Alternative to Example 10
Find the hydronium-ion concentration of a cleaning solution with a pH of 3.4.
■ 3.98×10^{-4} mol/L

EXAMPLE 10 Find the Hydronium-Ion Concentration

A sample of blood has a pH of 7.3. Find the hydronium-ion concentration of the blood.

Solution

$$\text{pH} = -\log[\text{H}^+]$$
$$7.3 = -\log[\text{H}^+] \qquad \text{• Substitute 7.3 for pH.}$$
$$-7.3 = \log[\text{H}^+] \qquad \text{• Multiply both sides by } -1.$$
$$10^{-7.3} = [\text{H}^+] \qquad \text{• Change to exponential form.}$$
$$5.0 \times 10^{-8} \approx [\text{H}^+]$$

The hydronium-ion concentration of the blood is about 5.0×10^{-8} mole per liter.

▶ Try Exercise 84, page 381

Answer graphs to Exercises 49–56 are on page AA18.

EXERCISE SET 4.4

Concept Check

In Exercises 1 to 4, use the given information to complete the sentence. Assume *b*, *M*, and *N* are positive real numbers and $b \neq 1$.

1. The product property of logarithms states that $\log_b(MN)$ equals ______. $\log_b M + \log_b N$
2. The quotient property of logarithms states that $\log_b\left(\frac{M}{N}\right)$ equals ______. $\log_b M - \log_b N$
3. The power property of logarithms states that $\log_b(M^p)$ equals ______. $p\log_b M$
4. The one-to-one property of logarithms states that $\log_b M = \log_b N$ implies ______. $M = N$

In Exercises 5 to 20, expand the given logarithmic expression. Assume all variable expressions represent positive real numbers. When possible, evaluate logarithmic expressions. Do not use a calculator.

5. $\log_b(xyz)$ $\log_b x + \log_b y + \log_b z$
6. $\ln \frac{z^3}{\sqrt{xy}}$ $3\ln z - \frac{1}{2}\ln x - \frac{1}{2}\ln y$
7. $\log_4\left(\frac{y^2}{16x}\right)$ $2\log_4 y - 2 - \log_4 x$
8. $\ln\left(\frac{e^3}{xy^2}\right)$ $3 - \ln x - 2\ln y$
9. $\log_2 \frac{\sqrt{x}}{y^3}$ $\frac{1}{2}\log_2 x - 3\log_2 y$
10. $\log_b(x\sqrt[3]{y})$ $\log_b x + \frac{1}{3}\log_b y$
11. $\log_7 \frac{\sqrt{xz}}{y^2}$ $\frac{1}{2}\log_7 x + \frac{1}{2}\log_7 z - 2\log_7 y$
12. $\ln \sqrt[3]{x^2\sqrt{y}}$ $\frac{2}{3}\ln x + \frac{1}{6}\ln y$
13. $\ln(e^2 z)$ $2 + \ln z$
14. $\ln(x^{1/2}y^{2/3})$ $\frac{1}{2}\ln x + \frac{2}{3}\ln y$
15. $\log_6\left(\frac{x^3\sqrt[3]{y}}{216z^4}\right)$ $3\log_6 x + \frac{1}{3}\log_6 y - 3 - 4\log_6 z$
16. $\log_5\left(\frac{x\sqrt{z}}{25y^2}\right)$ $\log_5 x + \frac{1}{2}\log_5 z - 2 - 2\log_5 y$
17. $\log\sqrt{x\sqrt{z}}$ $\frac{1}{2}\log x + \frac{1}{4}\log z$
18. $\ln\left(\frac{\sqrt[3]{x^2}}{z^2}\right)$ $\frac{2}{3}\ln x - 2\ln z$
19. $\ln(\sqrt[3]{z\sqrt{e}})$ $\frac{1}{3}\ln z + \frac{1}{6}$
20. $\ln\left[\frac{x^2\sqrt{z}}{y^{-3}}\right]$ $2\ln x + 3\ln y + \frac{1}{2}\ln z$

In Exercises 21 to 36, write each expression as a single logarithm with a coefficient of 1. Assume all variable expressions represent positive real numbers.

