4.4 WS 2

KEY

Use the properties of logarithms to expand the following logarithmic expressions. Assume all variable expressions represent positive real numbers. When possible, evaluate logarithmic expressions.

1. $\log \dfrac{xz}{y^2}$

$\log x + \log z - 2\log y$

2. $\ln(xy)^2$

$2\ln x + 2\ln y$

3. $\ln\sqrt{x\sqrt{y}}$

$\frac{1}{2}\ln x + \frac{1}{4}\ln y$

4. $\ln(e^2 z)$

$2\ln e + \ln z$

$\boxed{2 + \ln z}$

5. $\log_5\left(\dfrac{x\sqrt{z}}{25y^2}\right)$

$\log_5 x + \frac{1}{2}\log_5 z - [\log_5 25 + 2\log_5 y]$

$\log_5 x + \frac{1}{2}\log_5 z - 2 - 2\log_5 y$

or

$\log_5 x + \frac{1}{2}\log_5 z - [2 + 2\log_5 y]$

6. $\ln\left(\sqrt[5]{x^3 y\sqrt{z}}\right)$

$\boxed{\frac{3}{5}\ln x + \frac{1}{5}\ln y + \frac{1}{10}\ln z}$

Use the properties of logarithms to rewrite each expression as a single logarithm with a coefficient of 1. Assume all variable expressions represent positive real numbers.

7. $\dfrac{1}{2}\ln y + \ln z - \dfrac{1}{2}\ln z$

$\ln\left(\dfrac{\sqrt{y}\, z}{\sqrt{z}}\right)$

$\boxed{\ln\sqrt{yz}}$

8. $2(\log_6 x + \log_6 y^2) - \log_6(x+2)$

$2(\log_6 xy^2) - \log_6(x+2)$

$\log_6 x^2y^4 - \log_6(x+2)$

$\boxed{\log_6\left[\dfrac{x^2y^4}{x+2}\right]}$

9. $\log_b(x^2+7x+12) - 2\log_b(x+4)$

$\log_b\left[\dfrac{x^2+7x+12}{(x+4)^2}\right]$

$\log_b\left[\dfrac{(x+4)(x+3)}{(x+4)^2}\right]$

$\boxed{\log_b\left[\dfrac{x+3}{x+4}\right]}$

Evaluate the logarithm. Round to the nearest ten-thousandth.

10. $\log_2 6$ $\boxed{2.5810}$

11. $\log_{\sqrt{13}} 31$ $\boxed{2.6776}$

12. $\log_{12} \sqrt{3}$ $\boxed{0.2211}$

13. $\log_8 e$ $\boxed{0.4809}$

Find the inverse of each function, then state the domain and range of $f^{-1}(x)$.

14. $f(x) = \sqrt{x+4}$

$D_x: x \geq -4$
$R_y: y \geq 0$

$y^2 = (\sqrt{x+4})^2$

$y^2 = x+4$

$x = y^2 - 4$

$f^{-1}(x) = x^2 - 4$
D_x of $f^{-1}(x)$: $\{x \mid x \geq 0\}$
R_y of $f^{-1}(x)$: $\{y \mid y \geq -4\}$

15. $f(x) = \frac{1}{2}x + 6$

$y = \frac{1}{2}x + 6$

$2(y-6) = \frac{1}{2}x(2)$

$x = 2y - 12$

$f^{-1}(x) = 2x - 12$
D_x of $f^{-1}(x)$: $(-\infty, \infty)$
R_y of $f^{-1}(x)$: $(-\infty, \infty)$

16. $f(x) = \frac{4}{3-x}$

D_x of $f(x)$: $\{x \mid x \neq 3\}$

$y = \frac{4}{3-x}$

$y(3-x) = 4$

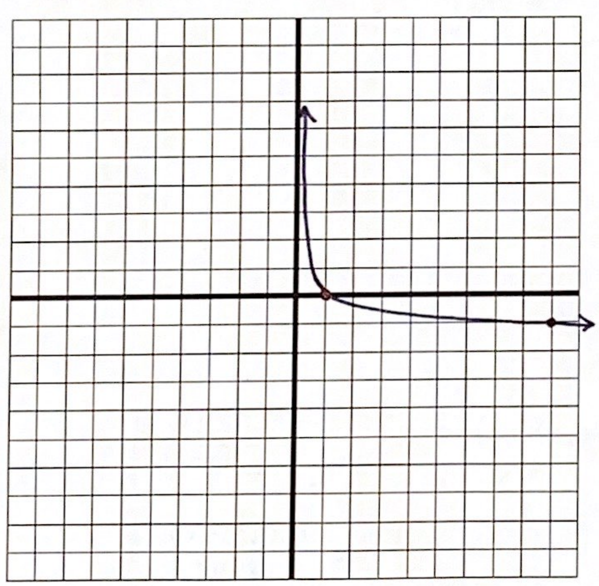
$3y - xy = 4$

$\frac{-xy}{-y} = \frac{4-3y}{-y}$

$x = \frac{-4+3y}{y}$

$f^{-1}(x) = \frac{3x-4}{x}$
D_x of $f^{-1}(x)$: $\{x \mid x \neq 0\}$
R_y of $f^{-1}(x)$: $\{y \mid y \neq 3\}$

17. Graph: $f(x) = \log_{1/9} x$



18. Graph: $f(x) = \log x$