b. The vertical asymptote $v = 175$ indicates that the velocity of the sky diver approaches, but never reaches or exceeds, 175 feet per second. In Figure 4.38, note that as $v \to 175$ from the left, $t \to \infty$.

▶ Try Exercise 78, page 391

Answer graphs to Exercises 73a, 74a, 75a, 76a, 77a, and 80a are on page AA18.

EXERCISE SET 4.5

Concept Check

1. Some exponential equations can be solved by using the Equality of Exponents Theorem. What is the Equality of Exponents Theorem? If $b^x = b^y$, then $x = y$, provided $b > 0$ and $b \neq 1$.
2. Name two methods that can be used to estimate the solutions of an equation of the form $f(x) = g(x)$, with the aid of a graphing utility. The Intersection Method and the Intercept Method, provided you rewrite the equation as $f(x) - g(x) = 0$
3. To solve $3^{2x+7} = 8^{x+6}$ without the aid of a graphing utility, would you apply the Equality of Exponents Theorem or the logarithm-of-each-side property? Logarithm-of-each-side property
4. A student has used the product property and the one-to-one property of logarithms to determine that 5 and -1 are possible solutions of $\ln(7x - 5) = \ln(2x + 5) + \ln(x - 3)$. Are both of these values solutions of the logarithmic equation? No. 5 is a solution, but -1 is not a solution.

In Exercises 5 to 52, use algebraic procedures to find the exact solution or solutions of the equation.

5. $2^x = 64$ 6
6. $3^x = 243$ 5
7. $49^x = \frac{1}{343}$ $-\frac{3}{2}$
8. $9^x = \frac{1}{243}$ $-\frac{5}{2}$
9. $3^{2x-1} = 81$ $\frac{5}{2}$
10. $2^{3x+2} = 1024$ $\frac{8}{3}$
11. $\left(\frac{2}{5}\right)^x = \frac{8}{125}$ 3
12. $\left(\frac{2}{5}\right)^x = \frac{25}{4}$ -2
13. $5^x = 70$ $\frac{\log 70}{\log 5}$
14. $6^x = 50$ $\frac{\ln 50}{\ln 6}$
15. $3^{x+1} = 251$ $\frac{\ln 251}{\ln 3} - 1$
16. $5^{-x} = 121$ $\frac{\ln 121}{\ln 5}$
17. $10^{2x+3} = 315$ $\frac{\log 315 - 3}{2}$
18. $10^{6-x} = 550$ $6 - \log 550$
19. $e^x = 10$ $\ln 10$
20. $e^{x+1} = 20$ $\ln 20 - 1$
21. $2^{1-x} = 3^{x+1}$ $\frac{\ln 2 - \ln 3}{\ln 6}$
22. $3^{x-2} = 4^{2x+1}$ $\frac{\ln 4 + 2\ln 3}{\ln 3 - 2\ln 4}$
23. $2^{2x-3} = 5^{-x-1}$ $\frac{3\log 2 - \log 5}{2\log 2 + \log 5}$
24. $5^{3x} = 3^{x+4}$ $\frac{4\ln 3}{3\ln 5 - \ln 3}$
25. $\log(4x - 18) = 1$ 7
26. $\log(x^2 + 19) = 2$ $-9, 9$
27. $\ln(x^2 - 9) = \ln(x + 11)$ $-4, 5$
28. $\log(x^2 - 6x) = \log 7$ $-1, 7$
29. $\log_2 x + \log_2(x - 4) = 2$ $2 + 2\sqrt{2}$
30. $\log_3 x + \log_3(x + 6) = 3$ 3
31. $\log(5x - 1) = 2 + \log(x - 2)$ $\frac{199}{95}$
32. $1 + \log(3x - 1) = \log(2x + 1)$ $\frac{11}{28}$
33. $\ln(1 - x) + \ln(3 - x) = \ln 8$ -1
34. $\log(4 - x) = \log(x + 8) + \log(2x + 13)$ -5
35. $\log\sqrt{x^3 - 17} = \frac{1}{2}$ 3
36. $\log(x^3) = (\log x)^2$ 1, 1000
37. $\ln(\ln x) = -1$ $e^{(e^{-1})}$
38. $\log(\log 100{,}000x) = 1$ 10^5
39. $\ln(e^{3x}) = 6$ 2
40. $\ln x = \frac{1}{2}\ln\left(2x + \frac{5}{2}\right) + \frac{1}{2}\ln 2$ 5
41. $\ln(2x + 5) = \ln(x + 3) + \ln(x - 1)$ $2\sqrt{2}$
42. $\log 14x - \log(x + 2) = 1$ 5