21. $\log(x + 5) + 2\log x$ $\log[x^2(x + 5)]$
22. $3\log_2 t - \frac{1}{3}\log_2 u + 4\log_2 v$ $\log_2 \frac{t^3v^4}{\sqrt[3]{u}}$
23. $\ln(x^2 - y^2) - \ln(x - y)$ $\ln(x + y)$
24. $\frac{1}{2}\log_8(x + 5) - 3\log_8 y$ $\log_8 \frac{\sqrt{x + 5}}{y^3}$
25. $2\log x + \log y - \frac{1}{2}\log y$ $\log(x^2\sqrt{y})$
26. $\ln(xy) + 3\ln\left(\frac{y}{z}\right) - 2\ln(xyz)$ $\ln\left(\frac{y^2}{xz^5}\right)$
27. $\log(xy^2) - \log z$ $\log\left(\frac{xy^2}{z}\right)$
28. $\ln(y^{1/2}z) - \ln z^{1/2}$ $\ln\sqrt{yz}$ or $\ln(y^{1/2}z^{1/2})$

Indicates Try It Exercises

29. $2(\log_6 x + \log_6 y^2) - \log_6(x + 2)$ $\log_6\left(\frac{x^2y^4}{x+2}\right)$

30. $\frac{1}{2}\log_3 x - \log_3 y + 2\log_3(x + 2)$ $\log_3\left[\frac{\sqrt{x}\,(x+2)^2}{y}\right]$

31. $2\ln(x + 4) - \ln x - \ln(x^2 - 3)$ $\ln\left[\frac{(x+4)^2}{x(x^2-3)}\right]$

32. $\log(3x) - (2\log x - \log y)$ $\log\left(\frac{3y}{x}\right)$

33. $\ln(2x + 5) - \ln y - 2\ln z + \frac{1}{2}\ln w$ $\ln\left[\frac{(2x+5)\sqrt{w}}{yz^2}\right]$

34. $\log_b x + \log_b(y + 3) + \log_b(y + 2) - \log_b(y^2 + 5y + 6)$ $\log_b x$

35. $\ln(x^2 - 9) - 2\ln(x - 3) + 3\ln y$ $\ln\left[\frac{(x+3)y^3}{x-3}\right]$

36. $\log_b(x^2 + 7x + 12) - 2\log_b(x + 4)$ $\log_b\left(\frac{x+3}{x+4}\right)$

In Exercises 37 to 48, use the change-of-base formula to approximate the logarithm accurate to the nearest ten-thousandth.

37. $\log_7 20$ 1.5395

38. $\log_5 37$ 2.2436

39. $\log_2 14$ 3.8074

40. $\log_{25} 15$ 0.8413

41. $\log_8\left(\frac{3}{5}\right)$ -0.2457

42. $\log_{11}\left(\frac{2}{3}\right)$ -0.1691

43. $\log_9\sqrt{17}$ 0.6447

44. $\log_4\sqrt{7}$ 0.7018

45. $\log_{\sqrt{2}} 17$ 8.1749

46. $\log_{\sqrt{3}} 5.5$ 3.1035

47. $\log_\pi e$ 0.8736

48. $\log_\pi\sqrt{15}$ 1.1828

In Exercises 49 to 56, use a graphing utility and the change-of-base formula to graph the logarithmic function.

49. $f(x) = \log_4 x$

50. $g(x) = \log_8(5 - x)$

51. $g(x) = \log_8(x - 3)$

52. $t(x) = \log_9(5 - x)$

53. $h(x) = \log_3(x - 3)^2$

54. $J(x) = \log_{12}(-x)$

55. $F(x) = -\log_5|x - 2|$

56. $n(x) = \log_2\sqrt{x - 8}$

In Exercises 57 to 66, determine whether the statement is true or false for all $x > 0$, $y > 0$. If it is false, write an example that disproves the statement.

