4.5 Solving Polynomial Equations

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Learning Target Solve polynomial equations and find zeros of polynomial functions.

Success Criteria

- I can explain how solutions of equations and zeros of functions are related.
- I can solve polynomial equations.
- I can write a polynomial function when given information about its zeros.

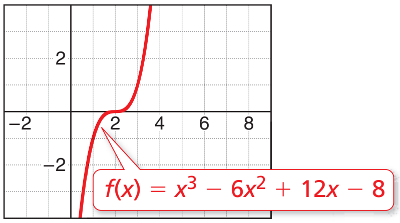
EXPLORE IT! Solving Polynomial Equations with Repeated Solutions

Work with a partner. Polynomial equations can have distinct solutions or *repeated solutions*. In parts (a)–(f), solve the equation algebraically. Then use the graph to describe the behavior of the related function near any repeated zeros. What do you notice?

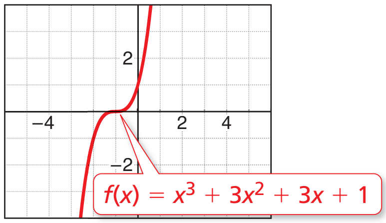
Math Practice

Make Conjectures
What conjecture can you make about the graph of a polynomial function f near the x-axis when a factor of $f(x)$ is raised to an odd power? to an even power?

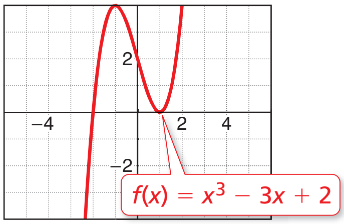
a. $x^3 - 6x^2 + 12x - 8 = 0$



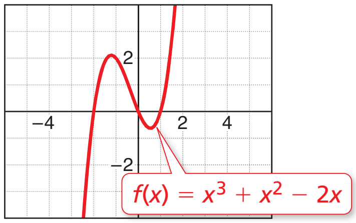
b. $x^3 + 3x^2 + 3x + 1 = 0$



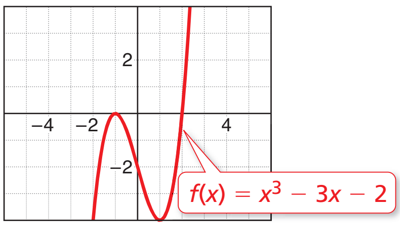
c. $x^3 - 3x + 2 = 0$



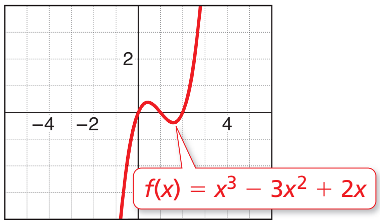
d. $x^3 + x^2 - 2x = 0$



e. $x^3 - 3x - 2 = 0$



f. $x^3 - 3x^2 + 2x = 0$



g. MP **CHOOSE TOOLS** Graph the related function for each quartic equation and describe the behavior of the function near its zeros.

i. $x^4 - 4x^3 + 5x^2 - 2x = 0$ **ii.** $x^4 - 2x^3 - x^2 + 2x = 0$

iii. $x^4 - 4x^3 + 4x^2 = 0$ **iv.** $x^4 + 3x^3 = 0$

h. Describe what it means when a polynomial equation has a repeated solution. How can you determine whether a polynomial equation has a repeated solution?

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Finding Solutions and Zeros

Vocabulary

repeated solution, *p. 184*

You have used the Zero-Product Property and factoring to solve quadratic equations. You can extend this technique to solve some higher-degree polynomial equations.

EXAMPLE 1 Solving a Polynomial Equation by Factoring

WATCH

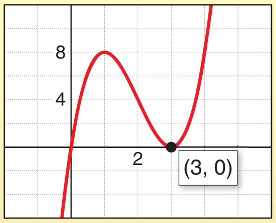
Solve $2x^3 - 12x^2 + 18x = 0$.

