# 5.2 Properties of Rational Exponents and Radicals

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**Learning Target** Simplify radical expressions.

**Success Criteria**

- I can simplify radical expressions with rational exponents.
- I can explain when radical expressions are in simplest form.
- I can simplify variable expressions containing rational exponents and radicals.

## EXPLORE IT! Reviewing Properties of Exponents

**Work with a partner.**

**a.** The Product Property of Square Roots states that the square root of a product equals the product of the square roots of the factors.

$$\sqrt{64x^2} = \sqrt{64} \cdot \sqrt{x^2} \quad \text{Product Property of Square Roots}$$
$$= 8x \quad \text{Simplify.}$$

Describe the behavior of the graphs of $y = \sqrt{64x^2}$ and $y = 8x$. What do you notice? Use technology to check your graphs and explain the results.

**b.** You can extend the Product Property of Square Roots to other radicals, such as cube roots.

$$\sqrt[3]{64x^3} = \sqrt[3]{64} \cdot \sqrt[3]{x^3} \quad \text{Product Property of Cube Roots}$$
$$= 4x \quad \text{Simplify.}$$

Describe the behavior of the graphs of $y = \sqrt[3]{64x^3}$ and $y = 4x$. What do you notice? Use technology to check your graphs and explain the results.

**c.** How can you change the function $y = 8x$ so that it coincides with the graph of $y = \sqrt{64x^2}$ for all values of $x$? Explain your reasoning.

**d.** Determine the values of $n$ for which $\sqrt[n]{x^n} = x$ and $\sqrt[n]{x^n} = |x|$.

**Math Practice**

**Use Technology to Explore**
How can you use technology to help you determine the values of *n* in part (d)?

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**Vocabulary** AZ VOCAB

simplest form of a radical, *p. 239*
like radicals, *p. 240*

## Properties of Rational Exponents

The properties of integer exponents can also be applied to rational exponents.

### KEY IDEA

**Properties of Rational Exponents**

Let $a$ and $b$ be real numbers and let $m$ and $n$ be rational numbers, such that the quantities in each property are real numbers.

| Property Name | Definition | Example |
|---|---|---|
| Product of Powers | $a^m \cdot a^n = a^{m+n}$ | $5^{1/2} \cdot 5^{3/2} = 5^{(1/2+3/2)} = 5^2 = 25$ |
| Power of a Power | $(a^m)^n = a^{mn}$ | $(3^{5/2})^2 = 3^{(5/2 \cdot 2)} = 3^5 = 243$ |
| Power of a Product | $(ab)^m = a^m b^m$ | $(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$ |
| Negative Exponent | $a^{-m} = \dfrac{1}{a^m}, a \neq 0$ | $36^{-1/2} = \dfrac{1}{36^{1/2}} = \dfrac{1}{6}$ |
| Zero Exponent | $a^0 = 1, a \neq 0$ | $213^0 = 1$ |
| Quotient of Powers | $\dfrac{a^m}{a^n} = a^{m-n}, a \neq 0$ | $\dfrac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)} = 4^2 = 16$ |
| Power of a Quotient | $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}, b \neq 0$ | $\left(\dfrac{27}{64}\right)^{1/3} = \dfrac{27^{1/3}}{64^{1/3}} = \dfrac{3}{4}$ |

**COMMON ERROR**

When you multiply powers, *do not* multiply the exponents, add them. For example, $3^2 \cdot 3^5 = 3^7$, not $3^{10}$.

### EXAMPLE 1 Using Properties of Exponents

WATCH

**WORDS AND MATH**

In everyday life, a property of an object or idea is a quality or attribute of it. A mathematical property describes attributes of numbers and expressions.

Use the properties of rational exponents to simplify each expression.

**a.** $7^{1/4} \cdot 7^{1/2} = 7^{(1/4+1/2)} = 7^{3/4}$ — Product of Powers Property

**b.** $(6^{1/2} \cdot 4^{1/3})^2 = (6^{1/2})^2 \cdot (4^{1/3})^2$ — Power of a Product Property

$= 6^{(1/2 \cdot 2)} \cdot 4^{(1/3 \cdot 2)}$ — Power of a Power Property

$= 6 \cdot 4^{2/3}$ — Simplify.

