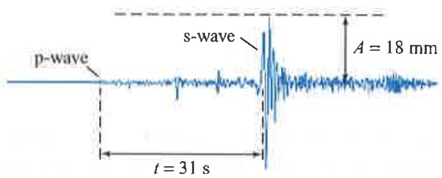
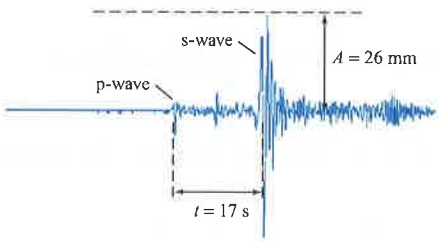
March 11, 2011, measured 9.0 on the Richter scale. In October 1989, an earthquake of magnitude 7.1 on the Richter scale struck the San Francisco Bay area. Compare the intensity of the larger earthquake to the intensity of the smaller earthquake by finding the ratio of the larger intensity to the smaller intensity.

78. **Comparison of Earthquakes** An earthquake that occurred in China in 1978 measured 8.2 on the Richter scale. In 1988, an earthquake in California measured 6.9 on the Richter scale. Compare the intensity of the larger earthquake to the intensity of the smaller earthquake by finding the ratio of the larger intensity to the smaller intensity.

79. **Earthquake Magnitude** Find the Richter scale magnitude of the earthquake that produced the seismogram in the following figure.



80. **Earthquake Magnitude** Find the Richter scale magnitude of the earthquake that produced the seismogram in the following figure.



81. **pH** Milk of magnesia has a hydronium-ion concentration of about 3.97×10^{-11} mole per liter. Determine the pH of milk of magnesia and state whether milk of magnesia is an acid or a base.

82. **pH** Vinegar has a hydronium-ion concentration of 1.26×10^{-3} mole per liter. Determine the pH of vinegar and state whether vinegar is an acid or a base.

83. **Hydronium-Ion Concentration** A morphine solution has a pH of 9.5. Determine the hydronium-ion concentration of the morphine solution.

84. **Hydronium-Ion Concentration** A rainstorm in New York City produced rainwater with a pH of 5.6. Determine the hydronium-ion concentration of the rainwater.

Decibel Level The range of sound intensities that the human ear can detect is so large that a special decibel scale (named after Alexander Graham Bell) is used to measure and compare sound intensities. The decibel level (dB) of a sound is given by

$$dB(I) = 10\log\left(\frac{I}{I_0}\right)$$

where I_0 is the intensity of sound that is barely audible to the human ear. Use the decibel level formula to work Exercises 85 to 88.

85. Find the decibel level for the following sounds. Round to the nearest tenth of a decibel.

	Sound	*Intensity*
a.	Automobile traffic	$I = 1.58 \times 10^8 \cdot I_0$
b.	Quiet conversation	$I = 10{,}800 \cdot I_0$
c.	Fender guitar	$I = 3.16 \times 10^{11} \cdot I_0$
d.	Jet engine	$I = 1.58 \times 10^{15} \cdot I_0$

86. A team in Arizona installed in a Ford Bronco a 48,000-watt sound system that it claims can output 175-decibel sound. The human pain threshold for sound is 125 decibels. How many times as great is the intensity of the sound from the Bronco than the human pain threshold for sound?

87. How many times as great is the intensity of a sound that measures 120 decibels than a sound that measures 110 decibels?

88. If the intensity of a sound is doubled, what is the increase in the decibel level? (*Hint:* Find $dB(2I) - dB(I)$.)

Enrichment Exercises

89. **Animated Maps** A software company that creates interactive maps for websites has designed an animated zooming feature such that when a user selects the zoom-in option, the map appears to expand on a location. This is accomplished by displaying several intermediate maps to give the illusion of motion. The company has determined that zooming in on a location is more informative and pleasing to observe when the scale of each step of the animation is determined using the equation

$$S_n = S_0 \cdot 10^{\frac{n}{N}(\log S_f - \log S_0)}$$

where S_n represents the scale of the current step n ($n = 0$ corresponds to the initial scale), S_0 is the starting scale of the map, S_f is the final scale, and N is the number of steps in the animation following the initial scale. (If the initial scale of the map is 1:200, then $S_0 = 200$.) Determine the scales to be used at each intermediate step if a map is to start with a scale of 1:1,000,000, and proceed through five intermediate steps to end with a scale of 1:500,000.

90. **Animated Maps** Use the equation in Exercise 89 to determine the scales for each stage of an animated map zoom that goes from a scale of 1:250,000 to a scale of 1:100,000 in four steps (following the initial scale).

91. Prove the quotient property of logarithms

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

(*Hint:* See the proof of the product property of logarithms on page 372.)

92. Prove the power property of logarithms

$$\log_b(M^p) = p \log_b M$$

See the hint given in Exercise 91.