SOLUTION

$2x^3 - 12x^2 + 18x = 0$	Write the equation.
$2x(x^2 - 6x + 9) = 0$	Factor common monomial.
$2x(x - 3)^2 = 0$	Perfect square trinomial pattern
$2x = 0$ or $(x - 3)^2 = 0$	Zero-Product Property
$x = 0$ or $x = 3$	Solve for x.

▶ The solutions, or roots, are $x = 0$ and $x = 3$.

Check



In Example 1, the factor $x - 3$ appears more than once. This creates a **repeated solution** of $x = 3$. Note that the graph of the related function touches the x-axis (but does not cross the x-axis) at the repeated zero $x = 3$, and crosses the x-axis at the zero $x = 0$. This concept can be generalized for a polynomial function f as follows.

- When a factor $x - k$ of $f(x)$ is raised to an odd power, the graph of f *crosses* the x-axis at $x = k$.
- When a factor $x - k$ of $f(x)$ is raised to an even power, the graph of f *touches* the x-axis (but does not cross the x-axis) at $x = k$.

EXAMPLE 2 Finding Zeros of a Polynomial Function

WATCH

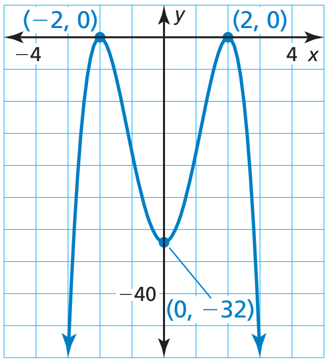
Find the zeros of $f(x) = -2x^4 + 16x^2 - 32$. Then sketch a graph of the function.

SOLUTION

$0 = -2x^4 + 16x^2 - 32$	Set $f(x)$ equal to 0.
$0 = -2(x^4 - 8x^2 + 16)$	Factor out -2.
$0 = -2(x^2 - 4)(x^2 - 4)$	Factor trinomial in quadratic form.
$0 = -2(x + 2)(x - 2)(x + 2)(x - 2)$	Difference of two squares pattern
$0 = -2(x + 2)^2(x - 2)^2$	Rewrite using exponents.

Because both factors $x + 2$ and $x - 2$ are raised to an even power, the graph of f touches the x-axis at the zeros $x = -2$ and $x = 2$.

By analyzing the original function, you can determine that the y-intercept is -32. Because the degree is even and the leading coefficient is negative, $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$. Use these characteristics to sketch a graph of f.



SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation.

1. $4x^4 - 40x^2 + 36 = 0$ **2.** $-3n^3 + 24n^2 - 48n = 0$ **3.** $2x^5 + 24x = 14x^3$

Find the zeros of the function. Then sketch a graph of the function.

4. $f(x) = 3x^4 - 6x^2 + 3$ **5.** $f(x) = x^3 + x^2 - 6x$ **6.** $h(x) = -x^3 - 2x^2 + 9x + 18$

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The Rational Root Theorem

The solutions of the equation $64x^3 + 152x^2 - 62x - 105 = 0$ are $-\frac{5}{2}$, $-\frac{3}{4}$, and $\frac{7}{8}$. Notice that the numerators (5, 3, and 7) of the zeros are factors of the constant term, -105. Also notice that the denominators (2, 4, and 8) are factors of the leading coefficient, 64. These observations are generalized by the *Rational Root Theorem*.

KEY IDEA

The Rational Root Theorem

If $f(x) = a_n x^n + \cdots + a_1 x + a_0$ has *integer* coefficients, then every rational solution of $f(x) = 0$ has the following form.

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

The Rational Root Theorem can be a starting point for finding solutions of polynomial equations. However, the theorem lists only *possible* solutions. In order to find the *actual* solutions, you must test values from the list of possible solutions.

EXAMPLE 3 Using the Rational Root Theorem

WATCH

Find all the real solutions of $x^3 - 8x^2 + 11x + 20 = 0$.

Math Practice

Look for Structure

What do you notice about the possible rational solutions when the leading coefficient is 1?