**c.** $(4^5 \cdot 3^5)^{-1/5} = [(4 \cdot 3)^5]^{-1/5}$ — Power of a Product Property

$= (12^5)^{-1/5}$ — Multiply.

$= 12^{[5 \cdot (-1/5)]}$ — Power of a Power Property

$= 12^{-1}$ — Simplify.

$= \dfrac{1}{12}$ — Definition of negative exponent

**d.** $\dfrac{5}{5^{1/3}} = \dfrac{5^1}{5^{1/3}} = 5^{(1-1/3)} = 5^{2/3}$ — Quotient of Powers Property

**e.** $\left(\dfrac{42^{1/3}}{6^{1/3}}\right)^2 = \left[\left(\dfrac{42}{6}\right)^{1/3}\right]^2$ — Power of a Quotient Property

$= (7^{1/3})^2$ — Divide.

$= 7^{2/3}$ — Power of a Power Property

**SELF-ASSESSMENT** 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

**Simplify the expression.**

**1.** $2^{3/4} \cdot 2^{1/2}$

**2.** $\dfrac{3}{3^{1/4}}$

**3.** $\left(\dfrac{20^{1/2}}{5^{1/2}}\right)^3$

**4.** $(5^{1/3} \cdot 7^{1/4})^3$

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## Simplifying Radical Expressions

The Power of a Product and Power of a Quotient properties can be expressed using radical notation when $m = \frac{1}{n}$ for some integer $n$ greater than 1.

### KEY IDEA

#### Properties of Radicals

Let $a$ and $b$ be real numbers such that the indicated roots are real numbers, and let $n$ be an integer greater than 1.

| Property Name | Definition | Example |
|---|---|---|
| Product Property | $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ | $\sqrt[3]{4} \cdot \sqrt[3]{2} = \sqrt[3]{8} = 2$ |
| Quotient Property | $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, $b \neq 0$ | $\frac{\sqrt[4]{162}}{\sqrt[4]{2}} = \sqrt[4]{\frac{162}{2}} = \sqrt[4]{81} = 3$ |

### EXAMPLE 2 Using Properties of Radicals

WATCH

Use the properties of radicals to simplify each expression.

**a.** $\sqrt[3]{12} \cdot \sqrt[3]{18} = \sqrt[3]{12 \cdot 18} = \sqrt[3]{216} = 6$ — Product Property of Radicals

**b.** $\frac{\sqrt[4]{80}}{\sqrt[4]{5}} = \sqrt[4]{\frac{80}{5}} = \sqrt[4]{16} = 2$ — Quotient Property of Radicals

An expression involving a radical with index $n$ is in **simplest form** when these three conditions are met.

- No radicands have perfect $n$th powers as factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

To meet the last two conditions, rationalize the denominator by multiplying the expression by an appropriate form of 1 that eliminates the radical from the denominator.

### EXAMPLE 3 Writing Radicals in Simplest Form

WATCH

Write the expressions (a) $\sqrt[3]{135}$ and (b) $\frac{\sqrt[5]{7}}{\sqrt[5]{8}}$ in simplest form.

**SOLUTION**

**a.**

$$\sqrt[3]{135} = \sqrt[3]{27 \cdot 5} \quad \text{Factor out perfect cube.}$$

$$= \sqrt[3]{27} \cdot \sqrt[3]{5} \quad \text{Product Property of Radicals}$$

$$= 3\sqrt[3]{5} \quad \text{Simplify.}$$

**b.**

$$\frac{\sqrt[5]{7}}{\sqrt[5]{8}} = \frac{\sqrt[5]{7}}{\sqrt[5]{8}} \cdot \frac{\sqrt[5]{4}}{\sqrt[5]{4}} \quad \text{Make the radicand in the denominator a perfect fifth power.}$$

$$= \frac{\sqrt[5]{28}}{\sqrt[5]{32}} \quad \text{Product Property of Radicals}$$

$$= \frac{\sqrt[5]{28}}{2} \quad \text{Simplify.}$$

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For a denominator that is a sum or difference involving square roots, multiply both the numerator and denominator by the *conjugate* of the denominator. The expressions $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ are conjugates of each other, where $a$, $b$, $c$, and $d$ are rational numbers.

### EXAMPLE 4 Writing a Radical Expression in Simplest Form

WATCH

Write $\dfrac{1}{5 + \sqrt{3}}$ in simplest form.