■ Indicates Try It Exercises

43. $e^{\ln(x-1)} = 4$ 5

44. $10^{\log(2x+7)} = 8$ $\frac{1}{2}$

45. $\dfrac{10^x - 10^{-x}}{2} = 20$ $\log(20 + \sqrt{401})$

46. $\dfrac{10^x + 10^{-x}}{2} = 8$ $\log(8 \pm 3\sqrt{7})$

47. $\dfrac{10^x + 10^{-x}}{10^x - 10^{-x}} = 5$ $\frac{1}{2}\log\left(\frac{3}{2}\right)$

48. $\dfrac{10^x - 10^{-x}}{10^x + 10^{-x}} = \dfrac{1}{2}$ $\frac{\log 3}{2}$

49. $\dfrac{e^x + e^{-x}}{2} = 15$ $\ln(15 \pm 4\sqrt{14})$

50. $\dfrac{e^x - e^{-x}}{2} = 15$ $\ln(15 + \sqrt{226})$

51. $\dfrac{1}{e^x - e^{-x}} = 4$ $\ln(1 + \sqrt{65}) - \ln 8$

52. $\dfrac{e^x + e^{-x}}{e^x - e^{-x}} = 3$ $\frac{\ln 2}{2}$

In Exercises 53 to 62, use a graphing utility to approximate the solution or solutions of the equation to the nearest hundredth.

53. $2^{-x+3} = x + 1$ 1.61

54. $3^{x-2} = -2x - 1$ −0.53

55. $e^{3-2x} - 2x = 1$ 0.96

56. $2e^{x+2} + 3x = 2$ −1.05

57. $3\log_2(x - 1) = -x + 3$ 2.20

58. $2\log_3(2 - 3x) = 2x - 1$ 0.38

59. $\ln(2x + 4) + \frac{1}{2}x = -3$ −1.93

60. $2\ln(3 - x) + 3x = 4$ 0.81, 2.91

61. $2^{x+1} = x^2 - 1$ −1.34

62. $\ln x = -x^2 + 4$ 1.84

63. **Population Growth** The population P of a city grows exponentially according to the function

$$P(t) = 8500(1.1)^t, \quad 0 \le t \le 8$$

where t is measured in years.

a. Find the population at time $t = 0$ and at time $t = 2$. 8500, 10,285

b. When, to the nearest year, will the population reach 15,000? In 6 years

64. **Physical Fitness** After a race, a runner's pulse rate R, in beats per minute, decreases according to the function

$$R(t) = 145e^{-0.092t}, \quad 0 \le t \le 15$$

where t is measured in minutes.

a. Find the runner's pulse rate at the end of the race and 1 minute after the end of the race. 145 bpm, 132 bpm

b. How long, to the nearest minute, after the end of the race will the runner's pulse rate be 80 beats per minute? 6 min

