^2 - 2x - 1$ and $g(x) = 3x + 4$ are shown. Which graph represents the function $f + g$? the function $f - g$? Explain.



A.



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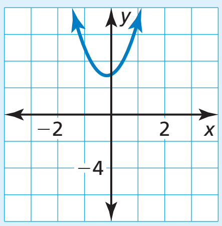


21. **MP REASONING** The table shows the outputs of the two functions f and g. Use the table to find each value.

x	0	1	2	3	4
$f(x)$	-2	-4	0	10	26
$g(x)$	-1	-3	-13	-31	-57

a. $(f+g)(3)$ **b.** $(f-g)(1)$

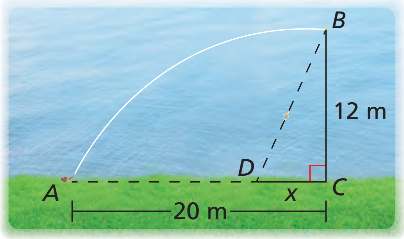
c. $(fg)(2)$ **d.** $\left(\frac{f}{g}\right)(0)$

22. **THOUGHT PROVOKING**
Is it possible to write two functions whose sum contains radicals, but whose product does not? Justify your answers.

23. **DIG DEEPER** For the functions f and g, $(f+g)(-1) = 4$ and $\left(\frac{f}{g}\right)(-1) = -\frac{3}{2}$. Find $f(-1)$ and $g(-1)$.

24. **MP PROBLEM SOLVING** You throw a tennis ball from point A along the water's edge of a lake to point B in the water, as shown. Your dog first runs from point A to point D and then swims to fetch the ball at point B.

GO DIGITAL



a. Your dog runs at a speed of about 6.4 meters per second and swims at a speed of about 0.9 meter per second. Write a function r in terms of x that represents the time he spends running. Write a function s in terms of x that represents the time he spends swimming.

b. Write a function t in terms of x that represents how long it takes your dog to reach the ball.

c. Use technology to graph t. Find the value of x that minimizes t. Explain the meaning of this value.

REVIEW & REFRESH

WATCH

In Exercises 25 and 26, solve the equation.

25. $3\sqrt{2x-5} = 9$ **26.** $\sqrt{-x-3} = x+5$

In Exercises 27 and 28, solve the literal equation for n.

27. $3xn - 9 = 6y$ **28.** $\frac{3+4n}{n} = 7b$

In Exercises 29 and 30, determine whether the relation is a function. Explain.

29. $(1, 6), (7, -3), (4, 0), (3, 0)$

30. $(3, 8), (2, 5), (9, 5), (2, -3)$

31. Let $f(x) = 8x^3$ and $g(x) = -2x^{3/2}$. Find $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$ and state the domain of each. Then evaluate fg and $\frac{f}{g}$ when $x = 4$.

In Exercises 32–35, simplify the expression.

32. $\sqrt[5]{243z^8}$ **33.** $\sqrt[4]{\frac{y^{12}}{625y^8}}$

34. $6\sqrt[3]{9} - 10\sqrt[3]{9}$ **35.** $3\sqrt{20} + 7\sqrt{5}$

In Exercises 36 and 37, describe the transformation of f represented by g. Then graph each function.

36. $f(x) = \sqrt{x}$, $g(x) = -\sqrt{x} + 2$

37. $f(x) = \sqrt[3]{x}$, $g(x) = 4\sqrt[3]{x} - 6$

38. Determine whether the table represents a *linear* or *nonlinear* function. Explain.

x	12	9	6	3
y	-1	0	1	2

39. **MODELING REAL LIFE** The number A of commercial drones sold (in thousands) can be modeled by the function $A = 19t^2 + 30t + 110$, where t represents the number of years after 2016.

a. In what year did commercial drone sales reach 200,000?

b. Find and interpret the average rate of change from 2016 to 2018.

c. Do you think this model will be accurate after 20 years? Explain your reasoning.