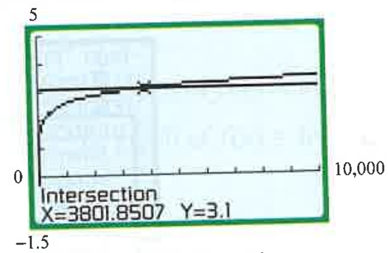
b. Graph $s(x) = 0.37 \ln x + 0.05$ and $s = 3.1$ in the same viewing window.



Xscl = 1000 Yscl = 1

The x value of the intersection point represents the population in thousands. The graphs indicate that a city with an average pedestrian walking speed of 3.1 miles per hour should have a population of about 3,800,000.

▶ Try Exercise 90, page 369

Answers to Exercises 47–54, 69–76, and 79–88 are on pages AA16–AA17.

EXERCISE SET 4.3

Concept Check

1. Is the function $f(x) = \log_b x$ a one-to-one function? Yes
2. Name two characteristics of the graph of $f(x) = \log_b x$, $b > 1$. Answers will vary.
3. What is the relationship between the graphs of $f(x) = \log_b x$ and $g(x) = b^x$? The graph of f is a reflection of the graph of g across the line given by $y = x$.
4. Explain how to use the graph of $f(x) = \log_b x$ to produce the graph of $f(x) = \log_b(x + 5)$. Shift the graph of f horizontally to the left 5 units.

In Exercises 5 to 16, write each equation in its exponential form.

5. $1 = \log 10$ $\quad 10^1 = 10$
6. $4 = \log 10{,}000$ $\quad 10^4 = 10{,}000$
7. $2 = \log_8 64$ $\quad 8^2 = 64$
8. $3 = \log_4 64$ $\quad 4^3 = 64$
9. $\log_5 125 = 3$ $\quad 5^3 = 125$
10. $-3 = \log_4 \frac{1}{64}$ $\quad 4^{-3} = \frac{1}{64}$
11. $\ln x = 7$ $\quad e^7 = x$
12. $\ln 5 = x$ $\quad e^x = 5$
13. $\ln 1 = 0$ $\quad e^0 = 1$
14. $\ln x = -3$ $\quad e^{-3} = x$
15. $3 = \log(5x + 2)$ $\quad 10^3 = 5x + 2$
16. $4 = \ln(3x + 1)$ $\quad e^4 = 3x + 1$

In Exercises 17 to 28, write each equation in its logarithmic form. Assume $y > 0$ and $b > 0$.

17. $3^2 = 9$ $\quad \log_3 9 = 2$
18. $5^3 = 125$ $\quad \log_5 125 = 3$
19. $7^2 = 49$ $\quad \log_7 49 = 2$
20. $2^5 = 32$ $\quad \log_2 32 = 5$
21. $b^x = y$ $\quad \log_b y = x$
22. $2^x = y$ $\quad \log_2 y = x$
23. $y = e^x$ $\quad \ln y = x$
24. $5^1 = 5$ $\quad \log_5 5 = 1$
25. $100 = 10^2$ $\quad \log 100 = 2$
26. $2^{-4} = \frac{1}{16}$ $\quad \log_2 \frac{1}{16} = -4$
27. $e^5 = 3x + 1$ $\quad 5 = \ln(3x + 1)$
28. $5^{x+2} = 612$ $\quad x + 2 = \log_5 612$

In Exercises 29 to 46, evaluate each logarithmic expression. Do not use a calculator.

29. $\log_4 16$ $\quad 2$
30. $\log_{3/2} \frac{8}{27}$ $\quad -3$
31. $\log_3 \frac{1}{243}$ $\quad -5$
32. $\log_b 1$ $\quad 0$
33. $\log_6 1296$ $\quad 4$
34. $\log_8 \left(\frac{1}{512}\right)$ $\quad -3$
35. $\log \frac{1}{100}$ $\quad -2$
36. $\log_{10}(10^6)$ $\quad 6$
37. $\log_{0.5} 16$ $\quad -4$
38. $\log_{0.3} \frac{100}{9}$ $\quad -2$
39. $4 \log 1000$ $\quad 12$
40. $\log_5 125^2$ $\quad 6$
41. $3 \log_7 49$ $\quad 6$
42. $2 \log_8 64$ $\quad 4$
43. $\log \sqrt{1000}$ $\quad \frac{3}{2}$
44. $10^{\log_{10} 7}$ $\quad 7$
45. $\ln e^4$ $\quad 4$
46. $\log_{12} \frac{1}{144}$ $\quad -2$

In Exercises 47 to 54, graph each function by using its exponential form.