65. **Rate of Cooling** A can of soda at 79°F is placed in a refrigerator that maintains a constant temperature of 36°F. The temperature T of the soda t minutes after it is placed in the refrigerator is given by

$$T(t) = 36 + 43e^{-0.058t}$$

a. Find the temperature, to the nearest degree, of the soda 10 minutes after it is placed in the refrigerator. 60°F

b. When, to the nearest minute, will the temperature of the soda be 45°F? 27 min

66. **Medicine** During surgery, a patient's circulatory system requires at least 50 milligrams of an anesthetic. The amount of anesthetic present t hours after 80 milligrams of anesthetic is administered is given by

$$T(t) = 80(0.727)^t$$

a. How much, to the nearest milligram, of the anesthetic is present in the patient's circulatory system 30 minutes after the anesthetic is administered? 68 mg

b. How long, to the nearest minute, can the operation last if the patient does not receive additional anesthetic? 88 min

Bertalanffy's Equation **In 1938, the biologist Ludwig von Bertalanffy developed the equation**

$$L = m - (m - L_0)e^{-rx}$$

which models the length L, in centimeters, of a fish as it grows under optimal conditions for a period of x years. In Bertalanffy's equation, m represents the maximum length, in centimeters, the fish is expected to attain; L_0 is the length, in centimeters, of the fish at birth; and r is a constant related to the growth rate of the fish species. Use Bertalanffy's equation to predict the age of the fish described in Exercises 67 and 68.

67. A barracuda has a length of 114 centimeters. Use Bertalanffy's equation to predict, to the nearest tenth of a year, the age of the barracuda. Assume $m = 198$ centimeters, $L_0 = 0.9$ centimeter, and $r = 0.23$. 3.7 years

68. A haddock has a length of 21 centimeters. Use Bertalanffy's equation to predict, to the nearest tenth of a year, the age of the haddock. Assume $m = 94$ centimeters, $L_0 = 0.6$ centimeter, and $r = 0.21$. 1.2 years

69. **Typing Speed** The following function models the average typing speed S, in words per minute, for a student who has been typing for t months.

$$S(t) = 5 + 29\ln(t + 1), \quad 0 \le t \le 9$$

Use S to determine how long it takes the student to achieve an average typing speed of 65 words per minute. Round to the nearest tenth of a month. 6.9 months

70. **Walking Speed** An approximate relation between the average pedestrian walking speed s, in miles per hour, and the population x, in thousands, of a city is given by the formula

$$s(x) = 0.37\ln x + 0.05$$

Unless otherwise noted, all content on this page is © Cengage Learning.

Use s to estimate the population of a city for which the average pedestrian walking speed is 2.9 miles per hour. Round to the nearest hundred thousand. 2,200,000

71. **Drag Racing** The quadratic function

$$s_1(x) = -2.25x^2 + 56.26x - 0.28, \quad 0 \le x \le 10$$

models the speed of a dragster from the start of a race until the dragster crosses the finish line 10 seconds later. This is the acceleration phase of the race.

The exponential function

$$s_2(x) = 8320(0.73)^x, \quad 10 < x \le 20$$

models the speed of the dragster during the 10-second period immediately following the time when the dragster crosses the finish line. This is the deceleration period.

How long after the start of the race did the dragster attain a speed of 275 miles per hour? Round to the nearest hundredth of a second. 6.67 s and 10.83 s

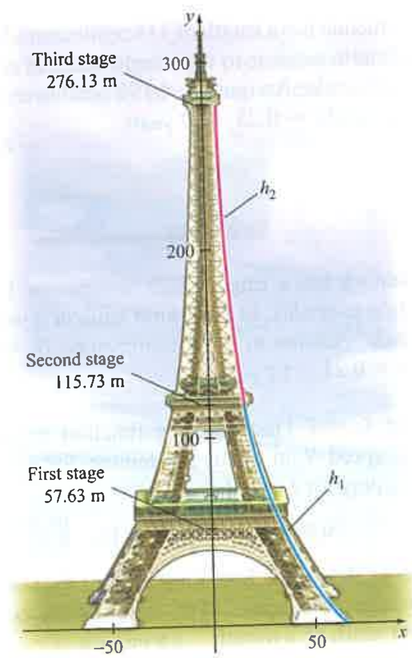
72. **Eiffel Tower** The functions

$$h_1(x) = 363.4 - 88.4 \ln x, \quad 16.47 < x \le 61.0$$

and

$$h_2(x) = 568.2 - 161.5 \ln x, \quad 6.1 \le x \le 16.47$$

approximate the height, in meters, of the Eiffel Tower x meters to the right of the center line, shown by the y-axis in the following figure.