MID-CHAPTER 4 QUIZ

1. Use composition of functions to verify that

$$f(x) = \frac{500 + 120x}{x} \quad \text{and} \quad g(x) = \frac{500}{x - 120}$$

are inverses of each other.

2. Find the inverse of $f(x) = \dfrac{24x + 5}{x - 4}, x \neq 4$. State any restrictions on the domain of $f^{-1}(x)$.

3. Evaluate $f(x) = e^x$, for $x = -2.4$. Round to the nearest ten-thousandth.

4. Write $\ln x = 6$ in exponential form.

5. Graph $f(x) = \log_3(x + 3)$.

6. Expand $\ln\left(\dfrac{xy^3}{e^2}\right)$. Assume x and y are positive real numbers.

7. Write $\log_3 x^4 - 2\log_3 z + \log_3(xy^2)$ as a single logarithm with a coefficient of 1. Assume all variables are positive real numbers.

8. Use the change-of-base formula to evaluate $\log_8 411$. Round to the nearest ten-thousandth.

9. What is the Richter scale magnitude of an earthquake with an intensity of $789{,}251 I_0$? Round to the nearest tenth.

10. How many times as great is the intensity of an earthquake that measures 7.9 on the Richter scale than the intensity of an earthquake that measures 5.1 on the Richter scale?

SECTION 4.5

Solving Exponential Equations
Solving Logarithmic Equations

Exponential and Logarithmic Equations

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A27.

PS1. Use the definition of a logarithm to write the exponential equation $3^6 = 729$ in logarithmic form. [4.3]

PS2. Use the definition of a logarithm to write the logarithmic equation $\log_5 625 = 4$ in exponential form. [4.3]

PS3. Use the definition of a logarithm to write the exponential equation $a^{x+2} = b$ in logarithmic form. [4.3]

PS4. Solve for x: $4a = 7bx + 2cx$ [1.2]

PS5. Solve for x: $165 = \dfrac{300}{1 + 12x}$ [1.4]

PS6. Solve for x: $A = \dfrac{100 + x}{100 - x}$ [1.4]

Solving Exponential Equations

If a variable appears in the exponent of a term of an equation, such as in $2^{x+1} = 32$, then the equation is called an **exponential equation**. Example 1 uses the following Equality of Exponents Theorem to solve $2^{x+1} = 32$.

Equality of Exponents Theorem

If $b^x = b^y$, then $x = y$, provided $b > 0$ and $b \neq 1$.

EXAMPLE 1 Solve an Exponential Equation

Use the Equality of Exponents Theorem to solve $2^{x+1} = 32$.

Solution

$$2^{x+1} = 32$$

$$2^{x+1} = 2^5 \quad \text{• Write each side as a power of 2.}$$

$$x + 1 = 5 \quad \text{• Equate the exponents.}$$

$$x = 4 \quad \text{• Solve for } x.$$

Check: Let $x = 4$. Then $2^{x+1} = 2^{4+1}$

$$= 2^5$$

$$= 32$$

▶ Try Exercise 6, page 388

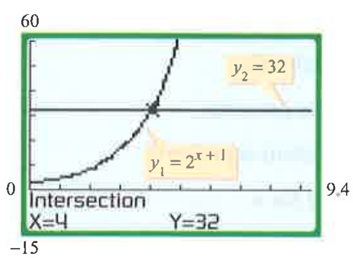
Integrating Technology

A graphing utility can also be used to find the solutions of an equation of the form $f(x) = g(x)$. Either of the following two methods can be employed.

Intersection Method Graph $y_1 = f(x)$ and $y_2 = g(x)$ on the same screen. The solutions of $f(x) = g(x)$ are the x-coordinates of the points of intersection of the graphs.

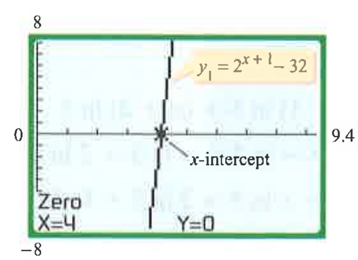
Intercept Method The solutions of $f(x) = g(x)$ are the x-coordinates of the x-intercepts of the graph of $y = f(x) - g(x)$.

Figure 4.36 and Figure 4.37 illustrate the graphical methods for solving $2^{x+1} = 32$.



Intersection method

Figure 4.36



Intercept method

Figure 4.37

In Example 1, we were able to write both sides of the equation as a power of the same base. If you find it difficult to write both sides of an exponential equation in terms of the same base, then try the procedure of taking the logarithm of each side of the equation. This procedure is used in Example 2.

