4.5 WS 2

KEY

Use algebraic procedures to find the solution or solutions of the equations. Round to the nearest hundredth.

1. $7^{x+2} = 231$

$(x+2)\log 7 = \log 231$

$x+2 = 2.80$

$\boxed{x = 0.80}$

2. $9^{5x-3} = 78,462$

$5x-3 = \frac{\log(78,462)}{\log 9}$

$5x-3 = 5.13$

$5x = 8.13$

$\boxed{x = 1.63}$

3. $2^{x+1} = 3^x$

$x\ln 2 + \ln 2 = x\ln 3$

$\ln 2 = x\ln 3 - x\ln 2$

$x = \frac{\ln 2}{\ln 3 - \ln 2}$

$\boxed{x = 1.71}$

4. $3^x = 68$

$x\log 3 = \log 68$

$x = \frac{\log 68}{\log 3}$

$\boxed{x = 3.84}$

5. $4^{x-1} = 600$

$x-1 = \frac{\log 600}{\log 4}$

$x-1 = 4.61$

$\boxed{x = 5.61}$

6. $5^{2x+1} = 9^{x+1}$

$2x\ln 5 + \ln 5 = x\ln 9 + \ln 9$

$2x\ln 5 - x\ln 9 = \ln 9 - \ln 5$

$x(2\ln 5 - \ln 9) = \ln 9 - \ln 5$

$x = \frac{\ln 9 - \ln 5}{2\ln 5 - \ln 9}$

$\boxed{x = .58}$

7. $\log 4 + \log(x-7) = 2$

$\log(4x-28) = 2$

$4x-28 = 100$

$4x = 128$

$\boxed{x = 32}$

8. $\log(2x-3) = \log 3$

$2x-3 = 3$

$2x = 6$

$\boxed{x = 3}$

9. $\ln(8x-4) = \ln 2 + \ln x$

$\ln(8x-4) = \ln(2x)$

$8x-4 = 2x$

$-4 = -6x$

$\frac{2}{3} = x$

$\boxed{x = \frac{2}{3}}$

10. $\log_9(x+6) + \log_9 x = \log_9 2$

$\log_9(x^2+6x) = \log_9 2$

$x^2+6x = 2$

$x^2+6x-2 = 0$

$x = \frac{-6 \pm \sqrt{36-4(1)(-2)}}{2}$

$x = \frac{-6 \pm 2\sqrt{11}}{2}$

$x = -3 \pm \sqrt{11}$ $\boxed{x = -3+\sqrt{11}}$

11. $\ln(x-3) - \ln(x-5) = \ln 5$

$\ln\left(\frac{x-3}{x-5}\right) = \ln 5$

$\frac{x-3}{x-5} = 5$

$x-3 = 5x-25$

$22 = 4x$

$\boxed{x = \frac{11}{2} \text{ or } 5.5}$

12. $\log(22x+113) = 3$

$22x+113 = 1000$

$22x = 887$

$\boxed{x = 40.32}$

13. $\log(x-1)-\log x=\log(x-3)$

$\log\left[\frac{x-1}{x}\right]=\log(x-3)$

$\frac{x-1}{x}=x-3$

$x-1=x^2-3x$

$0=x^2-4x+1$

$x=\frac{4\pm\sqrt{16-4(1)(1)}}{2}=\frac{4\pm\sqrt{12}}{2}=\frac{4\pm2\sqrt{3}}{2}$

$X=2\pm\sqrt{3}$

$\boxed{X=2+\sqrt{3}}$

14. $\frac{10^x+10^{-x}}{2}=5$

$10^x(10^x+10^{-x})=(10)10^x$

$10^{2x}+10^0=10(10^x)$

$(10^x)^2-10(10^x)+1=0$

$u=10^x$

$u^2-10u+1=0$

$u=\frac{10\pm\sqrt{100-4(1)(1)}}{2}=\frac{10\pm\sqrt{96}}{2}=\frac{10\pm4\sqrt{6}}{2}$

$10^x=5\pm2\sqrt{6}$

$x=\frac{\log(5\pm2\sqrt{6})}{\log 10}$

$\boxed{x=1,-1}$

Use the properties of logarithms to expand the following logarithmic expressions. Assume all variable expressions represent positive real numbers. When possible, evaluate logarithmic expressions.

15. $\log_4\left[\frac{64x^3\sqrt{y}}{z^2}\right]$

$\log_4 64+3\log_4 x+\frac{1}{2}\log_4 y-2\log_4 z$

$\boxed{3+3\log_4 x+\frac{1}{2}\log_4 y-2\log_4 z}$

16. $\ln\sqrt{xy^5\sqrt[5]{z}}$

$\boxed{\frac{1}{2}\ln x+\frac{5}{2}\ln y+\frac{1}{10}\ln z}$

17. $\log_3\left(\frac{x^{-1/3}y}{z^{-2}}\right)^4$

$4\log_3\left[\frac{yz^2}{\sqrt[3]{x}}\right]$

$\boxed{4\log_3 y+8\log_3 z-\frac{4}{3}\log_3 x}$

Find the domain of each of the following logarithmic functions.

18. $\log(x+13)$

$x+13>0$

$x>-13$

$\boxed{D_x:\{x\mid x>-13\}}$

or

$\boxed{D_x:(-13,\infty)}$

19. $\log_{12}(x^2-11x+28)$

$x^2-11x+28>0$

$(x-7)(x-4)>0$

c.v. 7, 4

		4		7	
$x-7$	−	|	−	|	+
$x-4$	−	|	+	|	+
	(+)	4	(−)	7	(+)

$\boxed{D_x:(-\infty,4)\cup(7,\infty)}$

20. $\log_2\left(\frac{10}{x+6}\right)$

$\frac{10}{x+6}>0$ c.v. -6

		−6	
10	+	|	+
$x+6$	−	|	+
	(−)	−6	(+)

$\boxed{D_x:(-6,\infty)}$