SOLUTION

The polynomial that defines the function $f(x) = x^3 - 8x^2 + 11x + 20$ is not easily factorable. Begin by using the Rational Root Theorem.

Step 1 List the possible rational solutions. The leading coefficient of $f(x)$ is 1 and the constant term is 20. So, the possible rational solutions of $f(x) = 0$ are

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{5}{1}, \pm\frac{10}{1}, \pm\frac{20}{1}.$$

STUDY TIP

You can use direct substitution to test possible solutions, but synthetic division helps you identify other factors of the polynomial.

Step 2 Test possible solutions using synthetic division until a solution is found.

Test $x = 1$:

$$\begin{array}{r|rrrr} 1 & 1 & -8 & 11 & 20 \\ & & 1 & -7 & 4 \\ \hline & 1 & -7 & 4 & 24 \end{array}$$

$f(1) \neq 0$, so $x - 1$ is not a factor of $f(x)$.

Test $x = -1$:

$$\begin{array}{r|rrrr} -1 & 1 & -8 & 11 & 20 \\ & & -1 & 9 & -20 \\ \hline & 1 & -9 & 20 & 0 \end{array}$$

$f(-1) = 0$, so $x + 1$ is a factor of $f(x)$.

Step 3 Factor completely using the result of the synthetic division.

$(x + 1)(x^2 - 9x + 20) = 0$ — Write as a product of factors.

$(x + 1)(x - 4)(x - 5) = 0$ — Factor the trinomial.

▶ So, the solutions are $x = -1$, $x = 4$, and $x = 5$.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

7. MP **STRUCTURE** Use the Rational Root Theorem to list the possible rational solutions of $x^3 + 14x^2 + 55x + 42 = 0$.

Find all the real solutions of the equation.

8. $x^3 + 2x^2 - 13x + 10 = 0$

9. $x^3 - 5x^2 - 2x + 24 = 0$

In Example 3, the leading coefficient of the polynomial is 1. When the leading coefficient is not 1, the list of possible rational solutions or zeros can increase dramatically. In such cases, the search can be shortened by using a graph.

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EXAMPLE 4 Finding Zeros of a Polynomial Function

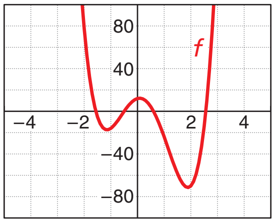
WATCH

Find all the real zeros of $f(x) = 10x^4 - 11x^3 - 42x^2 + 7x + 12$.

SOLUTION

Step 1 List the possible rational zeros of f: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{4}{5}, \pm\frac{6}{5}, \pm\frac{12}{5}, \pm\frac{1}{10}, \pm\frac{3}{10}$.

Step 2 Use the graph of f to choose reasonable values from the list in Step 1. The values $x = -\frac{3}{2}$, $x = -\frac{1}{2}$, $x = \frac{3}{5}$, and $x = \frac{12}{5}$ appear to be reasonable.



Step 3 Test the values using synthetic division until a zero is found.

$$\begin{array}{r|rrrrr} -\frac{3}{2} & 10 & -11 & -42 & 7 & 12 \\ & & -15 & 39 & \frac{9}{2} & -\frac{69}{4} \\ \hline & 10 & -26 & -3 & \frac{23}{2} & -\frac{21}{4} \end{array}$$

$-\frac{3}{2}$ is *not* a zero.

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 10 & -11 & -42 & 7 & 12 \\ & & -5 & 8 & 17 & -12 \\ \hline & 10 & -16 & -34 & 24 & 0 \end{array}$$

$-\frac{1}{2}$ is a zero.

Step 4 Factor out a binomial using the result of the synthetic division.

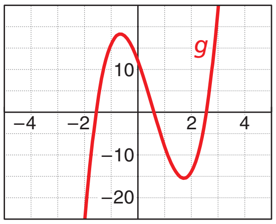
$f(x) = \left(x + \frac{1}{2}\right)(10x^3 - 16x^2 - 34x + 24)$ — Write as a product of factors.