The graph of h_1 models the shape of the tower from ground level up to the second stage in the figure, and the graph of h_2 models the shape of the tower from the second stage up to the third stage.

Determine the horizontal distance across the Eiffel Tower, rounded to the nearest tenth of a meter, at a height of

a. 50 meters 69.3 m

b. 125 meters 31.1 m

73. **Psychology** Industrial psychologists study employee training programs to assess the effectiveness of the instruction. In one study, the percent score P on a test for a person who had completed t hours of training was given by

$$P = \frac{100}{1 + 30e^{-0.088t}}$$

a. Use a graphing utility to graph the equation for $t \ge 0$.

b. Use the graph to estimate (to the nearest hour) the number of hours of training necessary to achieve a 70% score on the test. 48 h

c. Use the graph to estimate the horizontal asymptote. $P = 100$

d. Write a sentence that explains the meaning of the horizontal asymptote. As the number of hours of training increases, the test scores approach 100%.

74. **Psychology** An industrial psychologist has determined that the average percent score for an employee on a test of the employee's knowledge of the company's product is given by

$$P = \frac{100}{1 + 40e^{-0.1t}}$$

where t is the number of weeks on the job and P is the percent score.

a. Graph the equation for $t \ge 0$.

b. Use the graph to estimate (to the nearest week) the expected number of weeks of employment that are necessary for an employee to earn a 70% score on the test. 45 weeks

c. Use the graph to estimate the horizontal asymptote of the graph. $P = 100$

d. Write a sentence that explains the meaning of the horizontal asymptote. The more experience a person has, the closer the person's score is to 100%.

75. **Ecology** A herd of bison was placed in a wildlife preserve that can support a maximum of 1000 bison. A population model for the bison is given by

$$B = \frac{1000}{1 + 30e^{-0.127t}}$$

where B is the number of bison in the preserve and t is time in years, with the year 1999 represented by $t = 0$.

a. Graph the equation for $t \ge 0$.

b. Estimate (to the nearest year) the number of years before the bison population reaches 500. In 27 years, or 2026

c. Determine the horizontal asymptote of the graph. $B = 1000$

d. Write a sentence that explains the meaning of the horizontal asymptote in the context of this application. As the number of years increases, the bison population increases, but never reaches or exceeds 1000.

Unless otherwise noted, all content on this page is © Cengage Learning.

76. **Population Growth** A yeast culture grows according to the equation

$$Y = \frac{50{,}000}{1 + 250e^{-0.305t}}$$

where Y is the number of yeast and t is time in hours.

a. Graph the equation for $t \geq 0$.

b. Use the graph to estimate (to the nearest hour) the number of hours before the yeast population reaches 35,000. 21 h

c. Use the graph to estimate the horizontal asymptote. $Y = 50{,}000$

d. Write a sentence that explains the meaning of the horizontal asymptote in the context of this application. The number of yeast increases, but never reaches or exceeds 50,000.