**SOLUTION**

$$\frac{1}{5+\sqrt{3}} = \frac{1}{5+\sqrt{3}} \cdot \frac{5-\sqrt{3}}{5-\sqrt{3}}$$ The conjugate of $5 + \sqrt{3}$ is $5 - \sqrt{3}$.

$$= \frac{1(5-\sqrt{3})}{5^2 - (\sqrt{3})^2}$$ Sum and Difference Pattern

$$= \frac{5-\sqrt{3}}{22}$$ Simplify.

Radical expressions with the same index and radicand are **like radicals**. To add or subtract like radicals, use the Distributive Property.

### EXAMPLE 5 Adding and Subtracting Like Radicals and Roots

WATCH

Simplify each expression.

**a.** $\sqrt[4]{10} + 7\sqrt[4]{10}$ **b.** $2(8^{1/5}) + 10(8^{1/5})$ **c.** $\sqrt[3]{54} - \sqrt[3]{2}$

**SOLUTION**

**a.** $\sqrt[4]{10} + 7\sqrt[4]{10} = (1 + 7)\sqrt[4]{10} = 8\sqrt[4]{10}$

**b.** $2(8^{1/5}) + 10(8^{1/5}) = (2 + 10)(8^{1/5}) = 12(8^{1/5})$

**c.** $\sqrt[3]{54} - \sqrt[3]{2} = \sqrt[3]{27} \cdot \sqrt[3]{2} - \sqrt[3]{2} = 3\sqrt[3]{2} - \sqrt[3]{2} = (3 - 1)\sqrt[3]{2} = 2\sqrt[3]{2}$

## SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

**Simplify the expression.**

**5.** $\sqrt[4]{27} \cdot \sqrt[4]{3}$ **6.** $\dfrac{\sqrt[3]{250}}{\sqrt[3]{2}}$ **7.** $\sqrt[3]{104}$ **8.** $\sqrt[5]{\dfrac{3}{4}}$

**9.** $\dfrac{3}{6 - \sqrt{2}}$ **10.** $7\sqrt[5]{12} - \sqrt[5]{12}$ **11.** $4(9^{2/3}) + 8(9^{2/3})$ **12.** $\sqrt[3]{5} + \sqrt[3]{40}$

## Simplifying Variable Expressions

The properties of rational exponents and radicals can also be applied to expressions involving variables. Because a variable can be positive, negative, or zero, sometimes absolute value is needed when simplifying a variable expression.

| | Rule | Example |
|---|---|---|
| **When $n$ is odd** | $\sqrt[n]{x^n} = x$ | $\sqrt[7]{5^7} = 5$ and $\sqrt[7]{(-5)^7} = -5$ |
| **When $n$ is even** | $\sqrt[n]{x^n} = \lvert x \rvert$ | $\sqrt[4]{3^4} = 3$ and $\sqrt[4]{(-3)^4} = 3$ |

Absolute value is not needed when all variables are assumed to be positive.

## EXAMPLE 6 Simplifying Variable Expressions

WATCH

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Simplify each expression.

**a.** $\sqrt[3]{64y^6}$ **b.** $\sqrt[4]{\dfrac{x^4}{y^8}}$

### SOLUTION

**a.** $\sqrt[3]{64y^6} = \sqrt[3]{4^3(y^2)^3} = \sqrt[3]{4^3} \cdot \sqrt[3]{(y^2)^3} = 4y^2$

**b.** $\sqrt[4]{\dfrac{x^4}{y^8}} = \dfrac{\sqrt[4]{x^4}}{\sqrt[4]{y^8}} = \dfrac{\sqrt[4]{x^4}}{\sqrt[4]{(y^2)^4}} = \dfrac{|x|}{y^2}$

**STUDY TIP**

You do not need to take the absolute value of $y^2$ because it is always positive.

## EXAMPLE 7 Writing Variable Expressions in Simplest Form

WATCH

Write each expression in simplest form. Assume all variables are positive.

