# ANSWER PRESENTATION TOOL

Algebra 2 - Student Edit 4 6 - Practice 2-34 ALL EVEN Show Solı

ODD

**2.** Because $z^3 + 10z^2 + 28z + 24 = 0$ is a polynomial equation of degree 3, it has three solutions. The possible rational solutions are $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$.

$$\begin{array}{r|rrrr} -2 & 1 & 10 & 28 & 24 \\ & & -2 & -16 & -24 \\ \hline & 1 & 8 & 12 & 0 \end{array}$$

So, $z + 2$ is a factor.

$z^3 + 10z^2 + 28z + 24 = 0$

$(z + 2)(z^2 + 8z + 12) = 0$

$(z + 2)(z + 6)(z + 2) = 0$

So, the solutions are $z = -2$, $z = -2$,and $z = -6$.

**4.** Because $-2x^6 - 8x^5 - x^4 = 0$ is a polynomial equation of degree 6, it has six solutions.

$$-2x^6 - 8x^5 - x^4 = 0$$

$$-x^4(2x^2 + 8x + 1) = 0$$

$$-x^4 = 0 \quad \text{or} \quad 2x^2 + 8x + 1 = 0$$

$$x = 0 \qquad x = \frac{-8 \pm \sqrt{8^2 - 4(2)(1)}}{2(2)}$$

$$x = \frac{-8 \pm \sqrt{56}}{4}$$

$$x = \frac{-8 \pm 2\sqrt{14}}{4}$$

$$x = -2 \pm \frac{\sqrt{14}}{2}$$

So, the solutions are $x = 0$, $x = 0$, $x = 0$, $x = 0$, $x = -2 - \frac{\sqrt{14}}{2}$ and $x = -2 + \frac{\sqrt{14}}{2}$.

**6.** Because $y^4 + 5y^3 - 125y = 625$ is a polynomial equation of degree 4, it has four solutions.

$$y^4 + 5y^3 - 125y = 625$$
$$y^4 + 5y^3 - 125y - 625 = 0$$
$$y^3(y + 5) - 125(y + 5) = 0$$
$$(y + 5)(y^3 - 125) = 0$$
$$(y + 5)(y - 5)(y^2 + 5y + 25) = 0$$
$$y + 5 = 0 \quad \text{or} \quad y - 5 = 0$$
$$y = -5 \qquad\qquad y = 5$$

or

$$y^2 + 5y + 25 = 0$$
$$y = \frac{-5 \pm \sqrt{5^2 - 4(1)(25)}}{2(1)}$$
$$y = \frac{-5 \pm \sqrt{-75}}{2}$$
$$y = \frac{-5 \pm 5i\sqrt{3}}{2}$$
$$y = -\frac{5}{2} \pm \frac{5i\sqrt{3}}{2}$$

So, the solutions are $y = -5$, $y = 5$, $y = -\frac{5}{2} + \frac{5i\sqrt{3}}{2}$, and $y = -\frac{5}{2} - \frac{5i\sqrt{3}}{2}$.

**8.** Find the rational zeros of $f$. Because $f$ is a polynomial function of degree 4, it has four zeros. The possible rational zeros are $\pm 1$, $\pm 2$, $\pm 3$, $\pm 5$, $\pm 6$, $\pm 10$, $\pm 15$, and $\pm 30$. Using synthetic division, you can determine that $-5$, $-3$, 1, and 2 are zeros.

**10.** Find the rational zeros of $f$. Because $f$ is a polynomial function of degree 3, it has three zeros. The possible rational zeros are $\pm 1$, $\pm 2$, $\pm 4$, $\pm 5$, $\pm 10$, and $\pm 20$. Using synthetic division, you can determine that $-5$, $-2$, and 2 are zeros.

**12. Step 1** Find the rational zeros of $h$. Because $h$ is a polynomial function of degree 4, it has four zeros. The possible rational zeros are $\pm 1$, $\pm 2$, $\pm 3$, $\pm 6$, $\pm 9$, and $\pm 18$. Using synthetic division, you can determine that $-1$ and 2 are zeros.

**Step 2** Write $h(x)$ in factored form. Dividing $h(x)$ by its known factors $x + 1$ and $x - 2$ gives a quotient of $x^2 + 9$. So, $h(x) = (x + 1)(x - 2)(x^2 + 9)$.

**Step 3** Find the complex zeros of $f$. Solving $x^2 + 9 = 0$, you get $\pm 3i$. This means $x^2 + 9 = (x - 3i)(x + 3i)$. So,

$h(x) = (x + 1)(x - 2)(x - 3i)(x + 3i)$.

From the factorization, the zeros are $-1$, 2, $-3i$, and $3i$.

**14. Step 1** Find the rational zeros of $f$. Because $f$ is a polynomial function of degree 5, it has five zeros. The possible rational zeros are $\pm 1$, $\pm 2$, $\pm 4$, $\pm 5$, $\pm 10$, and $\pm 20$. Using synthetic division, you can determine that $-5$, 1, and 4 are zeros.

**Step 2** Write $f(x)$ in factored form. Dividing $f(x)$ by its known factors $x + 5$, $x - 1$, and $x - 4$ gives a quotient of $x^2 + 1$. So, $f(x) = (x + 5)(x - 1)(x - 4)(x^2 + 1)$.

**Step 3** Find the complex zeros of $f$. Solving $x^2 + 1 = 0$, you get $\pm i$. This means $x^2 + 1 = (x - i)(x + i)$. So,

$f(x) = (x + 5)(x - 1)(x - 4)(x - i)(x + i)$.

From the factorization, the zeros are $-5$, 1, 4, $-i$, and $i$.