57. $\log_b(x + y) = \log_b x + \log_b y$
False; $\log 10 + \log 10 = 2$ but $\log(10 + 10) = \log 20 \neq 2$.

58. $\log_b(xy) = \log_b x \cdot \log_b y$
False; $\log(10 \cdot 10) = 2$ but $\log 10 \cdot \log 10 = 1$.

59. $\log_b(xy) = \log_b x + \log_b y$ True

60. $\log_b x \cdot \log_b y = \log_b x + \log_b y$
False; $\log 10 \cdot \log 10 = 1$ but $\log 10 + \log 10 = 2$.

61. $\log_b x - \log_b y = \log_b(x - y), \quad x > y$
False; $\log 100 - \log 10 = 1$ but $\log(100 - 10) = \log 90 \neq 1$.

62. $\log_b\frac{x}{y} = \frac{\log_b x}{\log_b y}$ False; $\log\frac{100}{10} = \log 10 = 1$ but $\frac{\log 100}{\log 10} = \frac{2}{1} = 2$.

63. $\frac{\log_b x}{\log_b y} = \log_b x - \log_b y$
False; $\frac{\log 100}{\log 10} = \frac{2}{1} = 2$ but $\log 100 - \log 10 = 1$.

64. $\log_b(x^n) = n\log_b x$
True

65. $(\log_b x)^n = n\log_b x$ False; $(\log 10)^2 = 1$ but $2\log 10 = 2$.

66. $\log_b\sqrt{x} = \frac{1}{2}\log_b x$ True

In Exercises 67 and 68, evaluate the given expression *without* using a calculator.

67. $\log_3 5 \cdot \log_5 7 \cdot \log_7 9$ 2

68. $\log_5 20 \cdot \log_{20} 60 \cdot \log_{60} 100 \cdot \log_{100} 125$ 3

69. Which is larger, 500^{501} or 506^{500}? These numbers are too large for most calculators to handle. (They each have 1353 digits!) (*Hint:* Compare the logarithms of each number.) 500^{501}

70. Which is smaller, $\frac{1}{50^{300}}$ or $\frac{1}{151^{233}}$? See the hint in Exercise 69. $\frac{1}{50^{300}}$

71. **Earthquake Magnitude** The Gulf of California earthquake of April 2012 had an intensity of $I = 7{,}943{,}300I_0$. What did this earthquake measure on the Richter scale? 6.9

72. **Earthquake Magnitude** The Oaxaca, Mexico, earthquake of March 2012 had an intensity of $I = 25{,}118{,}900I_0$. What did this earthquake measure on the Richter scale? 7.4

73. **Earthquake Intensity** The earthquake that occurred off the coast of Northern Sumatra in April 2012 had a Richter scale magnitude of 8.6. Find the intensity of this earthquake. $10^{8.6}I_0$, or about $398{,}107{,}170.6I_0$

74. **Earthquake Intensity** The earthquake that occurred just south of Concepción, Chile, in 1960 had a Richter scale magnitude of 9.5. Find the intensity of this earthquake. $3{,}162{,}277{,}660I_0$

75. **Comparison of Earthquakes** Compare the intensity of an earthquake that measures 5.0 on the Richter scale to the intensity of an earthquake that measures 3.0 on the Richter scale by finding the ratio of the larger intensity to the smaller intensity. 100 to 1

76. **Comparison of Earthquakes** How many times as great was the intensity of the 1960 earthquake in Chile, which measured 9.5 on the Richter scale, than the San Francisco earthquake of 1906, which measured 8.3 on the Richter scale? $10^{1.2} \approx 15.8$ times as great

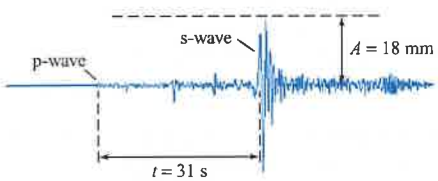
77. **Comparison of Earthquakes** The earthquake that occurred near the east coast of Honshu, Japan, on