47. $f(x) = \log_4 x$
48. $f(x) = \log_6 x$
49. $f(x) = \log_{12} x$
50. $f(x) = \log_8 x$
51. $f(x) = \log_{1/2} x$
52. $f(x) = \log_{1/4} x$
53. $f(x) = \log_{5/2} x$
54. $f(x) = \log_{7/3} x$

Indicates Try It Exercises

In Exercises 55 to 68, find the domain of the function. Write the domain using interval notation.

55. $f(x) = \log_5(x - 3)$ $(3, \infty)$

56. $k(x) = \log_4(5 - x)$ $(-\infty, 5)$

57. $k(x) = \log_{2/3}(11 - x)$ $(-\infty, 11)$

58. $H(x) = \log_{1/4}(x^2 + 1)$ $(-\infty, \infty)$

59. $P(x) = \ln(x^2 - 4)$ $(-\infty, -2) \cup (2, \infty)$

60. $J(x) = \ln\left(\dfrac{x - 3}{x}\right)$ $(-\infty, 0) \cup (3, \infty)$

61. $h(x) = \ln\left(\dfrac{x^2}{x - 4}\right)$ $(4, \infty)$

62. $R(x) = \ln(x^4 - x^2)$ $(-\infty, -1) \cup (1, \infty)$

63. $N(x) = \log_2(x^3 - x)$ $(-1, 0) \cup (1, \infty)$

64. $s(x) = \log_7(x^2 + 7x + 10)$ $(-\infty, -5) \cup (-2, \infty)$

65. $g(x) = \log\sqrt{2x - 11}$ $\left(\frac{11}{2}, \infty\right)$

66. $m(x) = \log|4x - 8|$ $(-\infty, 2) \cup (2, \infty)$

67. $t(x) = 2\ln(3x - 7)$ $\left(\frac{7}{3}, \infty\right)$

68. $v(x) = \ln(x - 4)^2$ $(-\infty, 4) \cup (4, \infty)$

In Exercises 69 to 76, explain how to use the graph of the first function to produce the graph of the second function.

69. $f(x) = \log_4 x$; $f(x) = \log_4(x - 3)$

70. $f(x) = \log_6 x$; $f(x) = \log_6(x + 3)$

71. $f(x) = \log_{12} x$; $f(x) = \log_{12} x + 2$

72. $f(x) = \log_8 x$; $f(x) = \log_8 x - 4$

73. $f(x) = \log_{1/2} x$; $f(x) = 3 + \log_{1/2} x$

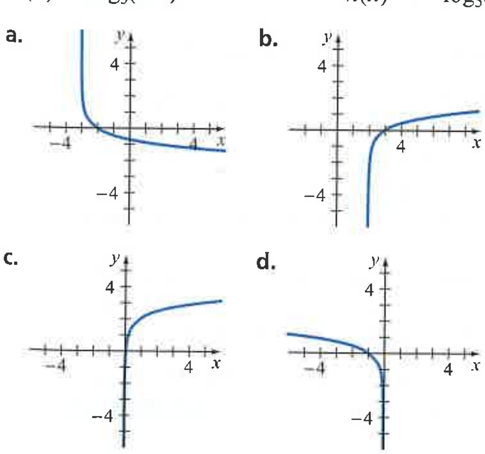
74. $f(x) = \log_{1/4} x$; $f(x) = 2 + \log_{1/4} x$

75. $f(x) = \log_{5/2} x$; $f(x) = 1 + \log_{5/2}(x - 4)$

76. $f(x) = \log_{7/3} x$; $f(x) = \log_{7/3}(x - 3) - 1$

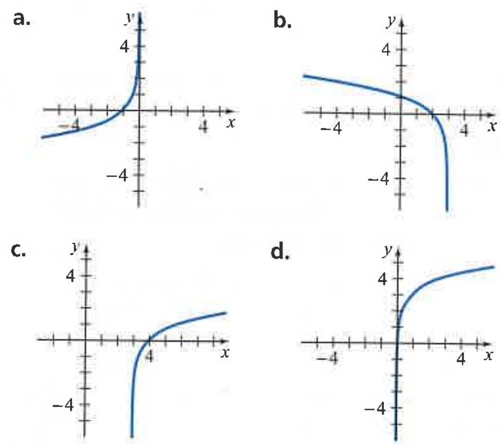
In Exercises 77 and 78, examine the four functions and the graphs labeled a, b, c, and d. For each graph, determine which function has been graphed.

77. $f(x) = \log_5(x - 2)$ $g(x) = 2 + \log_5 x$
$h(x) = \log_5(-x)$ $k(x) = -\log_5(x + 3)$



a. $k(x)$
b. $f(x)$
c. $g(x)$
d. $h(x)$

78. $f(x) = \ln x + 3$ $g(x) = \ln(x - 3)$
$h(x) = \ln(3 - x)$ $k(x) = -\ln(-x)$



a. $k(x)$
b. $h(x)$
c. $g(x)$
d. $f(x)$

In Exercises 79 to 88, use a graphing utility to graph the function.