**16.** There are four imaginary zeros because the degree is 5 and there is one $x$-intercept.

**18.** There are zero imaginary zeros because the degree is 3 and there are three $x$-intercepts.

**20.** Use the three zeros and the Factor Theorem to write $f(x)$ as a product of three factors.

$$\begin{aligned} f(x) &= (x + 2)(x - 1)(x - 3) \\ &= (x^2 + x - 2)(x - 3) \\ &= x^3 - 2x^2 - 5x + 6 \end{aligned}$$

**22.** Because the coefficients are rational and $5 - i$ is a zero, $5 + i$ must also be a zero by the Complex Conjugates Theorem. Use the three zeros and the Factor Theorem to write $f(x)$ as a product of three factors.

$$\begin{aligned} f(x) &= (x - 2)[x - (5 - i)][x - (5 + i)] \\ &= (x - 2)[(x - 5) + i][(x - 5) - i] \\ &= (x - 2)[(x - 5)^2 - i^2] \\ &= (x - 2)[x^2 - 10x + 25 - (-1)] \\ &= (x - 2)(x^2 - 10x + 25 + 1) \\ &= (x - 2)(x^2 - 10x + 26) \\ &= x^3 - 12x^2 + 46x - 52 \end{aligned}$$

**24.** Because the coefficients are rational and $2-\sqrt{3}$ is a zero, $2+\sqrt{3}$ must also be a zero by the Irrational Conjugates Theorem. Use the three zeros and the Factor Theorem to write $f(x)$ as a product of three factors.

$$\begin{aligned} f(x) &= (x-3)\left[x-\left(2-\sqrt{3}\right)\right]\left[x-\left(2+\sqrt{3}\right)\right] \\ &= (x-3)\left[(x-2)+\sqrt{3}\right]\left[(x-2)-\sqrt{3}\right] \\ &= (x-3)[(x-2)^2-3] \\ &= (x-3)(x^2-4x+4-3) \\ &= (x-3)(x^2-4x+1) \\ &= x^3-4x^2+x-3x^2+12x-3 \\ &= x^3-7x^2+13x-3 \end{aligned}$$

**26.** Because the coefficients are rational and $4+2i$ and $1+\sqrt{7}$ are zeros, $4-2i$ and $1-\sqrt{7}$ must also be zeros by the Complex Conjugates Theorem. Use the five zeros and the Factor Theorem to write $f(x)$ as a product of five factors.

$$\begin{aligned} f(x) &= (x-3)[x-(4+2i)][x-(4-2i)] \\ &\quad \left[x-\left(1+\sqrt{7}\right)\right]\left[x-\left(1-\sqrt{7}\right)\right] \\ &= (x-3)[(x-4)-2i][(x-4)+2i] \\ &\quad \left[(x-1)-\sqrt{7}\right]\left[(x-1)+\sqrt{7}\right] \\ &= (x-3)\left[(x-4)^2-(2i^2)\right]\left[(x-1)^2-\left(\sqrt{7}\right)^2\right] \\ &= (x-3)\left[x^2-8x+16-4(-1)\right](x^2-2x+1-7) \\ &= (x-3)(x^2-8x+20)(x^2-2x-6) \\ &= (x^3-8x^2+20x-3x^2+24x-60)(x^2-2x-6) \\ &= (x^3-11x^2+44x-60)(x^2-2x-6) \\ &= x^5-2x^4-6x^3-11x^4+22x^3+66x^2 \\ &\quad +44x^3-88x^2-264x-60x^2+120x+360 \\ &= x^5-13x^4+60x^3-82x^2-144x+360 \end{aligned}$$

**28.** In the second factor, the same zero is being added, instead of subtracting the conjugate of the zero.

$$\begin{aligned} f(x) &= [x - (2 + i)][x - (2 - i)] \\ &= [(x - 2) - i][(x - 2) + i] \\ &= (x - 2)^2 - i^2 \\ &= x^2 - 4x + 4 - (-1) \\ &= x^2 - 4x + 4 + 1 \\ &= x^2 - 4x + 5 \end{aligned}$$

**30.** The coefficients in $f(x)$ have 1 sign change, so $f$ has 1 positive real zero. The coefficients in $f(-x)$ have 0 sign changes, so $f$ has 0 negative real zeros. The possible number of zeros for $f$ are summarized in the table below.

| Positive real zeros | Negative real zeros | Imaginary zeros | Total zeros |
|---|---|---|---|
| 1 | 0 | 2 | 3 |

**32.** The coefficients in $f(x)$ have 2 sign changes, so $f$ has 2 or 0 positive real zeros. The coefficients in $f(-x)$ have 3 sign changes, so $f$ has 3 or 1 negative real zeros. The possible number of zeros for $f$ are summarized in the table below.

| Positive real zeros | Negative real zeros | Imaginary zeros | Total zeros |
|---|---|---|---|
| 2 | 3 | 0 | 5 |
| 2 | 1 | 2 | 5 |
| 0 | 3 | 2 | 5 |
| 0 | 1 | 4 | 5 |

**34.** The coefficients in $f(x)$ have 3 sign changes, so $f$ has 3 or 1 positive real zeros. The coefficients in $f(-x)$ have 2 sign changes, so $f$ has 2 or 0 negative real zeros. The possible number of zeros for $f$ are summarized in the table below.

| Positive real zeros | Negative real zeros | Imaginary zeros | Total zeros |
|---|---|---|---|
| 3 | 2 | 0 | 5 |
| 3 | 0 | 2 | 5 |
| 1 | 2 | 2 | 5 |
| 1 | 0 | 4 | 5 |