4.2 WS 2

Key

Evaluate the exponential function for the given x values.

1. $f(x) = 5^x$ a. $x = 4$ 625 b. $x = -3$ $\frac{1}{125}$

2. $g(x) = 7^x$ a. $x = 4$ 2401 b. $x = -2$ $\frac{1}{49}$

3. $f(x) = \left(\frac{2}{3}\right)^x$ a. $x = -3$ $\frac{27}{8}$ b. $x = 2$ $\frac{4}{9}$

4. $g(x) = \left(\frac{1}{4}\right)^x$ a. $x = -2$ 16 b. $x = 4$ $\frac{1}{256}$

Use a calculator to evaluate the exponential function for the given x value. Round to the nearest hundredth.

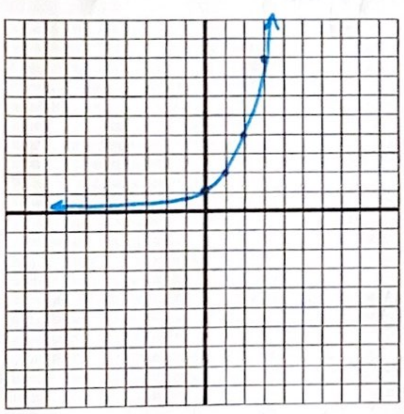
5. $f(x) = 6^x$; $x = 2.5$ 88.18

6. $h(x) = e^x$; $x = \sqrt{2}$ 4.11

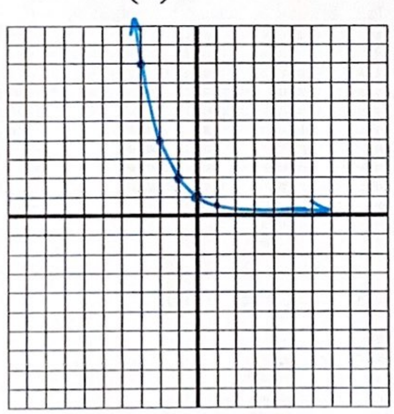
7. $g(x) = 4.6^x$; $x = -3$ 0.01

Sketch the graph of each function.

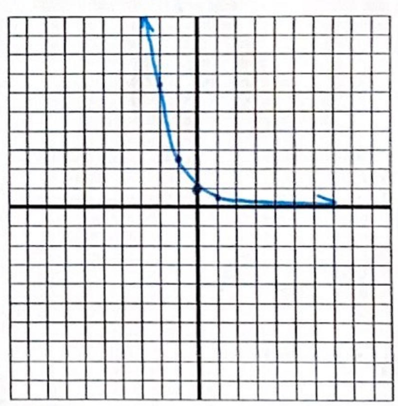
8. $f(x) = 2^x$



9. $g(x) = \left(\frac{1}{2}\right)^x$



10. $h(x) = \left(\frac{2}{5}\right)^x$



Explain how to use the graph of the first function f to produce the graph of the second function F.

11. $f(x) = 2^x; F(x) = 2^{x+7}$ Shift left 7 units

12. $f(x) = 4^x; F(x) = 4^x - 4$ Shift down 4 units

13. $f(x) = (3)^x; F(x) = 5(3)^x$ Vertical stretch away from the x-axis by a factor of 5.

14. $f(x) = \left(\frac{1}{3}\right)^x; F(x) = \left(\frac{1}{3}\right)^{-x}$ Reflection across the y-axis

15. The monthly income I, in dollars, from a new product is given by $I(t) = 8600 - 5500e^{-0.005t}$ where t is the time, in months, since the product was first put on the market.

a. What was the monthly income after the 10^{th} month and after the 100^{th} month?

$I(10) = 3368.24$

$I(100) = 5264.08$

b. What will the monthly income from the product approach as the time increases without bound?

\$8600

Use the composition of functions to determine whether f and g are inverses of one another.

16. $f(x) = \frac{4}{-x-2} + 2$; $g(x) = -\frac{1}{x+3}$

$$g(f(x)) = -\frac{1}{\frac{4}{-x-2}+3} = \frac{-1}{\frac{4+3(-x-2)}{-x-2}} = \frac{-1}{\frac{4-3x-6}{-x-2}}$$

$$\frac{-1}{\frac{-3x-2}{-x-2}} = -1 \cdot \frac{-x-2}{-3x-2}$$

$\boxed{\text{No}}$

17. $f(x) = \frac{x+7}{2}$; $g(x) = 2x - 7$

$$g(f(x)) = 2\left(\frac{x+7}{2}\right) - 7 = x+7-7 = x$$

$$f(g(x)) = \frac{2x-7+7}{2} = \frac{2x}{2} = x$$

$\boxed{\text{yes}}$

Find the inverse of each function, then state the domain and range of $f^{-1}(x)$.

18. $f(x) = \sqrt[3]{2x-4}$

$y^3 = 2x - 4$

$y^3 + 4 = 2x$

$\frac{1}{2}y^3 + 2 = x$

$f^{-1}(x) = \frac{1}{2}x^3 + 2$

D_x of $f^{-1}(x)$: $(-\infty, \infty)$

R_y of $f^{-1}(x)$: $(-\infty, \infty)$

19. $f(x) = \sqrt{x+8}$

$y^2 = x + 8$

$y^2 - 8 = x$

$f^{-1}(x) = x^2 - 8$

D_x of $f^{-1}(x)$: $[0, \infty)$

R_y of $f^{-1}(x)$: $[-8, \infty)$

$x \geq -8$

20. $f(x) = \frac{x-9}{x}$

$yx = x - 9$

$yx - x = -9$

$x(y-1) = -9$

$x = \frac{-9}{y-1}$

$f^{-1}(x) = \frac{-9}{x-1}$

D_x of $f^{-1}(x)$: $(-\infty, 1) \cup (1, \infty)$

R_y of $f^{-1}(x)$: $(-\infty, 0) \cup (0, \infty)$