$= \left(x + \frac{1}{2}\right)(2)(5x^3 - 8x^2 - 17x + 12)$ — Factor 2 out of the second factor.

$= (2x + 1)(5x^3 - 8x^2 - 17x + 12)$ — Multiply the first factor by 2.

Step 5 Repeat the steps above for $g(x) = 5x^3 - 8x^2 - 17x + 12$. Any zero of g will also be a zero of f. The possible rational zeros of g are

$$x = \pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{4}{5}, \pm\frac{6}{5}, \pm\frac{12}{5}.$$



The graph of g shows that $\frac{3}{5}$ may be a zero. Synthetic division shows that $\frac{3}{5}$ *is* a zero and $g(x) = \left(x - \frac{3}{5}\right)(5x^2 - 5x - 20) = (5x - 3)(x^2 - x - 4)$. It follows that $f(x) = (2x + 1) \bullet g(x) = (2x + 1)(5x - 3)(x^2 - x - 4)$.

Step 6 Find the remaining zeros of f by solving $x^2 - x - 4 = 0$.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-4)}}{2(1)}$$ Substitute 1 for *a*, −1 for *b*, and −4 for *c* in the Quadratic Formula.

$$x = \frac{1 \pm \sqrt{17}}{2}$$ Simplify.

▶ The real zeros of f are $-\frac{1}{2}$, $\frac{3}{5}$, $\frac{1 + \sqrt{17}}{2}$, and $\frac{1 - \sqrt{17}}{2}$.

The Irrational Conjugates Theorem

In Example 4, notice that the irrational zeros are *conjugates* of the form $a + \sqrt{b}$ and $a - \sqrt{b}$. This illustrates the theorem below.

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KEY IDEA

The Irrational Conjugates Theorem

Let f be a polynomial function with rational coefficients, and let a and b be rational numbers such that $\sqrt{b}$ is irrational. If $a + \sqrt{b}$ is a zero of f, then $a - \sqrt{b}$ is also a zero of f.

EXAMPLE 5 Using Zeros to Write a Polynomial Function

WATCH

Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the zeros 3 and $2 + \sqrt{5}$.

SOLUTION

Because the coefficients are rational and $2 + \sqrt{5}$ is a zero, $2 - \sqrt{5}$ must also be a zero by the Irrational Conjugates Theorem. Use the three zeros and the Factor Theorem to write $f(x)$ as a product of three factors.

$$f(x) = (x - 3)\left[x - \left(2 + \sqrt{5}\right)\right]\left[x - \left(2 - \sqrt{5}\right)\right] \quad \text{Write } f(x) \text{ in factored form.}$$

$$= (x - 3)\left[(x - 2) - \sqrt{5}\right]\left[(x - 2) + \sqrt{5}\right] \quad \text{Regroup terms.}$$

$$= (x - 3)\left[(x - 2)^2 - 5\right] \quad \text{Multiply.}$$

$$= (x - 3)\left[(x^2 - 4x + 4) - 5\right] \quad \text{Expand binomial.}$$

$$= (x - 3)(x^2 - 4x - 1) \quad \text{Simplify.}$$

$$= x^3 - 4x^2 - x - 3x^2 + 12x + 3 \quad \text{Multiply.}$$

$$= x^3 - 7x^2 + 11x + 3 \quad \text{Combine like terms.}$$

Check You can check this result by evaluating f at each of the given zeros.

$$f(3) = 3^3 - 7(3)^2 + 11(3) + 3 = 27 - 63 + 33 + 3 = 0 \checkmark$$

$$f\left(2 + \sqrt{5}\right) = \left(2 + \sqrt{5}\right)^3 - 7\left(2 + \sqrt{5}\right)^2 + 11\left(2 + \sqrt{5}\right) + 3$$

$$= 38 + 17\sqrt{5} - 63 - 28\sqrt{5} + 22 + 11\sqrt{5} + 3$$

$$= 0 \checkmark$$

Because $f\left(2 + \sqrt{5}\right) = 0$, by the Irrational Conjugates Theorem $f\left(2 - \sqrt{5}\right) = 0$. ✓

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Find all the real zeros of the function.