79. $f(x) = -2\ln x$

80. $f(x) = -\log x$

81. $f(x) = |\ln x|$

82. $f(x) = \ln|x|$

83. $f(x) = \log\sqrt[3]{x}$

84. $f(x) = \ln\sqrt{x}$

85. $f(x) = \log(x + 10)$

86. $f(x) = \ln(x + 3)$

87. $f(x) = 3\log|2x + 10|$

88. $f(x) = \frac{1}{2}\ln|x - 4|$

89. **Money Market Rates** The function

$$r(t) = 0.69607 + 0.60781\ln t$$

gives the annual interest rate r, as a percent, a bank will pay on its money market accounts, where t is the term (the time the money is invested) in months.

a. What interest rate, to the nearest tenth of a percent, will the bank pay on a money market account with a term of 9 months? 2.0%

b. What is the minimum number of complete months during which a person must invest to receive an interest rate of at least 3%? 45 months

90. **Average Typing Speed** The following function models the average typing speed S, in words per minute, of a student who has been typing for t months.

$$S(t) = 5 + 29\ln(t + 1), \quad 0 \le t \le 16$$

a. What was the student's average typing speed, to the nearest word per minute, when the student first started to type? What was the student's average typing speed, to the nearest word per minute, after 3 months?
5 words/min; 45 words/min

b. Use a graph of S to determine how long, to the nearest tenth of a month, it will take the student to achieve an average typing speed of 65 words per minute. 6.9 months

91. **Advertising Costs and Sales** The function

$$N(x) = 2750 + 180 \ln\left(\frac{x}{1000} + 1\right)$$

models the relationship between the dollar amount x spent on advertising a product and the number of units N that a company can sell.

a. Find the number of units that will be sold with advertising expenditures of \$20,000, \$40,000, and \$60,000. 3298 units; 3418 units; 3490 units

b. How many units will be sold if the company does not pay to advertise the product? 2750 units

92. **A Horse's Age in Human Years** Many infants start to walk when they are approximately a year old. Most horses are able to run within a few hours after birth. The fact that horses mature much more quickly than humans makes it difficult to estimate the age of a horse in human years during the horse's first year. However, the following function models the age, in human years h, of a horse that is x years old, for $1 < x < 24$.

$$h(x) = -2.18 + 19.27 \ln x$$

Use h to:

a. Estimate the age, in human years, of a 16-year-old horse. Round to the nearest year. 51 years

b. Estimate the age of a horse whose age in human years is 32 years. Round to the nearest year. 6 years

93. **Men's 100-Meter Freestyle** The function

$$t(x) = 76.42 - 6.25 \ln x$$

models the world record time t, in seconds, in the men's 100-meter freestyle for the years from 1910 to 2010, where x is the number of years after 1910.

Use t to :

a. Estimate the world record time in 2000 ($x = 90$). Round to the nearest hundredth of a second. 48.30 s

b. Predict the world record time in 2030 ($x = 120$). Round to the nearest hundredth of a second. 46.50 s

Enrichment Exercises

94. **Astronomy** Astronomers measure the apparent brightness of a star by a unit called the **apparent magnitude**. This unit was created in the second century B.C. when the Greek astronomer Hipparchus classified the relative brightness of several stars. In his list, he assigned the number 1 to the stars that appeared to be the brightest (Sirius, Vega, and Deneb). They are first-magnitude stars. Hipparchus assigned the number 2 to all the stars in the Big Dipper. They are second-magnitude stars. The following table shows the relationship between a star's brightness relative to a first-magnitude star and the star's apparent magnitude. Notice from the table that a first-magnitude star appears to be about 2.51 times as bright as a second-magnitude star.

Brightness x, relative to a first-magnitude star	Apparent magnitude $M(x)$
1	1
$\frac{1}{2.51}$	2
$\frac{1}{6.31} \approx \frac{1}{2.51^2}$	3
$\frac{1}{15.85} \approx \frac{1}{2.51^3}$	4
$\frac{1}{39.82} \approx \frac{1}{2.51^4}$	5
$\frac{1}{100} \approx \frac{1}{2.51^5}$	6

The following logarithmic function gives the apparent magnitude $M(x)$ of a star as a function of its brightness x.

$$M(x) = -2.51 \log x + 1, \quad 0 < x \le 1$$

a. Use $M(x)$ to find the apparent magnitude of a star that is $\frac{1}{10}$ as bright as a first-magnitude star. Round to the nearest hundredth. 3.51

b. Find the apparent magnitude of a star that is $\frac{1}{400}$ as bright as a first-magnitude star. Round to the nearest hundredth. 7.53

c. Which star appears brighter: a star with an apparent magnitude of 12 or a star with an apparent magnitude of 15? Apparent magnitude of 12

d. Is $M(x)$ an increasing function or a decreasing function? Decreasing function