**a.** $\sqrt[4]{16a^7b^{11}c^4}$ **b.** $\dfrac{x}{\sqrt[3]{y^8}}$ **c.** $\dfrac{14xy^{1/3}}{2x^{3/4}z^{-6}}$

### SOLUTION

**a.** $\sqrt[4]{16a^7b^{11}c^4} = \sqrt[4]{16a^4a^3b^8b^3c^4}$ Factor out perfect fourth powers.

$= \sqrt[4]{16a^4b^8c^4} \cdot \sqrt[4]{a^3b^3}$ Product Property of Radicals

$= 2ab^2c\sqrt[4]{a^3b^3}$ Simplify.

**b.** $\dfrac{x}{\sqrt[3]{y^8}} = \dfrac{x}{\sqrt[3]{y^8}} \cdot \dfrac{\sqrt[3]{y}}{\sqrt[3]{y}}$ Make denominator a perfect cube.

$= \dfrac{x\sqrt[3]{y}}{\sqrt[3]{y^9}}$ Product Property of Radicals

$= \dfrac{x\sqrt[3]{y}}{y^3}$ Simplify.

**c.** $\dfrac{14xy^{1/3}}{2x^{3/4}z^{-6}} = 7x^{(1-3/4)}y^{1/3}z^{-(-6)} = 7x^{1/4}y^{1/3}z^6$

**COMMON ERROR**

You must multiply both the numerator *and* denominator of the fraction by $\sqrt[3]{y}$ so that the value of the fraction does not change.

## EXAMPLE 8 Adding and Subtracting Variable Expressions

WATCH

Perform each indicated operation. Assume all variables are positive.

**a.** $5\sqrt{y} + 6\sqrt{y}$ **b.** $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2}$

### SOLUTION

**a.** $5\sqrt{y} + 6\sqrt{y} = (5 + 6)\sqrt{y} = 11\sqrt{y}$

**b.** $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2} = 12z\sqrt[3]{2z^2} - 3z\sqrt[3]{2z^2} = (12z - 3z)\sqrt[3]{2z^2} = 9z\sqrt[3]{2z^2}$

## SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

**13.** Simplify (a) $\sqrt[3]{27q^3}$ and (b) $\sqrt[4]{\dfrac{a^8}{256b^4}}$.

**Simplify the expression. Assume all variables are positive.**

**14.** $\sqrt[6]{36p^6q^8r^{10}}$ **15.** $\sqrt[5]{\dfrac{x^{10}}{y^5}}$ **16.** $\dfrac{6xy^{3/4}}{3x^{1/2}y^{1/2}}$ **17.** $\sqrt{9w^5} - w\sqrt{w^3}$

# 5.2 Practice WITH CalcChat® AND CalcView®

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**In Exercises 1–10, use the properties of rational exponents to simplify the expression.** ▶ *Example 1*

