ANSWER PRESENTATION TOOL

Algebra 2 - Student Edit 4 6 - Practice 1-33 ALL EVEN Show Solı

ODD

1. Because $x^3 + 4x^2 - 11x - 30 = 0$ is a polynomial equation of degree 3, it has three solutions. The possible rational solutions are $\pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30$.

$$\begin{array}{r|rrrr} 3 & 1 & 4 & -11 & -30 \\ & & 3 & 21 & 30 \\ \hline & 1 & 7 & 10 & 0 \end{array}$$

So, $x - 3$ is a factor.

$$x^3 + 4x^2 - 11x - 30 = 0$$
$$(x - 3)(x^2 + 7x + 10) = 0$$
$$(x - 3)(x + 5)(x + 2) = 0$$

So, the solutions are $x = 3$, $x = -5$ and $x = -2$.

3. Because $4x^5 - 8x^4 + 6x^3 = 0$ is a polynomial equation of degree 5, it has five solutions.

$$4x^5 - 8x^4 + 6x^3 = 0$$

$$2x^3(2x^2 - 4x + 3) = 0$$

$$2x^3 = 0 \quad \text{or} \quad 2x^2 - 4x + 3 = 0$$

$$x = 0 \qquad x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{4 \pm \sqrt{-8}}{4}$$

$$x = \frac{4 \pm 2i\sqrt{2}}{4}$$

$$x = 1 \pm \frac{i\sqrt{2}}{2}$$

So, the solutions are $x = 0$, $x = 0$, $x = 0$, $x = 1 - \frac{i\sqrt{2}}{2}$ and $x = 1 + \frac{i\sqrt{2}}{2}$.

5. Because $t^4 - 2t^3 + t = 2$ is a polynomial equation of degree 4, it has four solutions.

$$t^4 - 2t^3 + t = 2$$
$$t^4 - 2t^3 + t - 2 = 0$$
$$t^3(t - 2) + (t - 2) = 0$$
$$(t - 2)(t^3 + 1) = 0$$
$$(t - 2)(t + 1)(t^2 - t + 1) = 0$$

$t - 2 = 0$ or $t + 1 = 0$

$t = 2$ $\quad$ $t = -1$

or

$t^2 - t + 1 = 0$

$$t = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$
$$t = \frac{1 \pm \sqrt{-3}}{2}$$
$$t = \frac{1 \pm i\sqrt{3}}{2}$$
$$t = \frac{1}{2} \pm \frac{i\sqrt{3}}{2}$$

So, the solutions are $t = -1$, $t = 2$, $t = \frac{1}{2} - \frac{i\sqrt{3}}{2}$, and $t = \frac{1}{2} + \frac{i\sqrt{3}}{2}$.

7. Find the rational zeros of f. Because f is a polynomial function of degree 4, it has four zeros. The possible rational zeros are ±1, ±2, ±4, and ±8. Using synthetic division, you can determine that -1, 1, 2, and 4 are zeros.

9. **Step 1** Find the rational zeros of f. Because f is a polynomial function of degree 4, it has four zeros. The possible rational zeros are ± 1, ± 2, ± 3, ± 4, ± 6, and ± 12. Using synthetic division, you can determine that -2, 1, and 3 are zeros.

Step 2 Write $f(x)$ in factored form. Dividing $f(x)$ by its known factors $x + 2$, $x - 1$, and $x - 3$ gives a quotient of $x + 2$. So, $f(x) = (x + 2)(x - 1)(x - 3)(x + 2)$.

From the factorization, the zeros are -2, -2, 1, and 3.

11. **Step 1** Find the rational zeros of g. Because g is a polynomial function of degree 4, it has four zeros. The possible rational zeros are ± 1, ± 2, ± 3, ± 4, ± 6, and ± 12. Using synthetic division, you can determine that -3 and -1 are zeros.

Step 2 Write $g(x)$ in factored form. Dividing $g(x)$ by its known factors $x + 3$ and $x + 1$ gives a quotient of $x^2 + 4$. So, $g(x) = (x + 3)(x + 1)(x^2 + 4)$.

Step 3 Find the complex zeros of f. Solving $x^2 + 4 = 0$, you get $\pm 2i$. This means $x^2 + 4 = (x - 2i)(x + 2i)$. So,

$g(x) = (x + 3)(x + 1)(x - 2i)(x + 2i)$.

From the factorization, the zeros are -3, -1, $-2i$, and $2i$.

13. Step 1 Find the rational zeros of g. Because g is a polynomial function of degree 5, it has five zeros. The possible rational zeros are ± 1, ± 2, ± 4, ± 8, and ± 16. Using synthetic division, you can determine that -4, -1, and 2 are zeros.

Step 2 Write $g(x)$ in factored form. Dividing $g(x)$ by its known factors $x + 4$, $x - 1$, and $x - 2$ gives a quotient of $x^2 + 2$. So, $g(x) = (x + 4)(x + 1)(x - 2)(x^2 + 2)$.