77. **Consumption of Natural Resources** A model for how long our coal resources will last is given by

$$T = \frac{\ln(300r + 1)}{\ln(r + 1)}$$

where r is the percent increase in consumption from current levels of use and T is the time, in years, before the resources are depleted.

a. Graph this equation.

b. If our consumption of coal increases by 3% per year, in how many years will we deplete our coal resources? 78 years

c. What percent increase in consumption of coal will deplete the resources in 100 years? Round to the nearest tenth of a percent. 1.9%

78. **Effects of Air Resistance on Velocity** If we assume that air resistance is proportional to the square of the velocity, then the time t, in seconds, required for an object to reach a velocity v in feet per second is given by

$$t = \frac{9}{24}\ln\frac{24 + v}{24 - v}, 0 \leq v < 24$$

a. Determine the velocity, to the nearest hundredth of a foot per second, of the object after 1.5 seconds. 23.14 ft/s

b. Determine the vertical asymptote for the graph of this function. $v = 24$

c. Write a sentence that explains the meaning of the vertical asymptote in the context of this application. The velocity of the object can never reach or exceed 24 ft/s.

79. **Terminal Velocity with Air Resistance** The velocity v, in feet per second, of an object t seconds after it has been dropped from a height above the surface of the Earth is given by the equation $v = 32t$, assuming no air resistance. If we assume that air resistance is proportional to the square of the velocity, then the velocity after t seconds is given by

$$v = 100\left(\frac{e^{0.64t} - 1}{e^{0.64t} + 1}\right)$$

a. In how many seconds will the velocity be 50 feet per second? $Y = 1.72$ s

b. Determine the horizontal asymptote for the graph of this function. $v = 100$

c. Write a sentence that explains the meaning of the horizontal asymptote in the context of this application. The object cannot fall faster than 100 ft/s.

80. **Effects of Air Resistance on Distance** The distance s, in feet, that the object in Exercise 79 will fall in t seconds is given by

$$s = \frac{100^2}{32}\ln\left(\frac{e^{0.32t} + e^{-0.32t}}{2}\right)$$

a. Graph this equation for $t \geq 0$.

b. How long does it take for the object to fall 100 feet? Round to the nearest tenth of a second. 2.6 s

81. **Retirement Planning** The retirement account for a graphic designer contains \$250,000 on January 1, 2013, and earns interest at a rate of 0.5% per month. On February 1, 2013, the designer withdraws \$2000 and plans to continue these withdrawals as retirement income each month. The value V of the account after x months is

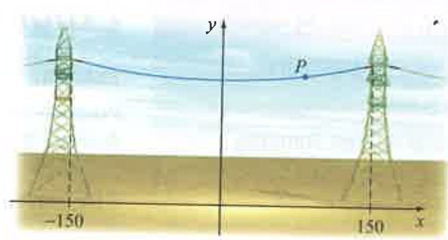
$$V = 400{,}000 - 150{,}000(1.005)^x$$

If the designer wishes to leave \$100,000 to a scholarship foundation, what is the maximum number of withdrawals the designer can make from this account and still have \$100,000 to donate? 138 withdrawals

82. **Transmission Cable** The height h, in feet, of any point P on the cable shown below is given by

$$h(x) = 40(e^{x/216.4} + e^{-x/216.4}), \quad -150 \leq x \leq 150$$

where $|x|$ is the horizontal distance, in feet, between P and the y-axis.



a. What is the lowest height of the cable? 80 ft

b. What is the height of the cable 100 feet to the right of the y-axis? Round to the nearest tenth of a foot. 88.7 ft

c. How far to the right of the y-axis is the cable 90 feet in height? Round to the nearest tenth of a foot. 107.1 ft

83. The following argument seems to indicate that $0.125 > 0.25$. Find the first incorrect statement in the argument.

$$3 > 2$$
$$3(\log 0.5) > 2(\log 0.5)$$
$$\log 0.5^3 > \log 0.5^2$$
$$0.5^3 > 0.5^2$$
$$0.125 > 0.25$$

The second step; because $\log 0.5 < 0$, the inequality sign must be reversed.

Unless otherwise noted, all content on this page is © Cengage Learning.

84. The following argument seems to indicate that $4 = 6$. Find the first incorrect statement in the argument.

$$4 = \log_2 16$$
$$4 = \log_2(8 + 8)$$
$$4 = \log_2 8 + \log_2 8$$
$$4 = 3 + 3$$
$$4 = 6$$

The third step; $\log_2(8 + 8)$ does not equal $\log_2 8 + \log_2 8$.

