4.4 WS 3

KEY

Use the properties of logarithms to expand the following logarithmic expressions. Assume all variable expressions represent positive real numbers. When possible, evaluate logarithmic expressions.

1. $\ln\left(x^{1/2}y^{2/3}\right)$

$\boxed{\frac{1}{2}\ln x + \frac{2}{3}\ln y}$

2. $\ln\left(\frac{\sqrt[3]{x^2}}{z^2}\right)$

$\boxed{\frac{2}{3}\ln x - 2\ln z}$

3. $\ln\sqrt[3]{z\sqrt{e}}$

$\frac{1}{3}\ln z + \frac{1}{6}\ln e$

$\boxed{\frac{1}{3}\ln z + \frac{1}{6}}$

4. $\log_6\left(\frac{x^3\sqrt[3]{y}}{216z^4}\right)$

$\boxed{3\log_6 x + \frac{1}{3}\log_6 y - 3 - 4\log_6 z}$

or

$\boxed{3\log_6 x + \frac{1}{3}\log_6 y - [3 + 4\log_6 z]}$

5. $\log\left(\frac{x^2\sqrt{z}}{y^{-3}}\right) = \log x^2\sqrt{z}\,y^3$

$\boxed{2\log x + 3\log y + \frac{1}{2}\log z}$

6. $\ln\left(\frac{e^2x}{\sqrt{y}}\right)^3$

$3\ln e^2 + 3\ln x + \frac{3}{2}\ln y$

$\boxed{6 + 3\ln x - \frac{3}{2}\ln y}$

Use the properties of logarithms to rewrite each expression as a single logarithm with a coefficient of 1. Assume all variable expressions represent positive real numbers.

7. $\log\left(xy^2\right) - \log z$

$\boxed{\log\left(\frac{xy^2}{z}\right)}$

8. $\ln\left(x^2-9\right) - 2\ln(x-3)$

$\ln\left(\frac{x^2-9}{(x-3)^2}\right)$

$\ln\left(\frac{(x+3)(x-3)}{(x-3)^2}\right)$

$\boxed{\ln\left(\frac{x+3}{x-3}\right)}$

9. $\frac{1}{2}\log_3 x - \log_3 y + 2\log_3(x+2)$

$\log_3\left(\frac{\sqrt{x}}{y}\right) + \log_3(x+2)^2$

$\boxed{\log_3\left(\frac{(x+2)^2\sqrt{x}}{y}\right)}$

Evaluate the logarithm. Round to the nearest ten-thousandth.

10. $\log_4 6^2$

2.5850

11. $\log_\pi 30$

2.9712

12. $\log_8\sqrt{19}$

0.7080

13. $\log_e 21$

3.0445

$x-12 \geq 0$

Find the inverse of each function, then state the domain and range of $f^{-1}(x)$.

14. $f(x)=\sqrt[3]{x-8}$ $D_x: (-\infty,\infty)$ $R_y: (-\infty,\infty)$

$y^3=(\sqrt[3]{x-8})^3$

$y^3=x-8$

$x=y^3+8$

$\boxed{f^{-1}(x)=x^3+8 \quad D_x \text{ of } f^{-1}(x): (-\infty,\infty) \quad R_y \text{ of } f^{-1}(x): (-\infty,\infty)}$

15. $f(x)=\dfrac{x}{5+x}$ $D_x: x \neq -5$

$(5+x)y=\dfrac{x}{5+x}(5+x)$

$5y+xy=x$

$5y=x-xy$

$\dfrac{5y}{1-y}=\dfrac{x(1-y)}{1-y}$

$x=\dfrac{5y}{1-y}$

$\boxed{f^{-1}(x)=\dfrac{5x}{1-x} \quad D_x \text{ of } f^{-1}(x): \{x \mid x\neq 1\} \quad R_y \text{ of } f^{-1}(x): \{y \mid y\neq -5\}}$

16. $f(x)=\sqrt{x-12}$ $D_x: x \geq 12$ $R_y: y \geq 0$

$y^2=(\sqrt{x-12})^2$

$y^2=x-12$

$x=y^2+12$

$\boxed{f^{-1}(x)=x^2+12 \quad D_x \text{ of } f^{-1}(x): \{x \mid x\geq 0\} \quad R_y \text{ of } f^{-1}(x): \{y \mid y\geq 12\}}$

17. The Gulf of California earthquake of April 2012 had an intensity of $I=7{,}943{,}300I_0$. What did this earthquake measure on the Richter scale?

$$M=\log\left(\frac{7{,}943{,}300\, I_0}{I_0}\right)=\log(7{,}943{,}300)=\boxed{6.9}$$

18. An earthquake that occurred in China in 1978 measured 8.2 on the Richter scale. In 1988, an earthquake in California measured 6.9 on the Richter scale. Compare the intensity of the larger earthquake to the intensity of the smaller earthquake by finding the ratio of the larger intensity of the smaller by finding the ratio of the larger intensity to the smaller intensity.

I_1 : China

$I_1=10^{8.2}I_0$

I_2: California

$I_2=10^{6.9}I_0$

The China earthquake is about 20 times more intense than the California earthquake.

$$\frac{I_1}{I_2}=\frac{10^{8.2}\cancel{I_0}}{10^{6.9}\cancel{I_0}}=\frac{10^{8.2}}{10^{6.9}}=10^{8.2-6.9}=10^{1.3}\approx 19.95$$

19. Milk of magnesia has a hydronium-ion concentration of about 3.97×10^{-11} mole per liter. Determine the pH of milk of magnesia and state whether milk of magnesia is an acid or a base.

$$pH=-\log(3.97\times10^{-11})=\boxed{10.4,\ \text{Base}}$$