**1.** $(9^2)^{1/3}$

**2.** $(12^2)^{1/4}$

**3.** $\frac{6}{6^{1/4}}$

**4.** $\frac{7}{7^{1/3}}$

**5.** $\left(\frac{8^4}{10^4}\right)^{-1/4}$

**6.** $\left(\frac{9^3}{6^3}\right)^{-1/3}$

**7.** $(3^{-2/3} \cdot 3^{1/3})^{-1}$

**8.** $(5^{1/2} \cdot 5^{-3/2})^{-1/4}$

**9.** $\frac{2^{2/3} \cdot 16^{2/3}}{4^{2/3}}$

**10.** $\frac{49^{3/8} \cdot 49^{7/8}}{7^{5/4}}$

**In Exercises 11–18, use the properties of radicals to simplify the expression.** ▶ *Example 2*

**11.** $\sqrt{2} \cdot \sqrt{72}$

**12.** $\sqrt[3]{16} \cdot \sqrt[3]{32}$

**13.** $\sqrt[4]{5} \cdot \sqrt[4]{125}$

**14.** $\sqrt[4]{2} \cdot \sqrt[4]{128}$

**15.** $\frac{\sqrt[5]{486}}{\sqrt[5]{2}}$

**16.** $\frac{\sqrt{2}}{\sqrt{32}}$

**17.** $\frac{\sqrt[3]{6} \cdot \sqrt[3]{72}}{\sqrt[3]{2}}$

**18.** $\frac{\sqrt[3]{3} \cdot \sqrt[3]{18}}{\sqrt[6]{2} \cdot \sqrt[6]{2}}$

**In Exercises 19–26, write the expression in simplest form.** ▶ *Example 3*

**19.** $\sqrt[4]{567}$

**20.** $\sqrt[5]{288}$

**21.** $\frac{\sqrt[3]{5}}{\sqrt[3]{4}}$

**22.** $\frac{\sqrt[4]{4}}{\sqrt[4]{27}}$

**23.** $\sqrt{\frac{3}{8}}$

**24.** $\sqrt[3]{\frac{7}{4}}$

**25.** $\sqrt[3]{\frac{64}{49}}$

**26.** $\sqrt[4]{\frac{1296}{25}}$

**In Exercises 27–34, write the expression in simplest form.** ▶ *Example 4*

**27.** $\frac{1}{1 + \sqrt{3}}$

**28.** $\frac{1}{2 + \sqrt{5}}$

**29.** $\frac{5}{3 - \sqrt{2}}$

**30.** $\frac{11}{9 - \sqrt{6}}$

**31.** $\frac{9}{\sqrt{3} + \sqrt{7}}$

**32.** $\frac{2}{\sqrt{8} + \sqrt{7}}$

**33.** $\frac{\sqrt{6}}{\sqrt{3} - \sqrt{5}}$

**34.** $\frac{\sqrt{7}}{\sqrt{10} - \sqrt{2}}$

**In Exercises 35–44, simplify the expression.** ▶ *Example 5*

**35.** $9\sqrt[3]{11} + 3\sqrt[3]{11}$

**36.** $8\sqrt[6]{5} - 12\sqrt[6]{5}$

**37.** $3(14^{1/4}) + 9(14^{1/4})$

**38.** $13(8^{3/4}) - 4(8^{3/4})$

**39.** $5\sqrt{12} - 19\sqrt{3}$

**40.** $27\sqrt{6} + 7\sqrt{150}$

**41.** $\sqrt[5]{224} + 3\sqrt[5]{7}$

**42.** $7\sqrt[3]{2} - \sqrt[3]{128}$

**43.** $5(24^{1/3}) - 4(3^{1/3})$

**44.** $5^{1/4} + 6(405^{1/4})$

**In Exercises 45–50, simplify the expression.** ▶ *Example 6*

**45.** $\sqrt[4]{81y^8}$

**46.** $\sqrt[3]{64r^3t^6}$

**47.** $\sqrt[5]{\frac{m^{10}}{n^5}}$

**48.** $\sqrt[4]{\frac{k^{16}}{16z^4}}$

**49.** $\sqrt[6]{\frac{g^6h}{h^7}}$

**50.** $\sqrt[8]{\frac{n^2p^{-1}}{n^{18}p^7}}$

**ERROR ANALYSIS** **In Exercises 51 and 52, describe and correct the error in simplifying the expression.**

**51.** ✗

$$3\sqrt[3]{12} + 5\sqrt[3]{12} = (3 + 5)\sqrt[3]{24}$$
$$= 8\sqrt[3]{24}$$
$$= 8\sqrt[3]{8 \cdot 3}$$
$$= 8 \cdot 2\sqrt[3]{3}$$
$$= 16\sqrt[3]{3}$$

**52.** ✗

$$\sqrt[6]{\frac{64}{w^6}} = \frac{\sqrt[6]{64}}{\sqrt[6]{w^6}}$$
$$= \frac{\sqrt[6]{2^6}}{\sqrt[6]{w^6}}$$
$$= \frac{2}{w}$$

**53.** **OPEN-ENDED** Write two variable expressions involving radicals, one that needs absolute value when simplifying and one that does not need absolute value. Justify your answers.

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**54.** **COLLEGE PREP** When each expression is simplified and written in radical form, which expressions are like radicals?