10. $h(x) = 16x^3 - 44x^2 + 8x + 5$

11. $f(x) = 3x^4 - 2x^3 - 37x^2 + 24x + 12$

Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the given zero(s).

12. $3 + \sqrt{2}$

13. $4, 1 - \sqrt{5}$

14. $0, 2, 6 + \sqrt{3}$

4.5 Practice WITH CalcChat® AND CalcView®

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In Exercises 1–10, solve the equation. *Example 1*

1. $z^3 - z^2 - 12z = 0$

2. $a^3 - 4a^2 + 4a = 0$

3. $2x^4 - 4x^3 = -2x^2$

4. $v^3 - 2v^2 - 16v = -32$

5. $5w^3 = 50w$

6. $9m^5 = 27m^3$

7. $2c^4 - 6c^3 = 12c^2 - 36c$

8. $p^4 + 40 = 14p^2$

9. $12n^2 + 48n = -n^3 - 64$

10. $y^3 - 27 = 9y^2 - 27y$

In Exercises 11–18, find the zeros of the function. Then sketch a graph of the function. *Example 2*

11. $h(x) = x^4 + x^3 - 6x^2$

12. $f(x) = x^4 - 18x^2 + 81$

13. $p(x) = x^6 - 11x^5 + 30x^4$

14. $g(x) = -2x^5 + 2x^4 + 40x^3$

15. $g(x) = -4x^4 + 8x^3 + 60x^2$

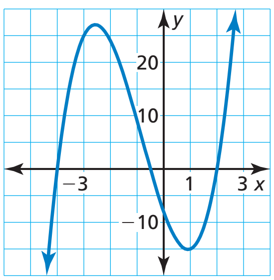
16. $h(x) = -x^3 - 2x^2 + 15x$

17. $h(x) = -x^3 - x^2 + 9x + 9$

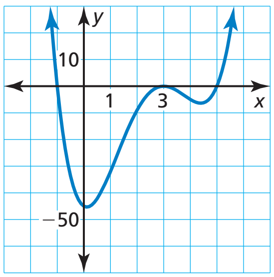
18. $p(x) = x^3 - 5x^2 - 4x + 20$

In Exercises 19 and 20, determine whether $f(x) = 0$ has any repeated real solutions. Explain your reasoning.

19.



20.



In Exercises 21–28, find all the real solutions of the equation. *Example 3*

21. $x^3 + x^2 - 17x + 15 = 0$

22. $x^3 - 2x^2 - 5x + 6 = 0$

23. $x^3 - 10x^2 + 19x + 30 = 0$

24. $x^3 + 4x^2 - 11x - 30 = 0$

25. $x^3 - 6x^2 - 7x + 60 = 0$

26. $x^3 - 16x^2 + 55x + 72 = 0$

27. $2x^3 - 3x^2 - 50x - 24 = 0$

28. $3x^3 + x^2 - 38x + 24 = 0$

29. **COLLEGE PREP** According to the Rational Root Theorem, which is *not* a possible solution of the equation $2x^4 - 5x^3 + 10x^2 - 9 = 0$?

Ⓐ $x = -9$

Ⓑ $x = -\frac{1}{2}$

Ⓒ $x = \frac{2}{9}$

Ⓓ $x = 3$

30. **COLLEGE PREP** According to the Rational Root Theorem, which is *not* a possible zero of the function $f(x) = 40x^5 - 42x^4 - 107x^3 + 107x^2 + 33x - 36$?

Ⓐ $-\frac{2}{3}$

Ⓑ $-\frac{3}{8}$

Ⓒ $\frac{3}{4}$

Ⓓ $\frac{4}{5}$

ERROR ANALYSIS **In Exercises 31 and 32, describe and correct the error in listing the possible rational zeros of the function.**

31.

✗ $f(x) = x^3 + 5x^2 - 9x - 45$

Possible rational zeros of f:

1, 3, 5, 9, 15, 45

32.