EXAMPLE 2 Solve an Exponential Equation

Solve: $5^x = 40$

Algebraic Solution

$$5^x = 40$$

$$\log(5^x) = \log 40 \quad \text{• Take the logarithm of each side.}$$

$$x \log 5 = \log 40 \quad \text{• Power property}$$

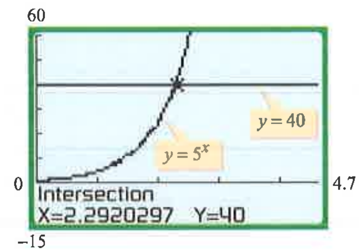
$$x = \frac{\log 40}{\log 5} \quad \text{• Exact solution}$$

$$x \approx 2.3 \quad \text{• Decimal approximation}$$

To the nearest tenth, the solution is 2.3.

Visualize the Solution

Intersection Method The solution of $5^x = 40$ is the x-coordinate of the point of intersection of $y = 5^x$ and $y = 40$.



▸ Try Exercise 14, page 388

An alternative approach to solving the equation in Example 2 is to rewrite the exponential equation in logarithmic form: $5^x = 40$ is equivalent to the logarithmic equation $\log_5 40 = x$. Using the change-of-base formula, we find that $x = \log_5 40 = \frac{\log 40}{\log 5}$. In Example 3, however, we must take logarithms of both sides to reach a solution.

EXAMPLE 3 Solve an Exponential Equation

Solve: $3^{2x-1} = 5^{x+2}$

Algebraic Solution

$$3^{2x-1} = 5^{x+2}$$

$$\ln 3^{2x-1} = \ln 5^{x+2} \quad \text{• Take the natural logarithm of each side.}$$

$$(2x - 1) \ln 3 = (x + 2) \ln 5 \quad \text{• Power property}$$

$$2x \ln 3 - \ln 3 = x \ln 5 + 2 \ln 5 \quad \text{• Distributive property}$$

$$2x \ln 3 - x \ln 5 = 2 \ln 5 + \ln 3 \quad \text{• Solve for } x.$$

$$x(2 \ln 3 - \ln 5) = 2 \ln 5 + \ln 3 \quad \text{• Factor.}$$

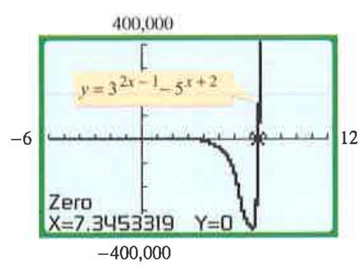
$$x = \frac{2 \ln 5 + \ln 3}{2 \ln 3 - \ln 5} \quad \text{• Exact solution}$$

$$x \approx 7.3 \quad \text{• Decimal approximation}$$

To the nearest tenth, the solution is 7.3.

Visualize the Solution

Intercept Method The solution of $3^{2x-1} = 5^{x+2}$ is the x-coordinate of the x-intercept of $y = 3^{2x-1} - 5^{x+2}$.



▸ Try Exercise 22, page 388

In Example 4, we solve an exponential equation that has two solutions.

EXAMPLE 4 Solve an Exponential Equation Involving $b^x + b^{-x}$

Solve: $\dfrac{2^x + 2^{-x}}{2} = 3$

Algebraic Solution

Multiplying each side by 2 produces

$$2^x + 2^{-x} = 6$$

$$2^{2x} + 2^0 = 6(2^x)$$ • Multiply each side by 2^x to clear negative exponents.

$$(2^x)^2 - 6(2^x) + 1 = 0$$ • Write in quadratic form.

$$(u)^2 - 6(u) + 1 = 0$$ • Substitute u for 2^x.

By the quadratic formula,

$$u = \frac{6 \pm \sqrt{36 - 4}}{2} = \frac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$$

$$2^x = 3 \pm 2\sqrt{2}$$ • Replace u with 2^x.

$$\log 2^x = \log(3 \pm 2\sqrt{2})$$ • Take the common logarithm of each side.

$$x \log 2 = \log(3 \pm 2\sqrt{2})$$ • Power property

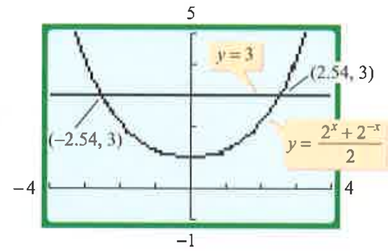
$$x = \frac{\log(3 \pm 2\sqrt{2})}{\log 2} \approx \pm 2.54$$ • Solve for x.

The approximate solutions are −2.54 and 2.54.

▶ Try Exercise 46, page 389

Visualize the Solution

Intersection Method The solutions of $\dfrac{2^x + 2^{-x}}{2} = 3$ are the x-coordinates of the points of intersection of $y = \dfrac{2^x + 2^{-x}}{2}$ and $y = 3$.



▶ Solving Logarithmic Equations

Equations that involve logarithms are called **logarithmic equations.** The properties of logarithms, along with the definition of a logarithm, are often used to find the solutions of a logarithmic equation.