March 11, 2011, measured 9.0 on the Richter scale. In October 1989, an earthquake of magnitude 7.1 on the Richter scale struck the San Francisco Bay area. Compare the intensity of the larger earthquake to the intensity of the smaller earthquake by finding the ratio of the larger intensity to the smaller intensity. $10^{1.9}$ to 1, or about 79.4 to 1

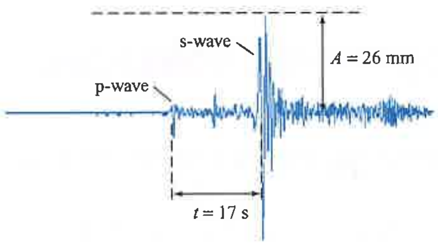
78. **Comparison of Earthquakes** An earthquake that occurred in China in 1978 measured 8.2 on the Richter scale. In 1988, an earthquake in California measured 6.9 on the Richter scale. Compare the intensity of the larger earthquake to the intensity of the smaller earthquake by finding the ratio of the larger intensity to the smaller intensity.

$10^{1.3}$ to 1, or about 20 to 1

79. **Earthquake Magnitude** Find the Richter scale magnitude of the earthquake that produced the seismogram in the following figure. 5.5



80. **Earthquake Magnitude** Find the Richter scale magnitude of the earthquake that produced the seismogram in the following figure. 4.9



81. **pH** Milk of magnesia has a hydronium-ion concentration of about 3.97×10^{-11} mole per liter. Determine the pH of milk of magnesia and state whether milk of magnesia is an acid or a base. 10.4; base

82. **pH** Vinegar has a hydronium-ion concentration of 1.26×10^{-3} mole per liter. Determine the pH of vinegar and state whether vinegar is an acid or a base. 2.9; acid

83. **Hydronium-Ion Concentration** A morphine solution has a pH of 9.5. Determine the hydronium-ion concentration of the morphine solution. 3.16×10^{-10} mol/L

84. **Hydronium-Ion Concentration** A rainstorm in New York City produced rainwater with a pH of 5.6. Determine the hydronium-ion concentration of the rainwater. 2.51×10^{-6} mol/L

Decibel Level The range of sound intensities that the human ear can detect is so large that a special decibel scale (named after Alexander Graham Bell) is used to measure and compare sound intensities. The decibel level (dB) of a sound is given by

$$dB(I) = 10\log\left(\frac{I}{I_0}\right)$$

where I_0 is the intensity of sound that is barely audible to the human ear. Use the decibel level formula to work Exercises 85 to 88.

85. Find the decibel level for the following sounds. Round to the nearest tenth of a decibel.

Sound	*Intensity*
a. Automobile traffic	$I = 1.58 \times 10^8 \cdot I_0$ 82.0 dB
b. Quiet conversation	$I = 10{,}800 \cdot I_0$ 40.3 dB
c. Fender guitar	$I = 3.16 \times 10^{11} \cdot I_0$ 115.0 dB
d. Jet engine	$I = 1.58 \times 10^{15} \cdot I_0$ 152.0 dB

86. A team in Arizona installed in a Ford Bronco a 48,000-watt sound system that it claims can output 175-decibel sound. The human pain threshold for sound is 125 decibels. How many times as great is the intensity of the sound from the Bronco than the human pain threshold for sound?

100,000 times as great

87. How many times as great is the intensity of a sound that measures 120 decibels than a sound that measures 110 decibels?
10 times as great

88. If the intensity of a sound is doubled, what is the increase in the decibel level? (*Hint:* Find $dB(2I) - dB(I)$.) ≈ 3.0103 dB

Enrichment Exercises

89. **Animated Maps** A software company that creates interactive maps for websites has designed an animated zooming feature such that when a user selects the zoom-in option, the map appears to expand on a location. This is accomplished by displaying several intermediate maps to give the illusion of motion. The company has determined that zooming in on a location is more informative and pleasing to observe when the scale of each step of the animation is determined using the equation

$$S_n = S_0 \cdot 10^{\frac{n}{N}(\log S_f - \log S_0)}$$

where S_n represents the scale of the current step n ($n = 0$ corresponds to the initial scale), S_0 is the starting scale of the map, S_f is the final scale, and N is the number of steps in the animation following the initial scale. (If the initial scale of the map is 1:200, then $S_0 = 200$.) Determine the scales to be used at each intermediate step if a map is to start with a scale of 1:1,000,000, and proceed through five intermediate steps to end with a scale of 1:500,000.
1:870,551; 1:757,858; 1:659,754; 1:574,349; 1:500,000