(A) $(5^{2/9})^{3/2}$ (B) $\dfrac{5^3}{5^{8/3}}$

(C) $\sqrt[3]{625}$ (D) $\sqrt[3]{960} - \sqrt[3]{120}$

(E) $\sqrt[3]{40} + 3\sqrt[3]{320}$ (F) $7\sqrt[4]{80} - 2\sqrt[4]{405}$

**In Exercises 55–62, write the expression in simplest form. Assume all variables are positive.** *Example 7*

**55.** $\sqrt{81a^7b^{12}c^9}$

**56.** $\sqrt[3]{125r^4s^9t^7}$

**57.** $\dfrac{c^3}{\sqrt[4]{d^7}}$

**58.** $\dfrac{w^7}{\sqrt[5]{z^{13}}}$

**59.** $\sqrt[5]{\dfrac{160m^6}{n^7}}$

**60.** $\sqrt[4]{\dfrac{405x^3y^3}{5x^{-1}y}}$

**61.** $\dfrac{18w^{1/3}v^{5/4}}{27w^{4/3}v^{1/2}}$

**62.** $\dfrac{7x^{3/4}y^{5/2}}{56x^{-1/2}y^{1/4}z^{-2/3}}$

**In Exercises 63–68, perform the indicated operation. Assume all variables are positive.** *Example 8*

**63.** $12\sqrt[3]{y} + 9\sqrt[3]{y}$

**64.** $11\sqrt{2z} - 5\sqrt{2z}$

**65.** $3x^{7/2} - 5x^{7/2}$

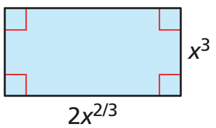
**66.** $7m^{7/3} + 3m^{7/3}$

**67.** $\sqrt[4]{16w^{10}} + 2w\sqrt[4]{w^6}$

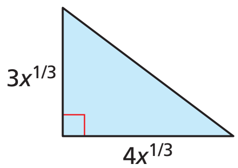
**68.** $\sqrt[3]{32p^{10}} - 9p^2\sqrt[3]{4p^4}$

**CONNECTING CONCEPTS** **In Exercises 69 and 70, find simplified expressions for the perimeter and area of the given figure.**

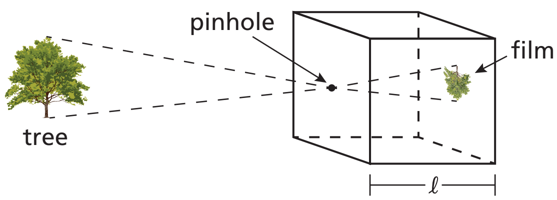
**69.**




**70.**




**71.** **MODELING REAL LIFE** The optimum diameter $d$ (in millimeters) of the pinhole in a pinhole camera can be modeled by $d = 1.9[(5.5 \times 10^{-4})\ell]^{1/2}$, where $\ell$ is the length (in millimeters) of the camera box. Find the optimum pinhole diameter for a camera box with a length of 10 centimeters.



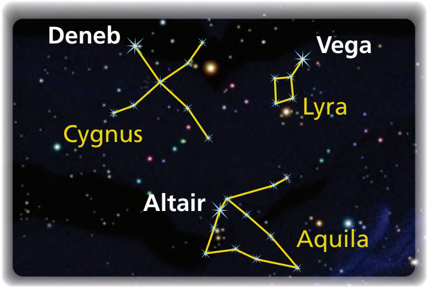

**72.** **MODELING REAL LIFE** The apparent magnitude of a star is a number that indicates how faint the star is in relation to other stars. The expression $\dfrac{2.512^{m_1}}{2.512^{m_2}}$ tells how many times fainter a star with apparent magnitude $m_1$ is than a star with apparent magnitude $m_2$.

| Star | Apparent magnitude | Constellation |
|---|---|---|
| Vega | 0.03 | Lyra |
| Altair | 0.77 | Aquila |
| Deneb | 1.25 | Cygnus |

**a.** How many times fainter is Altair than Vega?

**b.** How many times fainter is Deneb than Altair?

**c.** How many times fainter is Deneb than Vega?




**73.** **MAKING AN ARGUMENT** Your friend claims it is not possible to simplify the expression $7\sqrt{11} - 9\sqrt{44}$ because it does not contain like radicals. Is your friend correct? Explain your reasoning.

**74.** **MP PROBLEM SOLVING** The surface area $S$ (in square centimeters) of a mammal can be modeled by $S = km^{2/3}$, where $m$ is the mass (in grams) of the mammal and $k$ is a constant. The table shows the values of $k$ for different mammals.

| Mammal | Rabbit | Human | Bat |
|---|---|---|---|
| Value of $k$ | 9.75 | 11.0 | 57.5 |

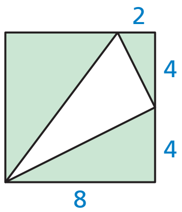
**a.** Find the surface area of a bat whose mass is 32 grams.

