4.4 WS 4

KEY

Use the properties of logarithms to expand the following logarithmic expressions. Assume all variable expressions represent positive real numbers. When possible, evaluate logarithmic expressions.

1. $\log\left(x\sqrt{y^2z}\right)$

$\log x + \log y + \frac{1}{2}\log z$

2. $\ln\left(\sqrt{\frac{x^3y}{x+10}}\right)$

$\frac{3}{2}\ln x + \frac{1}{2}\ln y - \frac{1}{2}\ln(x+10)$

3. $\log_2\sqrt{xy\sqrt[3]{8}}$

$\frac{1}{2}\log_2 x + \frac{1}{2}\log_2 y + \frac{1}{2}$

4. $\log\left(\frac{x}{100\sqrt{z^3}}\right)$

$\log x - \left[2 + \frac{3}{2}\log z\right]$

OR

$\log x - 2 - \frac{3}{2}\log z$

5. $\log\left(\frac{x^3z^{-1/2}}{y^{-5}}\right) = \log\left(\frac{x^3y^5}{z^{1/2}}\right)$

$3\log x + 5\log y - \frac{1}{2}\log z$

6. $\ln\left(\frac{ex}{\sqrt[4]{y}}\right)^5$

$5 + 5\ln x - \frac{5}{4}\ln y$

Use the properties of logarithms to rewrite each expression as a single logarithm with a coefficient of 1. Assume all variable expressions represent positive real numbers.

7. $4\log x - \frac{1}{3}\log z$

$\log\left(\frac{x^4}{\sqrt[3]{z}}\right)$

8. $\ln\left(x^2+5x+6\right) - 2\ln(x+3)$

$\ln\left(\frac{x^2+5x+6}{(x+3)^2}\right)$

$\ln\left(\frac{(x+3)(x+2)}{(x+3)^2}\right)$

$\ln\left(\frac{x+2}{x+3}\right)$

9. $\log\left(x^2-4\right) + \log y - 3\log(x+2)$

$\log\left(\frac{(x^2-4)y}{(x+2)^3}\right)$

$\log\left(\frac{(x+2)(x-2)y}{(x+2)^3}\right)$

$\log\left(\frac{(x-2)y}{(x+2)^2}\right)$

Evaluate the logarithm. Round to the nearest ten-thousandth.

10. $\log_2 5$

2.3219

11. $\log_e \pi$

1.1447

12. $\log_6\sqrt{11}$

0.6691

13. $\log_e 13$

2.5649

Find the inverse of each function, then state the domain and range of $f^{-1}(x)$.

14. $f(x)=\sqrt{7-x}$ $D_x: x \le 7$ $R_y: y \ge 0$

$y^2=(\sqrt{7-x})^2$

$y^2=7-x$

$-x=y^2-7$

$x=-y^2+7$

$f^{-1}(x)=-x^2+7$

D_x of $f^{-1}(x)$: $\{x \mid x \ge 0\}$

R_y of $f^{-1}(x)$: $\{y \mid y \le 7\}$

15. $f(x)=\dfrac{7}{3+x}$ $D_x: x \ne -3$

$(3+x)y=\dfrac{7}{3+x}(3+x)$

$3y+xy=7$

$\dfrac{xy}{y}=\dfrac{7-3y}{y}$

$x=\dfrac{7-3y}{y}$

$f^{-1}(x)=\dfrac{7-3x}{x}$

D_x of $f^{-1}(x)$: $\{x \mid x \ne 0\}$

R_y of $f^{-1}(x)$: $\{y \mid y \ne -3\}$

16. $f(x)=\dfrac{1}{3}x+4$ $D_x: (-\infty,\infty)$ $R_y: (-\infty,\infty)$

$y=\frac{1}{3}x+4$

$3(y-4)=\frac{1}{3}x(3)$

$x=3y-12$

$f^{-1}(x)=3x-12$

D_x of $f^{-1}(x)$: $(-\infty,\infty)$

R_y of $f^{-1}(x)$: $(-\infty,\infty)$

17. Compare the intensity of an earthquake that measures 5.0 on the Richter scale to the intensity of an earthquake that measures 3.0 on the Richter scale by finding the ratio of the larger intensity to the smaller intensity.

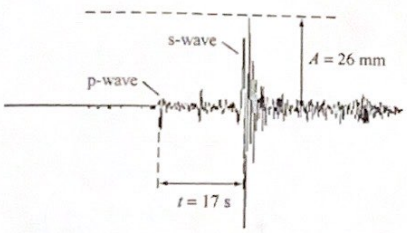
$\dfrac{I_1}{10^{5.0}=\frac{I_1}{I_0}}$ $\quad I_1=10^5 I_0$

$\dfrac{I_2}{I_2=10^3 I_0}$

$\dfrac{I_1}{I_2}=\dfrac{10^5 I_0}{10^3 I_0}=\dfrac{10^5}{10^3}=10^{5-3}=10^2=100$

$\frac{100}{1}$ or 100 times more intense

18. Find the Richter scale magnitude of the earthquake that produced the seismogram in the following figure.



$M=\log(26)+3\log(8\cdot 17)-2.92$

$M=4.9$

19. Vinegar has a hydronium-ion concentration of 1.26×10^{-3} mole per liter. Determine the pH of vinegar of magnesia and state whether vinegar is an acid or a base.

$pH=-\log(1.26\times10^{-3})$

pH = 2.9, Acid