EXAMPLE 5 Solve a Logarithmic Equation

Solve: $\log(3x - 5) = 2$

Solution

$$\log(3x - 5) = 2$$

$$3x - 5 = 10^2$$ • Definition of a logarithm

$$3x = 105$$ • Solve for x.

$$x = 35$$

Check: $\log[3(35) - 5] = \log 100 = 2$

▶ Try Exercise 26, page 388

EXAMPLE 6 Solve a Logarithmic Equation

Solve: $\log 2x - \log(x - 3) = 1$

Solution

$$\log 2x - \log(x-3) = 1$$

$$\log \frac{2x}{x-3} = 1 \qquad \text{• Quotient property}$$

$$\frac{2x}{x-3} = 10^1 \qquad \text{• Definition of a logarithm}$$

$$2x = 10x - 30 \qquad \text{• Multiply each side by } x - 3.$$

$$-8x = -30 \qquad \text{• Solve for } x.$$

$$x = \frac{15}{4}$$

Check the solution by substituting $\frac{15}{4}$ into the original equation.

▶ Try Exercise 30, page 388

In Example 7 we use the one-to-one property of logarithms to find the solution of a logarithmic equation.

EXAMPLE 7 Solve a Logarithmic Equation

Solve: $\ln(3x + 8) = \ln(2x + 2) + \ln(x - 2)$

Algebraic Solution

$$\ln(3x+8) = \ln(2x+2) + \ln(x-2)$$

$$\ln(3x+8) = \ln[(2x+2)(x-2)] \qquad \text{• Product property}$$

$$\ln(3x+8) = \ln(2x^2 - 2x - 4)$$

$$3x + 8 = 2x^2 - 2x - 4 \qquad \text{• One-to-one property of logarithms}$$

$$0 = 2x^2 - 5x - 12 \qquad \text{• Subtract } 3x + 8 \text{ from each side.}$$

$$0 = (2x+3)(x-4) \qquad \text{• Factor.}$$

$$x = -\frac{3}{2} \quad \text{or} \quad x = 4 \qquad \text{• Solve for } x.$$

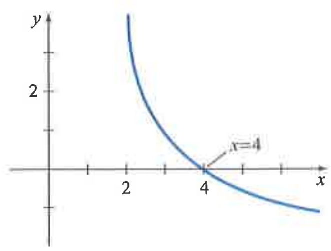
A check will show that 4 is a solution but that $-\frac{3}{2}$ is not a solution.

Visualize the Solution

The graph of

$y = \ln(3x + 8) - \ln(2x + 2) - \ln(x - 2)$

has only one x-intercept. Thus there is only one real solution.



▶ Try Exercise 40, page 388

Question • Why does $x = -\frac{3}{2}$ not check in Example 7?

EXAMPLE 8 Velocity of a Sky Diver Experiencing Air Resistance

During the free fall portion of a jump, the time t, in seconds, required for a sky diver to reach a velocity v, in feet per second is given by

$$t = -\frac{175}{32}\ln\left(1 - \frac{v}{175}\right), 0 \le v < 175$$

a. Determine the velocity of the diver after 5 seconds.

b. The graph of t has a vertical asymptote at $v = 175$. Explain the meaning of the vertical asymptote in the context of this example.

Solution

a. Substitute 5 for t and solve for v.

$$t = -\frac{175}{32}\ln\left(1 - \frac{v}{175}\right)$$

$$5 = -\frac{175}{32}\ln\left(1 - \frac{v}{175}\right)$$ • Replace t with 5.

$$\left(-\frac{32}{175}\right)5 = \ln\left(1 - \frac{v}{175}\right)$$ • Multiply each side by $-\frac{32}{175}$.

$$-\frac{32}{35} = \ln\left(1 - \frac{v}{175}\right)$$ • Simplify.

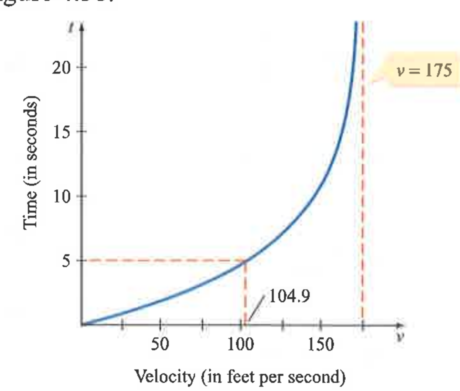
$$e^{-32/35} = 1 - \frac{v}{175}$$ • Write in exponential form.

$$e^{-32/35} - 1 = -\frac{v}{175}$$ • Subtract 1 from each side.

$$v = 175(1 - e^{-32/35})$$ • Multiply each side by -175.

$$v \approx 104.86$$

After 5 seconds the velocity of the sky diver will be about 104.9 feet per second. See Figure 4.38.



$t = -\frac{175}{32}\ln(1 - \frac{v}{175})$

Figure 4.38

(continued)

Note

If air resistance is not considered, then the time in seconds required for a sky diver to reach a given velocity (in feet per second) is $t = \frac{v}{32}$. The function in Example 8 is a more realistic model of the time required to reach a given velocity during the free fall of a sky diver who is experiencing air resistance.