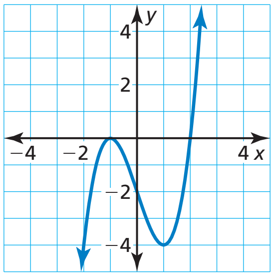
✗ $f(x) = 3x^3 + 13x^2 - 41x + 8$

Possible rational zeros of f:

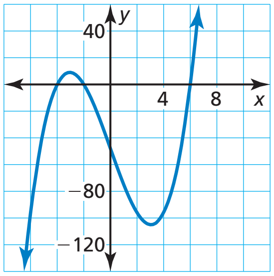
$\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{1}{8}, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm\frac{3}{8}$

In Exercises 33–40, find all the real zeros of the function. ▶ *Example 4*

33. $f(x) = x^3 - 3x - 2$



34. $f(x) = x^3 - 28x - 48$



35. $p(x) = 2x^3 - x^2 - 27x + 36$

36. $g(x) = 3x^3 - 25x^2 + 58x - 40$

37. $f(x) = 4x^3 - 20x + 16$

38. $f(x) = 4x^3 - 49x - 60$

39. $h(x) = 64x^4 + 32x^3 - 44x^2 - 12x + 9$

40. $f(x) = 3x^4 - 2x^3 - 25x^2 - 20x - 4$

In Exercises 41–48, write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the given zero(s). ▶ *Example 5*

41. $-2, 3, 6$

42. $-4, -2, 5$

43. $6 - \sqrt{7}$

44. $-2 + \sqrt{5}$

45. $-2, 1 + \sqrt{7}$

46. $4, 6 - \sqrt{7}$

47. $-6, 0, 3 - \sqrt{5}$

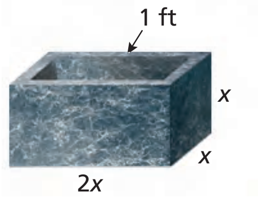
48. $0, 5, -5 + \sqrt{8}$

49. MP **REASONING** According to the Rational Root Theorem, ± 1 and ± 2 are possible rational zeros of a function f. The graph of f does *not* have x-intercepts of ± 1 or ± 2. Does this contradict the Rational Root Theorem? Explain.

50. **COMPARING METHODS** Solve the equation $x^3 - 4x^2 - 9x + 36 = 0$ using two different methods. Which method do you prefer? Explain your reasoning.

51. **MODELING REAL LIFE** Archaeologists discovered several huge concrete blocks at the ruins of Caesarea. One of the blocks has a volume of 180 cubic meters and the dimensions shown. Find the value of x.

52. MP **PROBLEM SOLVING** You are designing a marble basin that will hold a fountain for a city park. The sides and bottom of the basin should be 1 foot thick. Its outer length should be twice its outer width and outer height. What should the outer dimensions of the basin be if it is to hold 36 cubic feet of water?



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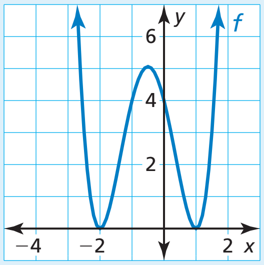
53. MP **REASONING** Is it possible for a cubic function to have more than three real zeros? Explain.

54. **HOW DO YOU SEE IT?** The graph of a quartic function f is shown.



a. What are the real zeros of f?

b. Write an equation of the function in factored form.

55. **MODELING REAL LIFE** During a 10-year period, the amount (in millions of dollars) of athletic equipment E sold domestically can be modeled by $E(t) = -20t^3 + 252t^2 - 280t + 21{,}614$, where t is in years. When is about \$24,014,000,000 of athletic equipment sold?

56. **THOUGHT PROVOKING** All the possible rational solutions and actual rational solutions of the equation below are shown. Complete the equation.

Possible: $x = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

Actual: $x = -1, 2$

$$(x + \square)(x + \square)(x^2 + \square) = 0$$

In Exercises 57–60, solve $f(x) = g(x)$ by graphing and algebraic methods.