85. A common mistake that students make is to write $\log(x + y)$ as $\log x + \log y$. If $\log(x + y) = \log x + \log y$, then what is the relationship between x and y? (*Hint:* Solve for x in terms of y.) $x = \dfrac{y}{y - 1}$

86. Let $f(x) = 2 \ln x$ and $g(x) = \ln x^2$. Does $f(x) = g(x)$ for all x? No

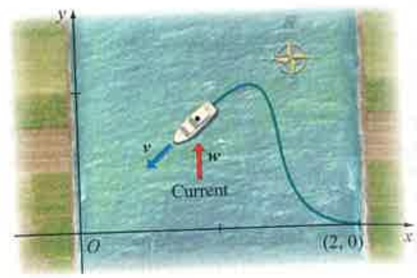
87. Explain why the functions $F(x) = 1.4^x$ and $G(x) = e^{0.336x}$ represent essentially the same function. $e^{0.336} \approx 1.4$

88. Find k such that $f(t) = 2.2^t$ and $g(t) = e^{-kt}$ represent essentially the same function. $k = -\ln 2.2 \approx -0.788$

Enrichment Exercises

89. **Navigating** The pilot of a boat is trying to cross a river to a point O, 2 miles due west of the boat's starting position, by always pointing the nose of the boat toward O. Suppose the speed of the current is w miles per hour and the speed of the boat is v miles per hour. If point O is the origin and the boat's starting position is $(2, 0)$, then the equation of the boat's path is given by

$$y = \left(\frac{x}{2}\right)^{1-(w/v)} - \left(\frac{x}{2}\right)^{1+(w/v)}$$



a. If the speed of the current and the speed of the boat are the same, can the pilot reach point O by always having the nose of the boat pointed toward O? If not, at what point will the pilot arrive? Explain.
No. If $w = v$, the equation becomes $y = 1 - (x^2/4)$ and when $x = 0$, $y = 1$. So the boat will arrive at $(0, 1)$.

b. If the speed of the current is greater than the speed of the boat, can the pilot reach point O by always pointing the nose of the boat toward point O? If not, where will the pilot arrive? Explain.
No. If $w > v$, then $1 - (w/v)$ is negative and the expression $(x/2)^{1-(w/v)}$ becomes larger and larger as x approaches 0. Thus the path of the boat curves north and never reaches O.

c. If the speed of the current is less than the speed of the boat, can the pilot reach O by always pointing the nose of the boat toward O? If not, where will the pilot arrive? Explain.
Yes. If $w < v$, then $y = 0$ when $x = 0$, and the boat reaches O.

SECTION 4.6

- Exponential Growth and Decay
- Carbon Dating
- Compound Interest Formulas
- Restricted Growth Models

Exponential Growth and Decay

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A27.

PS1. Evaluate $A = 1000\left(1 + \dfrac{0.1}{12}\right)^{12t}$ for $t = 2$. Round to the nearest hundredth. [4.2] 1220.39

PS2. Evaluate $A = 600\left(1 + \dfrac{0.04}{4}\right)^{4t}$ for $t = 8$. Round to the nearest hundredth. [4.2] 824.96

PS3. Solve $0.5 = e^{14k}$ for k. Round to the nearest ten-thousandth. [4.5] −0.0495

PS4. Solve $0.85 = 0.5^{t/5730}$ for t. Round to the nearest ten. [4.5] 1340

PS5. Solve $6 = \dfrac{70}{5 + 9e^{-k \cdot 12}}$ for k. Round to the nearest thousandth. [4.5] 0.02

PS6. Solve $2{,}000{,}000 = \dfrac{3^{n+1} - 3}{2}$ for n. Round to the nearest tenth. [4.5] 12.8

Unless otherwise noted, all content on this page is © Cengage Learning.