Answer • If $x = -\frac{3}{2}$, the original equation becomes $\ln\left(\frac{7}{2}\right) = \ln(-1) + \ln\left(-\frac{7}{2}\right)$. This cannot be true because the function $f(x) = \ln x$ is not defined for negative values of x.

b. The vertical asymptote $v = 175$ indicates that the velocity of the sky diver approaches, but never reaches or exceeds, 175 feet per second. In Figure 4.38, note that as $v \to 175$ from the left, $t \to \infty$.

Try Exercise 78, page 391

EXERCISE SET 4.5

Concept Check

1. Some exponential equations can be solved by using the Equality of Exponents Theorem. What is the Equality of Exponents Theorem?

2. Name two methods that can be used to estimate the solutions of an equation of the form $f(x) = g(x)$, with the aid of a graphing utility.

3. To solve $3^{2x+7} = 8^{x+6}$ without the aid of a graphing utility, would you apply the Equality of Exponents Theorem or the logarithm-of-each-side property?

4. A student has used the product property and the one-to-one property of logarithms to determine that 5 and -1 are possible solutions of $\ln(7x - 5) = \ln(2x + 5) + \ln(x - 3)$. Are both of these values solutions of the logarithmic equation?

In Exercises 5 to 52, use algebraic procedures to find the exact solution or solutions of the equation.

5. $2^x = 64$
6. $3^x = 243$
7. $49^x = \frac{1}{343}$
8. $9^x = \frac{1}{243}$
9. $3^{2x-1} = 81$
10. $2^{3x+2} = 1024$
11. $\left(\frac{2}{5}\right)^x = \frac{8}{125}$
12. $\left(\frac{2}{5}\right)^x = \frac{25}{4}$
13. $5^x = 70$
14. $6^x = 50$
15. $3^{x+1} = 251$
16. $5^{-x} = 121$
17. $10^{2x+3} = 315$
18. $10^{6-x} = 550$
19. $e^x = 10$
20. $e^{x+1} = 20$
21. $2^{1-x} = 3^{x+1}$
22. $3^{x-2} = 4^{2x+1}$
23. $2^{2x-3} = 5^{-x-1}$
24. $5^{3x} = 3^{x+4}$
25. $\log(4x - 18) = 1$
26. $\log(x^2 + 19) = 2$
27. $\ln(x^2 - 9) = \ln(x + 11)$
28. $\log(x^2 - 6x) = \log 7$
29. $\log_2 x + \log_2(x - 4) = 2$
30. $\log_3 x + \log_3(x + 6) = 3$
31. $\log(5x - 1) = 2 + \log(x - 2)$
32. $1 + \log(3x - 1) = \log(2x + 1)$
33. $\ln(1 - x) + \ln(3 - x) = \ln 8$
34. $\log(4 - x) = \log(x + 8) + \log(2x + 13)$
35. $\log \sqrt{x^3 - 17} = \frac{1}{2}$
36. $\log(x^3) = (\log x)^2$
37. $\ln(\ln x) = -1$
38. $\log(\log 100{,}000x) = 1$
39. $\ln(e^{3x}) = 6$
40. $\ln x = \frac{1}{2}\ln\left(2x + \frac{5}{2}\right) + \frac{1}{2}\ln 2$
41. $\ln(2x + 5) = \ln(x + 3) + \ln(x - 1)$
42. $\log 14x - \log(x + 2) = 1$

Indicates Try It Exercises

43. $e^{\ln(x-1)} = 4$

44. $10^{\log(2x+7)} = 8$

45. $\dfrac{10^x - 10^{-x}}{2} = 20$

46. $\dfrac{10^x + 10^{-x}}{2} = 8$

47. $\dfrac{10^x + 10^{-x}}{10^x - 10^{-x}} = 5$

48. $\dfrac{10^x - 10^{-x}}{10^x + 10^{-x}} = \dfrac{1}{2}$

49. $\dfrac{e^x + e^{-x}}{2} = 15$

50. $\dfrac{e^x - e^{-x}}{2} = 15$

51. $\dfrac{1}{e^x - e^{-x}} = 4$

52. $\dfrac{e^x + e^{-x}}{e^x - e^{-x}} = 3$

In Exercises 53 to 62, use a graphing utility to approximate the solution or solutions of the equation to the nearest hundredth.

53. $2^{-x+3} = x + 1$

54. $3^{x-2} = -2x - 1$

55. $e^{3-2x} - 2x = 1$

56. $2e^{x+2} + 3x = 2$

57. $3 \log_2(x - 1) = -x + 3$

58. $2 \log_3(2 - 3x) = 2x - 1$

59. $\ln(2x + 4) + \dfrac{1}{2}x = -3$

60. $2 \ln(3 - x) + 3x = 4$

61. $2^{x+1} = x^2 - 1$

62. $\ln x = -x^2 + 4$

63. **Population Growth** The population P of a city grows exponentially according to the function

$$P(t) = 8500(1.1)^t, \quad 0 \le t \le 8$$

where t is measured in years.

a. Find the population at time $t = 0$ and at time $t = 2$.

b. When, to the nearest year, will the population reach 15,000?