90. **Animated Maps** Use the equation in Exercise 89 to determine the scales for each stage of an animated map zoom that goes from a scale of 1:250,000 to a scale of 1:100,000 in four steps (following the initial scale).
1:198,818; 1:158,114; 1:125,743; 1:100,000

91. Prove the quotient property of logarithms

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

(*Hint:* See the proof of the product property of logarithms on page 372.)

92. Prove the power property of logarithms

$$\log_b(M^p) = p \log_b M$$

See the hint given in Exercise 91.

MID-CHAPTER 4 QUIZ

1. Use composition of functions to verify that

$$f(x) = \frac{500 + 120x}{x} \quad \text{and} \quad g(x) = \frac{500}{x - 120}$$

are inverses of each other. [4.1]

2. Find the inverse of $f(x) = \dfrac{24x + 5}{x - 4}, x \neq 4$. State any restrictions on the domain of $f^{-1}(x)$. $f^{-1}(x) = \dfrac{4x + 5}{x - 24}, x \neq 24$ [4.1]

3. Evaluate $f(x) = e^x$, for $x = -2.4$. Round to the nearest ten-thousandth. 0.0907 [4.2]

4. Write $\ln x = 6$ in exponential form. $e^6 = x$ [4.3]

5. Graph $f(x) = \log_3(x + 3)$. Answer on page AA18. [4.3]

6. Expand $\ln\left(\dfrac{xy^3}{e^2}\right)$. Assume x and y are positive real numbers.
$\ln x + 3\ln y - 2$ [4.4]

7. Write $\log_3 x^4 - 2\log_3 z + \log_3(xy^2)$ as a single logarithm with a coefficient of 1. Assume all variables are positive real numbers. $\log_3\left(\dfrac{x^5y^2}{z^2}\right)$ [4.4]

8. Use the change-of-base formula to evaluate $\log_8 411$. Round to the nearest ten-thousandth. 2.8943 [4.4]

9. What is the Richter scale magnitude of an earthquake with an intensity of $789{,}251I_0$? Round to the nearest tenth. 5.9 [4.4]

10. How many times as great is the intensity of an earthquake that measures 7.9 on the Richter scale than the intensity of an earthquake that measures 5.1 on the Richter scale?
$10^{2.8} \approx 631$ times as great [4.4]

SECTION 4.5

Solving Exponential Equations
Solving Logarithmic Equations

Exponential and Logarithmic Equations

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A27.

PS1. Use the definition of a logarithm to write the exponential equation $3^6 = 729$ in logarithmic form. [4.3] $\log_3 729 = 6$

PS2. Use the definition of a logarithm to write the logarithmic equation $\log_5 625 = 4$ in exponential form. [4.3] $5^4 = 625$

PS3. Use the definition of a logarithm to write the exponential equation $a^{x+2} = b$ in logarithmic form. [4.3] $\log_a b = x + 2$

PS4. Solve for x: $4a = 7bx + 2cx$ [1.2] $x = \dfrac{4a}{7b + 2c}$

PS5. Solve for x: $165 = \dfrac{300}{1 + 12x}$ [1.4] $x = \dfrac{3}{44}$

PS6. Solve for x: $A = \dfrac{100 + x}{100 - x}$ [1.4] $x = \dfrac{100(A - 1)}{A + 1}$

Solving Exponential Equations

If a variable appears in the exponent of a term of an equation, such as in $2^{x+1} = 32$, then the equation is called an **exponential equation**. Example 1 uses the following Equality of Exponents Theorem to solve $2^{x+1} = 32$.