

4.5 WS 4

KEY

Solve the following equations. Be sure to check your answers.

1. $3^{x-1} = 81$

$$\log 3^{x-1} = \log 81$$

$$x-1 = \frac{\log 81}{\log 3}$$

$$x = \frac{\log 81}{\log 3} + 1$$

$$x = 5$$

2. $8^x = 4$

$$x \log 8 = \log 4$$

$$x = \frac{\log 4}{\log 8}$$

$$x = \frac{2}{3}$$

3. $-14 + 3e^x = 11$

$$3e^x = 25$$

$$e^x = \frac{25}{3}$$

$$x = \ln\left(\frac{25}{3}\right)$$

$$x = 2.12$$

4. $\log(3x+1) = 2$

$$3x+1 = 100$$

$$3x = 99$$

$$x = 33$$

5. $-6 + \ln 3x = 0$

$$\ln 3x = 6$$

$$3x = e^6$$

$$x = \frac{e^6}{3}$$

$$x = 134.48$$

6. $\ln x - \ln 3 = 4$

$$\ln\left(\frac{x}{3}\right) = 4$$

$$\frac{x}{3} = e^4$$

$$x = 3e^4$$

$$x = 163.79$$

7. $\log_2 x + \log_2(x-3) = 2$

$$\log_2(x(x-3)) = 2$$

$$x^2 - 3x = 4$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x = 4, -1$$

$$x = 4$$

8. $2\log_6 4x = 0$

$$\log_6 4x = 0$$

$$4x = 6^0$$

$$4x = 1$$

$$x = \frac{1}{4}$$

9. $\log x - \log 6 = 2\log 4$

$$\log\left[\frac{x}{6}\right] = \log 4^2$$

$$\frac{x}{6} = 4^2$$

$$\frac{x}{6} = 16$$

$$x = 96$$

10. $\log_2 2x = \log_2 100$

$2x = 100$

$x = 50$

11. $2\log_4 x - \log_4(x-1) = 1$

$\log_4 x^2 - \log_4(x-1) = 1$

$\log_4 \left[\frac{x^2}{x-1} \right] = 1$

$(x-1) \frac{x^2}{x-1} = 4(x-1)$

$x^2 = 4x - 4$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x = 2$

12. $2e^{2x} - 5e^x - 3 = 0$

$u = e^x$

$2u^2 - 5u - 3 = 0$

$u = 3 \quad u = \frac{1}{2}$

$e^x = 3 \quad e^x = \frac{1}{2}$

$x = \ln 3 \quad x = \ln \left(\frac{1}{2} \right)$

$x = 1.10$

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Find the inverse of each function, then state the domain and range of $f^{-1}(x)$.

13. $f(x) = 3x - 12$

$y = 3x - 12$

$\frac{y+12}{3} = \frac{3x}{3}$

$f^{-1}(x) = \frac{1}{3}x + 4$

$D_x \text{ of } f^{-1}(x) = (-\infty, \infty)$

$R_y \text{ of } f^{-1}(x) = (-\infty, \infty)$

14. $g(x) = \sqrt{7-x} \quad 7-x \geq 0$

$y^2 = 7-x \quad -x \geq -7$

$x \leq 7$

$y^2 - 7 = -x$

$g^{-1}(x) = -x^2 + 7$

$D_x \text{ of } g^{-1}(x) = [0, \infty)$

$R_y \text{ of } g^{-1}(x) = \{x \mid x \leq 7\}$

or $(-\infty, 7]$

15. $f(x) = \frac{x}{x-5}$

$yx - 5y = x$

$-5y = x - yx$

$-5y = x(1-y)$

$x = \frac{-5y}{1-y}$

$f^{-1}(x) = \frac{-5x}{1-x}$

$D_x \text{ of } f^{-1}(x) = \{x \mid x \neq 1\}$

$R_y \text{ of } f^{-1}(x) = \{x \mid x \neq 5\}$

Use the properties of logarithms to expand the following logarithmic expressions. Assume all variable expressions represent positive real numbers. When possible, evaluate logarithmic expressions.

16. $\log \left(\frac{100x\sqrt{y}}{z^3} \right)$

$\log 100 + \log x + \frac{1}{2} \log y - 3 \log z$

$2 + \log x + \frac{1}{2} \log y - 3 \log z$

17. $\ln(e^3 y^2)$

$2 \ln(e^3 y)$

$2 + 6 \ln x + 2 \ln y$

18. $\ln \sqrt{x\sqrt{yz^3}}$

$\frac{1}{2} \ln(x\sqrt{yz^3})$

$\frac{1}{2} \ln x + \frac{1}{4} \ln y + \frac{3}{4} \ln z$