64. **Physical Fitness** After a race, a runner's pulse rate R, in beats per minute, decreases according to the function

$$R(t) = 145e^{-0.092t}, \quad 0 \le t \le 15$$

where t is measured in minutes.

a. Find the runner's pulse rate at the end of the race and 1 minute after the end of the race.

b. How long, to the nearest minute, after the end of the race will the runner's pulse rate be 80 beats per minute?

65. **Rate of Cooling** A can of soda at 79°F is placed in a refrigerator that maintains a constant temperature of 36°F. The temperature T of the soda t minutes after it is placed in the refrigerator is given by

$$T(t) = 36 + 43e^{-0.058t}$$

a. Find the temperature, to the nearest degree, of the soda 10 minutes after it is placed in the refrigerator.

b. When, to the nearest minute, will the temperature of the soda be 45°F?

66. **Medicine** During surgery, a patient's circulatory system requires at least 50 milligrams of an anesthetic. The amount of anesthetic present t hours after 80 milligrams of anesthetic is administered is given by

$$T(t) = 80(0.727)^t$$

a. How much, to the nearest milligram, of the anesthetic is present in the patient's circulatory system 30 minutes after the anesthetic is administered?

b. How long, to the nearest minute, can the operation last if the patient does not receive additional anesthetic?

Bertalanffy's Equation **In 1938, the biologist Ludwig von Bertalanffy developed the equation**

$$L = m - (m - L_0)e^{-rx}$$

which models the length L, in centimeters, of a fish as it grows under optimal conditions for a period of x years. In Bertalanffy's equation, m represents the maximum length, in centimeters, the fish is expected to attain; L_0 is the length, in centimeters, of the fish at birth; and r is a constant related to the growth rate of the fish species. Use Bertalanffy's equation to predict the age of the fish described in Exercises 67 and 68.

67. A barracuda has a length of 114 centimeters. Use Bertalanffy's equation to predict, to the nearest tenth of a year, the age of the barracuda. Assume $m = 198$ centimeters, $L_0 = 0.9$ centimeter, and $r = 0.23$.

68. A haddock has a length of 21 centimeters. Use Bertalanffy's equation to predict, to the nearest tenth of a year, the age of the haddock. Assume $m = 94$ centimeters, $L_0 = 0.6$ centimeter, and $r = 0.21$.

69. **Typing Speed** The following function models the average typing speed S, in words per minute, for a student who has been typing for t months.

$$S(t) = 5 + 29 \ln(t + 1), \quad 0 \le t \le 9$$

Use S to determine how long it takes the student to achieve an average typing speed of 65 words per minute. Round to the nearest tenth of a month.

70. **Walking Speed** An approximate relation between the average pedestrian walking speed s, in miles per hour, and the population x, in thousands, of a city is given by the formula

$$s(x) = 0.37 \ln x + 0.05$$

Use s to estimate the population of a city for which the average pedestrian walking speed is 2.9 miles per hour. Round to the nearest hundred thousand.

71. Drag Racing The quadratic function

$$s_1(x) = -2.25x^2 + 56.26x - 0.28, \quad 0 \le x \le 10$$

models the speed of a dragster from the start of a race until the dragster crosses the finish line 10 seconds later. This is the acceleration phase of the race.

The exponential function

$$s_2(x) = 8320(0.73)^x, \quad 10 < x \le 20$$

models the speed of the dragster during the 10-second period immediately following the time when the dragster crosses the finish line. This is the deceleration period.

How long after the start of the race did the dragster attain a speed of 275 miles per hour? Round to the nearest hundredth of a second.

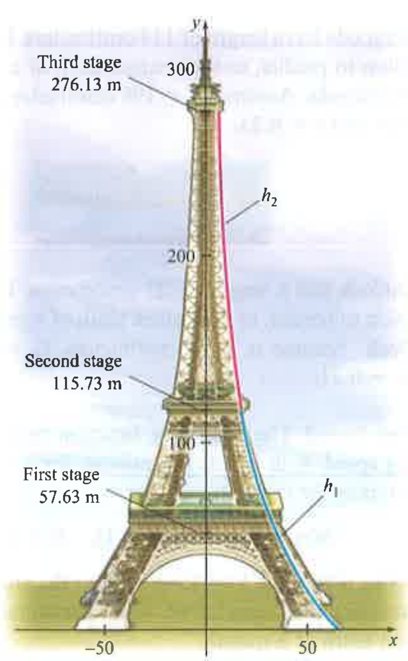
72. Eiffel Tower The functions

$$h_1(x) = 363.4 - 88.4 \ln x, \quad 16.47 < x \le 61.0$$

and

$$h_2(x) = 568.2 - 161.5 \ln x, \quad 6.1 \le x \le 16.47$$

approximate the height, in meters, of the Eiffel Tower x meters to the right of the center line, shown by the y-axis in the following figure.