Step 3 Find the complex zeros of g. Solving $x^2 + 2 = 0$, you get $\pm i\sqrt{2}$. This means

$x^2 + 2 = (x - i\sqrt{2})(x + i\sqrt{2})$. So,

$g(x) = (x + 4)(x + 1)(x - 2)(x - i\sqrt{2})(x + i\sqrt{2})$.

From the factorization, the zeros are -4, -1, 2, $-i\sqrt{2}$, and $i\sqrt{2}$.

15. There are two imaginary zeros because the degree is 4 and there are two x-intercepts.

17. There are two imaginary zeros because the degree is 2 and there are no x-intercepts.

19. Use the three zeros and the Factor Theorem to write $f(x)$ as a product of three factors.

$$\begin{aligned} f(x) &= (x + 5)(x + 1)(x - 2) \\ &= (x^2 + 6x + 5)(x - 2) \\ &= x^3 + 4x^2 - 7x - 10 \end{aligned}$$

21. Because the coefficients are rational and $4 + i$ is a zero, $4 - i$ must also be a zero by the Complex Conjugates Theorem. Use the three zeros and the Factor Theorem to write $f(x)$ as a product of three factors.

$$\begin{aligned} f(x) &= (x-3)[x-(4+i)][x-(4-i)] \\ &= (x-3)[(x-4)-i][(x-4)+i] \\ &= (x-3)[(x-4)^2-i^2] \\ &= (x-3)[x^2-8x+16-(-1)] \\ &= (x-3)(x^2-8x+16+1) \\ &= (x-3)(x^2-8x+17) \\ &= x^3-11x^2+41x-51 \end{aligned}$$

23. Because the coefficients are rational and $-\sqrt{5}$ is a zero, $\sqrt{5}$ must also be a zero by the Irrational Conjugates Theorem. Use the three zeros and the Factor Theorem to write $f(x)$ as a product of three factors.

$$\begin{aligned} f(x) &= (x-4)(x-\sqrt{5})(x+\sqrt{5}) \\ &= (x-4)(x^2-\sqrt{5}^2) \\ &= (x-4)(x^2-5) \\ &= x^3-4x^2-5x+20 \end{aligned}$$

25. Because the coefficients are rational and $1 + i$ and $2 - \sqrt{3}$ are zeros, $1 - i$ and $2 + \sqrt{3}$ must also be zeros by the Complex Conjugates Theorem and the Irrational Conjugates Theorem. Use the five zeros and the Factor Theorem to write $f(x)$ as a product of five factors.

$$
\begin{aligned}
f(x) &= (x-2)[x-(1+i)][x-(1-i)] \\
&\quad [x-(2+\sqrt{3})][x-(2-\sqrt{3})] \\
&= (x-2)[(x-1)-i][(x-1)+i] \\
&\quad [(x-2)-\sqrt{3}][(x-2)+\sqrt{3}] \\
&= (x-2)[(x-1)^2-i^2][(x-2)^2-(\sqrt{3})^2] \\
&= (x-2)[x^2-2x+1-(-1)](x^2-4x+4-3) \\
&= (x-2)(x^2-2x+2)(x^2-4x+1) \\
&= (x^3-2x^2+2x-2x^2+4x-4)(x^2-4x+1) \\
&= (x^3-4x^2+6x-4)(x^2-4x+1) \\
&= x^5-4x^4+x^3-4x^4+16x^3-4x^2 \\
&\quad +6x^3-24x^2+6x-4x^2+16x-4 \\
&= x^5-8x^4+23x^3-32x^2+22x-4
\end{aligned}
$$

27. The complex conjugate of $1 + i$ was not used as a factor.

$$
\begin{aligned}
f(x) &= (x-2)[x-(1+i)][x-(1-i)] \\
&= (x-2)[(x-1)-i][(x-1)+i] \\
&= (x-2)[(x-1)^2-i^2] \\
&= (x-2)[x^2-2x+1-(-1)] \\
&= (x-2)(x^2-2x+2) \\
&= x^3-2x^2+2x-2x^2+4x-4 \\
&= x^3-4x^2+6x-4
\end{aligned}
$$

29. The coefficients in $f(x)$ have 1 sign change, so f has 1 positive real zero. The coefficients in $f(-x)$ have 1 sign change, so f has 1 negative real zero. The possible number of zeros for f are summarized in the table below.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
1	1	2	4

31. The coefficients in $f(x)$ have 2 sign changes, so f has 2 or 0 positive real zeros. The coefficients in $f(-x)$ have 1 sign change, so f has 1 negative real zero. The possible number of zeros for f are summarized in the table below.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
2	1	0	3
0	1	2	3

33. The coefficients in $f(x)$ have 3 sign changes, so f has 3 or 1 positive real zeros. The coefficients in $f(-x)$ have 2 sign changes, so f has 2 or 0 negative real zeros. The possible number of zeros for f are summarized in the table below.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
3	2	0	5
3	0	2	5
1	2	2	5
1	0	4	5