4.1 WS

KEY

Use composition of functions to determine whether *f* and *g* are inverses of one another.

1. $f(x) = 2x + 7;\ g(x) = \frac{1}{2}x - \frac{7}{2}$

$f(g(x)) = 2\left(\frac{1}{2}x - \frac{7}{2}\right) + 7$

$= x - 7 + 7$

$= x$

$g(f(x)) = \frac{1}{2}(2x + 7) - \frac{7}{2}$

$= x + \frac{7}{2} - \frac{7}{2}$

$= x$

$\boxed{\text{yes}}$

2. $f(x) = -\frac{1}{2}x - \frac{1}{2};\ g(x) = -2x + 1$

$f(g(x)) = \frac{-1}{2}(-2x + 1) - \frac{1}{2}$

$= x - \frac{1}{2} - \frac{1}{2}$

$= x - 1$

$\boxed{\text{No}}$

3. $f(x) = x^3 + 2;\ g(x) = \sqrt[3]{x - 2}$

$f(g(x)) = \left(\sqrt[3]{x-2}\right)^3 + 2$

$= x - 2 + 2$

$= x$

$g(f(x)) = \sqrt[3]{x^3 + 2 - 2}$

$= \sqrt[3]{x^3}$

$= x$

$\boxed{\text{yes}}$

4. $f(x) = \frac{5}{x - 3};\ g(x) = \frac{5}{x} + 3$

$f(g(x)) = \dfrac{5}{\frac{5}{x} + 3 - 3}$

$= \dfrac{5}{\frac{5}{x}} = 5\left(\frac{x}{5}\right) = x$

$g(f(x)) = \dfrac{5}{\frac{5}{x-3}} + 3$

$= 5\left(\frac{x-3}{5}\right) + 3$

$= x - 3 + 3$

$= x$

$\boxed{\text{yes}}$

Find $f^{-1}(x)$. State the domain and range of $f^{-1}(x)$.

5. $f(x) = 2x + 4$

D_x of $f(x)$: $(-\infty, \infty)$

$y = 2x + 4$

$y - 4 = 2x$

$x = \frac{y-4}{2}$

$f^{-1}(x) = \frac{1}{2}x - 2$

D_x of $f^{-1}(x)$: $(-\infty, \infty)$

R_y of $f^{-1}(x)$: $(-\infty, \infty)$

6. $f(x) = -3x - 8$

D_x of $f(x) = (-\infty, \infty)$

$y = -3x - 8$

$y + 8 = -3x$

$x = \frac{y+8}{-3}$

$f^{-1}(x) = \frac{-1}{3}x - \frac{8}{3}$

D_x of $f^{-1}(x)$: $(-\infty, \infty)$

R_y of $f^{-1}(x)$: $(-\infty, \infty)$

7. $f(x) = -\frac{1}{2}x - \frac{3}{4}$

D_x of $f(x)$: $(-\infty, \infty)$

$y = \frac{-1}{2}x - \frac{3}{4}$

$y + \frac{3}{4} = \frac{1}{2}x$

$x = -2y - \frac{3}{2}$

$f^{-1}(x) = -2x - \frac{3}{2}$

D_x of $f^{-1}(x)$: $(-\infty, \infty)$

R_y of $f^{-1}(x)$: $(-\infty, \infty)$

8. $f(x) = \frac{2x}{x-1}$

D_x of $f(x)$: $\{x \mid x \neq 1\}$

$xy - y = 2x$

$-y = 2x - xy$

$-y = x(2 - y)$

$f^{-1}(x) = \frac{-x}{2-x}$

D_x of $f^{-1}(x)$: $\{x \mid x \neq 2\}$

R_y of $f^{-1}(x)$: $\{y \mid y \neq 1\}$

9. $f(x) = \frac{x-1}{x+1}$

D_x of $f(x)$: $\{x \mid x \neq -1\}$

$y = \frac{x-1}{x+1}$

$xy + y = x - 1$

$xy - x = -y - 1$

$x(y - 1) = -y - 1$

$x = \frac{-y-1}{y-1}$

$f^{-1}(x) = \frac{-x-1}{x-1}$

D_x of $f^{-1}(x)$: $\{x \mid x \neq 1\}$

R_y of $f^{-1}(x)$: $\{y \mid y \neq -1\}$

10. $f(x) = x^2 + 1$, $x \geq 0$

D_x of $f(x)$: $\{x \mid x \geq 0\}$

R_y of $f(x)$: $\{y \mid y \geq 1\}$

$y = x^2 + 1$

$\sqrt{y-1} = \sqrt{x^2}$

$x = \sqrt{y-1}$

$f^{-1}(x) = \sqrt{x-1}$

D_x of $f^{-1}(x)$: $\{x \mid x \geq 1\}$

R_y of $f^{-1}(x)$: $\{y \mid y \geq 0\}$