The graph of h_1 models the shape of the tower from ground level up to the second stage in the figure, and the graph of h_2 models the shape of the tower from the second stage up to the third stage.

Determine the horizontal distance across the Eiffel Tower, rounded to the nearest tenth of a meter, at a height of

a. 50 meters

b. 125 meters

73. Psychology Industrial psychologists study employee training programs to assess the effectiveness of the instruction. In one study, the percent score P on a test for a person who had completed t hours of training was given by

$$P = \frac{100}{1 + 30e^{-0.088t}}$$

a. Use a graphing utility to graph the equation for $t \ge 0$.

b. Use the graph to estimate (to the nearest hour) the number of hours of training necessary to achieve a 70% score on the test.

c. Use the graph to estimate the horizontal asymptote.

d. Write a sentence that explains the meaning of the horizontal asymptote.

74. Psychology An industrial psychologist has determined that the average percent score for an employee on a test of the employee's knowledge of the company's product is given by

$$P = \frac{100}{1 + 40e^{-0.1t}}$$

where t is the number of weeks on the job and P is the percent score.

a. Graph the equation for $t \ge 0$.

b. Use the graph to estimate (to the nearest week) the expected number of weeks of employment that are necessary for an employee to earn a 70% score on the test.

c. Use the graph to estimate the horizontal asymptote of the graph.

d. Write a sentence that explains the meaning of the horizontal asymptote.

75. Ecology A herd of bison was placed in a wildlife preserve that can support a maximum of 1000 bison. A population model for the bison is given by

$$B = \frac{1000}{1 + 30e^{-0.127t}}$$

where B is the number of bison in the preserve and t is time in years, with the year 1999 represented by $t = 0$.

a. Graph the equation for $t \ge 0$.

b. Estimate (to the nearest year) the number of years before the bison population reaches 500.

c. Determine the horizontal asymptote of the graph.

d. Write a sentence that explains the meaning of the horizontal asymptote in the context of this application.

76. **Population Growth** A yeast culture grows according to the equation

$$Y = \frac{50{,}000}{1 + 250e^{-0.305t}}$$

where Y is the number of yeast and t is time in hours.

a. Graph the equation for $t \geq 0$.

b. Use the graph to estimate (to the nearest hour) the number of hours before the yeast population reaches 35,000.

c. Use the graph to estimate the horizontal asymptote.

d. Write a sentence that explains the meaning of the horizontal asymptote in the context of this application.

77. **Consumption of Natural Resources** A model for how long our coal resources will last is given by

$$T = \frac{\ln(300r + 1)}{\ln(r + 1)}$$

where r is the percent increase in consumption from current levels of use and T is the time, in years, before the resources are depleted.

a. Graph this equation.

b. If our consumption of coal increases by 3% per year, in how many years will we deplete our coal resources?

c. What percent increase in consumption of coal will deplete the resources in 100 years? Round to the nearest tenth of a percent.

78. **Effects of Air Resistance on Velocity** If we assume that air resistance is proportional to the square of the velocity, then the time t, in seconds, required for an object to reach a velocity v in feet per second is given by

$$t = \frac{9}{24}\ln\frac{24 + v}{24 - v}, 0 \leq v < 24$$

a. Determine the velocity, to the nearest hundredth of a foot per second, of the object after 1.5 seconds.

b. Determine the vertical asymptote for the graph of this function.

c. Write a sentence that explains the meaning of the vertical asymptote in the context of this application.

79. **Terminal Velocity with Air Resistance** The velocity v, in feet per second, of an object t seconds after it has been dropped from a height above the surface of the Earth is given by the equation $v = 32t$, assuming no air resistance. If we assume that air resistance is proportional to the square of the velocity, then the velocity after t seconds is given by

$$v = 100\left(\frac{e^{0.64t} - 1}{e^{0.64t} + 1}\right)$$

a. In how many seconds will the velocity be 50 feet per second?

b. Determine the horizontal asymptote for the graph of this function.

c. Write a sentence that explains the meaning of the horizontal asymptote in the context of this application.

80. **Effects of Air Resistance on Distance** The distance s, in feet, that the object in Exercise 79 will fall in t seconds is given by

$$s = \frac{100^2}{32}\ln\left(\frac{e^{0.32t} + e^{-0.32t}}{2}\right)$$

a. Graph this equation for $t \geq 0$.

b. How long does it take for the object to fall 100 feet? Round to the nearest tenth of a second.

