

# 4.1 WS 3

# KEY

Use the composition of functions to determine whether  $f$  and  $g$  are inverses of one another.

1.  $f(x) = -2x + 5$ ;  $g(x) = \frac{x-5}{-2}$

$$f(g(x)) = -2\left(\frac{x-5}{-2}\right) + 5 = x - 5 + 5 = x$$

$$g(f(x)) = \frac{-2x + 5 - 5}{-2} = \frac{-2x}{-2} = x$$

yes

2.  $f(x) = \sqrt[3]{x-5}$ ;  $g(x) = (x+5)^3$

$$f(g(x)) = \sqrt[3]{(x+5)^3 - 5} = x + 5 - 5 = x$$

$$g(f(x)) = ((\sqrt[3]{x-5} + 5)^3) = (\sqrt[3]{x-5})^3 = x$$

yes

3.  $f(x) = 4x + 1$ ;  $g(x) = \frac{1}{2}x + \frac{1}{4}$

$$f(g(x)) = 4\left(\frac{1}{2}x + \frac{1}{4}\right) + 1 = 2x + 1 + 1 = 2x + 2$$

No

4.  $f(x) = \frac{2x}{x-1}$ ;  $g(x) = \frac{x}{x-3}$

$$f(g(x)) = \frac{2\left(\frac{x}{x-3}\right)}{\frac{x}{x-3} - \frac{x-3}{x-3}} = \frac{\frac{2x}{x-3}}{\frac{3}{x-3}} = \frac{2x}{x-3} \left(\frac{x-3}{3}\right) = \frac{2}{3}x$$

No

Find the inverse function of the one-to-one function given.

5.  $f(x) = \{(-3, 1), (-2, 2), (1, 5), (4, -7)\}$

$$f^{-1}(x) = \{(1, -3), (2, -2), (5, 1), (-7, 4)\}$$

6.  $g(x) = \{(0, 1), (1, 2), (2, 4), (3, 8), (4, 16)\}$

$$g^{-1}(x) = \{(1, 0), (2, 1), (4, 2), (8, 3), (16, 4)\}$$

Find the inverse of each function, then state the domain and range of  $f^{-1}(x)$ .

7.  $f(x) = 4x - 8$

$$\frac{y+8}{4} = x$$

$$f^{-1}(x) = \frac{1}{4}x + 2$$

$$D_x \text{ of } f^{-1}(x) = (-\infty, \infty)$$

$$R_y \text{ of } f^{-1}(x) = (-\infty, \infty)$$

8.  $f(x) = \frac{x}{x-2}$

$$yx - 2y = x$$

$$-2y = x - yx$$

$$-2y = x(1-y)$$

$$x = \frac{-2y}{1-y}$$

$$f^{-1}(x) = \frac{-2x}{1-x}$$

$$D_x \text{ of } f^{-1}(x) = (-\infty, 1) \cup (1, \infty)$$

$$R_y \text{ of } f^{-1}(x) = (-\infty, 2) \cup (2, \infty)$$

9.  $f(x) = \sqrt{x-2}$

$$y^2 = x - 2$$

$$y^2 + 2 = x$$

$$f^{-1}(x) = x^2 + 2$$

$$D_x \text{ of } f^{-1}(x) = [0, \infty)$$

$$R_y \text{ of } f^{-1}(x) = [2, \infty)$$

10.  $f(x) = \frac{x-7}{2x-1}$

$$2xy - y = x - 7$$

$$-y + 7 = x - 2xy$$

$$-y + 7 = x(1-2y)$$

$$f^{-1}(x) = \frac{-x+7}{1-2x}$$

$$D_x \text{ of } f^{-1}(x) = (-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$$

$$R_y \text{ of } f^{-1}(x) = (-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$$

11. The function  $K(x) = 1.3x - 4.7$  converts a men's shoe size in the United States to the equivalent shoe size in the United Kingdom. Determine the function  $K^{-1}(x)$  that can be used to convert a U.K. men's shoesize to its equivalent U.S. shoe size.

$$y = 1.3x - 4.7$$

$$y + 4.7 = 1.3x$$

$$\frac{y + 4.7}{1.3} = x$$

$$K^{-1}(x) = \frac{x + 4.7}{1.3}$$

12. A catering service uses the function  $c(x) = \frac{300 + 12x}{x}$  to determine the amount, in dollars, it charges per person for a sit-down dinner; where  $x$  is the number people in attendance.

a. Find  $c(30)$  and explain what it represents.  $c(30) = \frac{300 + 12(30)}{30} = 22 \text{ people}$

b. Find  $c^{-1}(x)$ .  $yx = 300 + 12x$   $x = \frac{300}{y-12}$   
 $yx - 12x = 300$   
 $x(y-12) = 300$   
 $c^{-1}(x) = \frac{300}{x-12}$

c. Use to determine how many people attended a dinner for which the cost person was \$15.

$$c^{-1}(x) = \frac{300}{15-12} = 100 \text{ people}$$