**b.** Find the surface area of a rabbit whose mass is 3.4 kilograms ($3.4 \times 10^3$ grams).

**c.** Which mammal has the greatest mass per square centimeter of surface area, the bat in part (a), the rabbit in part (b), or a human whose mass is 59 kilograms?

**d.** Rewrite the formula so that one side is $\dfrac{m}{S}$. Use this formula to justify your answer in part (c).

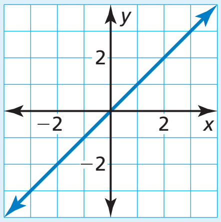
**75.** **CONNECTING CONCEPTS** Find a simplified radical expression for the perimeter of the triangle inscribed in the square. Is the inscribed triangle a right triangle?



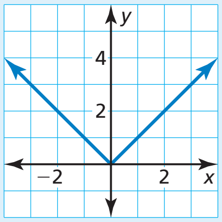

**76.** **HOW DO YOU SEE IT?**

Without finding points, match the functions $f(x) = \sqrt{x^2}$ and $g(x) = \sqrt[3]{x^3}$ with their graphs. Explain your reasoning.

**A.**




**B.**




**77.** **REWRITING A FORMULA** You fill two round balloons with water. One balloon contains twice as much water as the other balloon.

**a.** Solve the formula for the volume of a sphere, $V = \frac{4}{3}\pi r^3$, for $r$.

**b.** Use your result from part (a) and the formula for the surface area of a sphere, $S = 4\pi r^2$, to show that $S = (4\pi)^{1/3}(3V)^{2/3}$.

**c.** Compare the surface areas of the two water balloons using the formula in part (b).

**78.** **THOUGHT PROVOKING**
Determine whether the expressions $(x^2)^{1/6}$ and $(x^{1/6})^2$ are equivalent for all values of $x$. Explain your reasoning.

**79.** **DRAWING CONCLUSIONS** Substitute different combinations of odd and even positive integers for $m$ and $n$ in the expression $\sqrt[n]{x^m}$. When you cannot assume $x$ is positive, explain when absolute value is needed in simplifying the expression.

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## REVIEW & REFRESH

WATCH

**In Exercises 80–83, write a rule for $g$. Describe the graph of $g$ as a transformation of the graph of $f$.**

**80.** $f(x) = x^4 - 3x^2 - 2x,\ g(x) = -f(x)$

**81.** $f(x) = x^3 - x,\ g(x) = f(x) - 3$

**82.** $f(x) = x^3 - 4,\ g(x) = f(x - 2)$

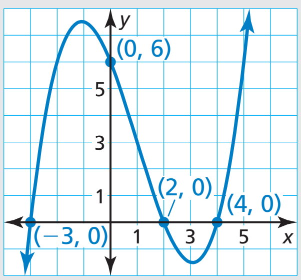
**83.** $f(x) = x^4 + 2x^3 - 4x^2,\ g(x) = f(2x)$

**In Exercises 84 and 85, identify the focus, directrix, and axis of symmetry of the parabola. Then graph the equation.**

**84.** $y = 2x^2$

**85.** $y^2 = -x$

**86.** Write the cubic function whose graph is shown.




**87.** Is $f(x) = 4x^3 + 2^x - 5x$ a polynomial function? Explain.

**88.** Determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

4, 12, 36, 108, . . .

**In Exercises 89–92, simplify the expression.**

**89.** $\left(\dfrac{48^{1/4}}{6^{1/4}}\right)^6$

**90.** $\sqrt[4]{3} \cdot \sqrt[4]{432}$

**91.** $\dfrac{1}{3 + \sqrt{2}}$

**92.** $\sqrt[3]{16} - 5\sqrt[3]{2}$

**In Exercises 93 and 94, determine whether the function is *even*, *odd*, or *neither*.**

**93.** $f(x) = 3x^4 - 5$

**94.** $g(x) = x^5 + 2x - 3$

**In Exercises 95 and 96, evaluate the expression without using technology.**

**95.** $16^{3/4}$

**96.** $125^{2/3}$

**97.** **MODELING REAL LIFE** While standing on an apartment balcony, you drop a pair of sunglasses from a height of 25 feet.

**a.** Write a function $h$ that gives the height (in feet) of the pair of sunglasses after $t$ seconds. How long do the sunglasses take to hit the ground?

**b.** Find and interpret $h(0.25) - h(1)$.