95. **Number of Digits in b^x** The number N of digits in the expansion of b^x, where both b and x are positive integers, is $N = \text{int}(x \log b) + 1$, where $\text{int}(x \log b)$ denotes the greatest integer of $x \log b$. (*Note:* See pages 174–176 for information on the greatest integer function.)

a. Because $2^{10} = 1024$, we know that 2^{10} has four digits. Use the equation $N = \text{int}(x \log b) + 1$ to verify this result. Answers will vary.

b. Find the number of digits in 3^{200}. 96 digits

c. Find the number of digits in 7^{4005}. 3385 digits

d. The largest known prime number as of January 25, 2013, was $2^{57,885,161} - 1$. Find the number of digits in this prime number. (*Hint:* Because $2^{57,885,161}$ is not a power of 10, both $2^{57,885,161}$ and $2^{57,885,161} - 1$ have the same number of digits.) 17,425,170 digits

96. **Number of Digits in $9^{(9^9)}$** A science teacher has offered 10 points extra credit to any student who will write out all the digits in the expansion of $9^{(9^9)}$.

a. Use the formula from Exercise 95 to determine the number of digits in this number. 369,693,100 digits

b. Assume that you can write 1000 digits per page and that 500 sheets of paper are in a ream of paper. How many reams of paper, to the nearest tenth of a ream, are required to write out the expansion of $9^{(9^9)}$? Assume that you write on only one side of each sheet. 739.4 reams

97. Use a graphing utility to graph $f(x) = \frac{e^x - e^{-x}}{2}$ and $g(x) = \ln(x + \sqrt{x^2 + 1})$ on the same screen. Use a square viewing window. What appears to be the relationship between f and g? *f* and *g* are inverse functions.

98. Use a graphing utility to graph $f(x) = \frac{e^x + e^{-x}}{2}$ for $x \geq 0$ and $g(x) = \ln(x + \sqrt{x^2 - 1})$ for $x \geq 1$ on the same screen. Use a square viewing window. What appears to be the relationship between f and g? *f* and *g* are inverse functions.

99. Use a graph of $f(x) = \frac{2}{e^x + e^{-x}}$ to determine the domain and range of f.
Domain: all real numbers; range: $\{y \mid 0 < y \leq 1\}$

SECTION 4.4

Properties of Logarithms
Change-of-Base Formula
Logarithmic Scales

Properties of Logarithms and Logarithmic Scales

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A26.

In Exercises PS1 to PS6, use a calculator to compare the values of the given expressions.

PS1. $\log 3 + \log 2$; $\log 6$ [4.3] ≈ 0.77815 for each expression

PS2. $\ln 8 - \ln 3$; $\ln\left(\frac{8}{3}\right)$ [4.3] ≈ 0.98083 for each expression

PS3. $3 \log 4$; $\log(4^3)$ [4.3] ≈ 1.80618 for each expression

PS4. $2 \ln 5$; $\ln(5^2)$ [4.3] ≈ 3.21888 for each expression

PS5. $\ln 5$; $\frac{\log 5}{\log e}$ [4.3] ≈ 1.60944 for each expression

PS6. $\log 8$; $\frac{\ln 8}{\ln 10}$ [4.3] ≈ 0.90309 for each expression

Properties of Logarithms

INSTRUCTOR NOTE
Stress the importance of the properties of logarithms. An understanding of these properties is essential to the study and use of logarithms.

In Section 4.3 we introduced the following basic properties of logarithms.

$$\log_b b = 1 \quad \text{and} \quad \log_b 1 = 0$$

Also, because exponential functions and logarithmic functions are inverses of each other, we observed the relationships

$$\log_b(b^x) = x \quad \text{and} \quad b^{\log_b x} = x$$

We can use the properties of exponents to establish the following additional logarithmic properties.

Caution

Pay close attention to these properties. Note that

$$\log_b(MN) \neq \log_b M \cdot \log_b N$$

and

$$\log_b \frac{M}{N} \neq \frac{\log_b M}{\log_b N}$$

Also,

$$\log_b(M + N) \neq \log_b M + \log_b N$$

In fact, the expression $\log_b(M + N)$ cannot be expanded.

Properties of Logarithms

In the following properties, b, M, and N are positive real numbers ($b \neq 1$).

Product property	$\log_b(MN) = \log_b M + \log_b N$
Quotient property	$\log_b \frac{M}{N} = \log_b M - \log_b N$
Power property	$\log_b(M^p) = p \log_b M$
Logarithm-of-each-side property	$M = N$ implies $\log_b M = \log_b N$
One-to-one property	$\log_b M = \log_b N$ implies $M = N$

A proof of the product property is given on the next page.