81. **Retirement Planning** The retirement account for a graphic designer contains \$250,000 on January 1, 2013, and earns interest at a rate of 0.5% per month. On February 1, 2013, the designer withdraws \$2000 and plans to continue these withdrawals as retirement income each month. The value V of the account after x months is

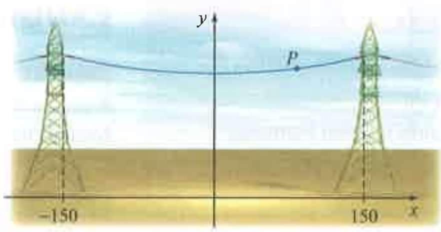
$$V = 400{,}000 - 150{,}000(1.005)^x$$

If the designer wishes to leave \$100,000 to a scholarship foundation, what is the maximum number of withdrawals the designer can make from this account and still have \$100,000 to donate?

82. **Transmission Cable** The height h, in feet, of any point P on the cable shown below is given by

$$h(x) = 40(e^{x/216.4} + e^{-x/216.4}), \quad -150 \leq x \leq 150$$

where $|x|$ is the horizontal distance, in feet, between P and the y-axis.



a. What is the lowest height of the cable?

b. What is the height of the cable 100 feet to the right of the y-axis? Round to the nearest tenth of a foot.

c. How far to the right of the y-axis is the cable 90 feet in height? Round to the nearest tenth of a foot.

83. The following argument seems to indicate that $0.125 > 0.25$. Find the first incorrect statement in the argument.

$$3 > 2$$
$$3(\log 0.5) > 2(\log 0.5)$$
$$\log 0.5^3 > \log 0.5^2$$
$$0.5^3 > 0.5^2$$
$$0.125 > 0.25$$

84. The following argument seems to indicate that $4 = 6$. Find the first incorrect statement in the argument.

$$4 = \log_2 16$$
$$4 = \log_2(8 + 8)$$
$$4 = \log_2 8 + \log_2 8$$
$$4 = 3 + 3$$
$$4 = 6$$

85. A common mistake that students make is to write $\log(x + y)$ as $\log x + \log y$. If $\log(x + y) = \log x + \log y$, then what is the relationship between x and y? (*Hint:* Solve for x in terms of y.)

86. Let $f(x) = 2 \ln x$ and $g(x) = \ln x^2$. Does $f(x) = g(x)$ for all x?

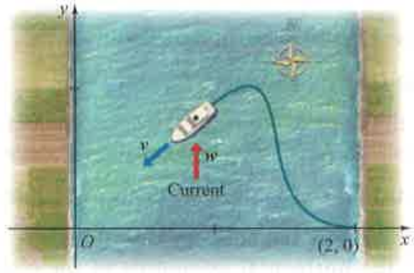
87. Explain why the functions $F(x) = 1.4^x$ and $G(x) = e^{0.336x}$ represent essentially the same function.

88. Find k such that $f(t) = 2.2^t$ and $g(t) = e^{-kt}$ represent essentially the same function.

Enrichment Exercises

89. **Navigating** The pilot of a boat is trying to cross a river to a point O, 2 miles due west of the boat's starting position, by always pointing the nose of the boat toward O. Suppose the speed of the current is w miles per hour and the speed of the boat is v miles per hour. If point O is the origin and the boat's starting position is $(2, 0)$, then the equation of the boat's path is given by

$$y = \left(\frac{x}{2}\right)^{1-(w/v)} - \left(\frac{x}{2}\right)^{1+(w/v)}$$



a. If the speed of the current and the speed of the boat are the same, can the pilot reach point O by always having the nose of the boat pointed toward O? If not, at what point will the pilot arrive? Explain.

b. If the speed of the current is greater than the speed of the boat, can the pilot reach point O by always pointing the nose of the boat toward point O? If not, where will the pilot arrive? Explain.

c. If the speed of the current is less than the speed of the boat, can the pilot reach O by always pointing the nose of the boat toward O? If not, where will the pilot arrive? Explain.

SECTION 4.6

Exponential Growth and Decay
Carbon Dating
Compound Interest Formulas
Restricted Growth Models

Exponential Growth and Decay

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A27.

PS1. Evaluate $A = 1000\left(1 + \frac{0.1}{12}\right)^{12t}$ for $t = 2$. Round to the nearest hundredth. [4.2]

PS2. Evaluate $A = 600\left(1 + \frac{0.04}{4}\right)^{4t}$ for $t = 8$. Round to the nearest hundredth. [4.2]

PS3. Solve $0.5 = e^{14k}$ for k. Round to the nearest ten-thousandth. [4.5]

PS4. Solve $0.85 = 0.5^{t/5730}$ for t. Round to the nearest ten. [4.5]

PS5. Solve $6 = \dfrac{70}{5 + 9e^{-k \cdot 12}}$ for k. Round to the nearest thousandth. [4.5]

PS6. Solve $2{,}000{,}000 = \dfrac{3^{n+1} - 3}{2}$ for n. Round to the nearest tenth. [4.5]