57. $f(x) = x^3 + x^2 - x - 1$; $g(x) = -x + 1$

58. $f(x) = x^3 - 4x^2 + 4x$; $g(x) = -2x + 4$

59. $f(x) = x^4 - 5x^3 + 2x^2 + 8x$; $g(x) = -x^2 + 6x - 8$

60. $f(x) = x^4 + 2x^3 - 11x^2 - 12x + 36$; $g(x) = -x^2 - 6x - 9$

61. **MP REASONING** Determine the value of k for each equation so that the given x-value is a solution.

a. $x^3 - 6x^2 - 7x + k = 0;\ x = 4$

b. $2x^3 + 7x^2 - kx - 18 = 0;\ x = -6$

c. $kx^3 - 35x^2 + 19x + 30 = 0;\ x = 5$

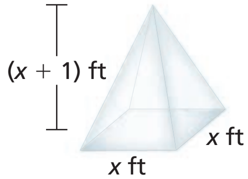
62. **MAKING AN ARGUMENT** A student solves the equation $9x = x^3$ as shown. Is the solution correct? Explain your reasoning.

$$9x = x^3$$
$$9 = x^2$$
$$\pm\sqrt{9} = x$$
$$\pm 3 = x$$

The solutions are $x = -3$ and $x = 3$.

63. **MODELING REAL LIFE** Some ice sculptures are made by filling a mold and then freezing it. You make the ice mold shown. The mold is in the shape of a square pyramid and can hold a maximum of 30 gallons of water. What are the dimensions of the mold?

GO DIGITAL



64. **DIG DEEPER** Let a_n be the leading coefficient of a polynomial function f and a_0 be the constant term. If a_n has r factors and a_0 has s factors, what is the greatest number of possible rational zeros of f that can be generated by the Rational Root Theorem? Explain your reasoning.

REVIEW & REFRESH

WATCH

In Exercises 65–70, find the product or quotient.

65. $(2b + 3)(2b - 3)$

66. $(5x + 8)^2$

67. $(4p - 2)^3$

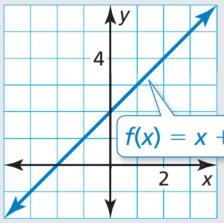
68. $(mn + 6)^3$

69. $(x^4 + 2x^3 - 25x^2 - 2x + 24) \div (x^2 + 5x - 6)$

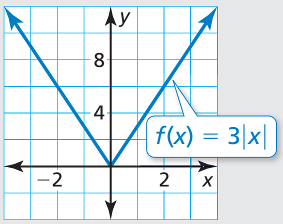
70. $(x^4 + 4x^3 - 10x^2 - 28x - 15) \div (x - 3)$

In Exercises 71 and 72, identify the function family to which f belongs. Describe the transformation from the graph of the parent function to the graph of f.

71.



72.



73. Write a function to model the data.

x	−1	0	1	2	3
y	4.5	8	10.5	12	12.5

74. Write the function $f(x) = x^2 + 7x + 13$ in vertex form. Then identify the vertex.

In Exercises 75 and 76, find all the real zeros of the function.

75. $f(x) = x^3 - 3x^2 - 6x + 8$

76. $g(x) = 45x^3 - 69x^2 + 32x - 4$

In Exercises 77–80, solve the equation.

77. $8x^2 - 1 = 0$

78. $7x^2 + 42 = 0$

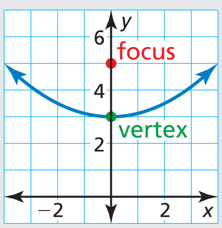
79. $-5 = 3x^2 + 7$

80. $9 = \frac{1}{2}(x - 4)^2 - 6$

81. **MODELING REAL LIFE** The profit P (in millions of dollars) for a manufacturer of cell phone grips can be modeled by $P = -\frac{1}{4}(x^3 - 49x)$, where x is the number (in tens of millions) of grips produced. Currently, the company produces 50 million grips and makes a profit of \$30 million. What lesser number of grips could the company produce and still make the same profit?

In Exercises 82 and 83, write an equation of the parabola.

82.



83.