4.5 WS KEY

Use algebraic procedures to find the solution or solutions of the equations. Round to the nearest hundredth.

1. $2^x = 64$

$x \log 2 = \log 64$

$x = \frac{\log 64}{\log 2}$

$\boxed{x = 6}$

2. $49^x = \frac{1}{343}$

$x \log 49 = \log\left(\frac{1}{343}\right)$

$x = \frac{\log(1/343)}{\log 49}$

$\boxed{x = -1.5}$

3. $3^{2x-1} = 81$

$(2x-1)\log 3 = \log 81$

$2x - 1 = \frac{\log 81}{\log 3}$

$2x - 1 = 4$

$2x = 5$

$\boxed{x = \frac{5}{2} \text{ or } 2.5}$

4. $5^x = 70$

$x \log 5 = \log 70$

$x = \frac{\log 70}{\log 5}$

$\boxed{x = 2.64}$

5. $3^{x-1} = 251$

$(x-1)\log 3 = \log 251$

$x - 1 = 5.03$

$\boxed{x = 6.03}$

★ 6. $2^{x-1} = 3^{x+1}$

$x \ln 2 - \ln 2 = x \ln 3 + \ln 3$

$x \ln 2 - x \ln 3 = \ln 3 + \ln 2$

$x(\ln 2 - \ln 3) = \ln 3 + \ln 2$

$x = \frac{\ln 3 + \ln 2}{\ln 2 - \ln 3}$

$\boxed{x = -4.42}$

7. $2^{2x-3} = 5^{-x-1}$

$2x \ln 2 - 3\ln 2 = -x \ln 5 - \ln 5$

$2x \ln 2 + x \ln 5 = 3\ln 2 - \ln 5$

$x(2\ln 2 + \ln 5) = 3\ln 2 - \ln 5$

$x = \frac{3\ln 2 - \ln 5}{2\ln 2 + \ln 5}$

$\boxed{x = 0.16}$

8. $\log(4x - 18) = 1$

$4x - 18 = 10$

$4x = 28$

$\boxed{x = 7}$

9. $\ln(x^2 - 9) = \ln(x + 11)$

$x^2 - 9 = x + 11$

$x^2 - x - 20 = 0$

$-5 \quad 4$

$(x-5)(x+4) = 0$

$\boxed{x = 5, -4}$

10. $\log_2 x + \log_2(x-4) = 2$

$\log_2(x(x-4)) = 2$

$\log_2(x^2 - 4x) = 2$

$4 = x^2 - 4x$

$0 = x^2 - 4x - 4$

$x = \frac{4 \pm \sqrt{16 - 4(1)(-4)}}{2} = \frac{4 \pm 4\sqrt{2}}{2}$

$\boxed{x = 2 + 2\sqrt{2}}$

★ 11. $\log(5x-1) = 2 + \log(2x+1)$

$\log\left[\frac{5x-1}{2x+1}\right] = 2$

$\frac{5x-1}{2x+1} = 100$

$5x - 1 = 200x + 100$

$-101 = 195x$

$x = \frac{-1}{1}$

$\boxed{\text{No Sol.}}$

12. $\log\sqrt{x^3 - 17} = \frac{1}{2}$

$\sqrt{10} = \sqrt{x^3 - 17}$

$10 = x^3 - 17$

$\sqrt[3]{27} = \sqrt[3]{x^3}$

$\boxed{x = 3}$

13. $\ln(2x+5) = \ln(x+3) + \ln(x-1)$

$\ln(2x+5) = \ln((x+3)(x-1))$

$2x+5 = (x+3)(x-1)$

$2x+5 = x^2+2x-3$

$0 = x^2-8$

$x = \frac{0 \pm \sqrt{0-4(1)(-8)}}{2}$

$x = \frac{\pm\sqrt{32}}{2} = \frac{\pm 4\sqrt{2}}{2}$

$x = \pm 2\sqrt{2}$

$\boxed{x = 2\sqrt{2}}$

14. $\frac{10^x + 10^{-x}}{2} = 20$

$10^x(10^x + 10^{-x}) = 40(10^x)$

$10^{2x} + 10^0 = 40(10^x)$

$(10^x)^2 + 1 = 40(10^x)$

$(10^x)^2 - 40(10^x) + 1 = 0$

$u = 10^x$

$u^2 - 40u + 1 = 0$

$u = \frac{40 \pm \sqrt{1600-4(1)(1)}}{2} = \frac{40 \pm \sqrt{1596}}{2}$

$u = \frac{40 \pm 2\sqrt{399}}{2} = 20 \pm \sqrt{399}$

$10^x = 20 \pm \sqrt{399}$

$x \log 10 = \log(20 \pm \sqrt{399})$

$x = \frac{\log(20 \pm \sqrt{399})}{\log 10}$

$\boxed{x = 1.60, -1.60}$

Use the properties of logarithms to expand the following logarithmic expressions. Assume all variable expressions represent positive real numbers. When possible, evaluate logarithmic expressions.

15. $\log\left[\frac{x^2 y}{1000\sqrt{z}}\right]$

$[2\log x + \log y] - [3 + \frac{1}{2}\log z]$

16. $\ln\sqrt[3]{exy^2\sqrt{z}}$

$\frac{1}{3}\ln(exy^2\sqrt{z})$

$\boxed{\frac{1}{3} + \frac{1}{3}\ln x + \frac{2}{3}\ln y + \frac{1}{6}\ln z}$

17. $\log_3\left[\sqrt{\frac{x+2}{y^{-3}z^5}}\right]$

$\frac{1}{2}\log_3\left[\frac{(x+2)y^3}{z^5}\right]$

$\boxed{\frac{1}{2}\log_3(x+2) + \frac{3}{2}\log_3 y - \frac{5}{2}\log_3 z}$

Find the domain of each of the following logarithmic functions.

18. $\log(5-x)$

$5-x > 0$

$-x > -5$

$x < 5$

$\boxed{D_x \text{ of } f(x): \{x \mid x<5\}}$

19. $\log_7(x^2+10x-39)$

$x^2+10x-39 > 0$

$(x+13)(x-3) > 0$

C.V. $-13, 3$

	$(-\infty, -13)$	$(-13, 3)$	$(3, \infty)$
$x+13$	$-$	$+$	$+$
$x-3$	$-$	$-$	$+$
	$+$	$-$	$+$

$\boxed{D_x \text{ of } f(x): (-\infty, -13) \cup (3, \infty)}$

20. $\log_2\left(\frac{9}{x+12}\right)$

$\frac{9}{x+12} > 0 \quad x \neq -12$

	$(-\infty, -12)$	$(-12, \infty)$
9	$+$	$+$
$x+12$	$-$	$+$
	$-$	$+$

$\boxed{D_x \text{ of } f(x): (-12, \infty)}$