

# 3.5 WS

KEY

Determine the vertical asymptotes of each rational function.

1.  $F(x) = \frac{2x-1}{x^2+3x} = \frac{2x-1}{x(x+3)}$

VA:  $x=0$  &  $x=-3$

2.  $F(x) = \frac{x^2+11}{6x^2-5x-4}$

$F(x) = \frac{x^2+11}{(3x-4)(2x+1)}$

VA:  $x = \frac{4}{3}$  &  $x = -\frac{1}{2}$

3.  $G(x) = \frac{5x^2-3}{4x^3-25x^2+6x}$

$x(4x^2-25x+6) = 0$   
 AC  
 24  
 $\frac{-24 \pm \sqrt{24^2 - 4(4)(6)}}{2(4)}$   
 $\frac{-24 \pm \sqrt{576 - 96}}{8}$   
 $\frac{-24 \pm \sqrt{480}}{8}$   
 $\frac{-24 \pm 4\sqrt{30}}{8}$   
 $\frac{-6 \pm \sqrt{30}}{2}$

VA:  $x=0, x=6, \text{ and } x = \frac{1}{4}$

4.  $H(x) = \frac{2x}{3x^2+5}$

$3x^2+5=0$   
 $x = \frac{0 \pm \sqrt{0-4(3)(5)}}{2(3)}$   
 $x = \frac{0 \pm \sqrt{-60}}{6}$

VA: NONE

Determine horizontal asymptotes of each rational function.

5.  $P(x) = \frac{4x^2+1}{x^2+x+1}$

HA:  $y=4$

6.  $H(x) = \frac{3x^3-27x^2+5x-11}{x^3-2x^3+7}$

HA:  $y=0$

7.  $F(x) = \frac{4x^2-11x+6}{4-x+\frac{1}{3}x^2}$

$y = \frac{4x^2}{\frac{1}{3}x^2} = \frac{4}{\frac{1}{3}} = 4(3) = 12$

HA:  $y=12$

Determine the vertical and horizontal asymptotes of each rational function.

8.  $F(x) = \frac{x}{x+4}$

VA:  $x=-4$   
 HA:  $y=1$

9.  $G(x) = \frac{1}{x^2-9} = \frac{1}{(x+3)(x-3)}$

VA:  $x=-3$  &  $x=3$   
 HA:  $y=0$

10.  $F(x) = \frac{x^2+x+4}{x^2+2x-1}$

$F(x) = \frac{x^2+x+4}{x^2+2x-1}$   
 $x^2+2x-1=0$   
 $x = \frac{-2 \pm \sqrt{4-4(1)(-1)}}{2}$   
 $x = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2}$

VA:  $x = -1 \pm \sqrt{2}$   
 HA:  $y=1$

11.  $P(x) = \frac{10}{x^2+2}$

$x^2+2=0$   
 $\sqrt{x^2} = \sqrt{-2}$   
 $x = \pm i\sqrt{2}$

VA: NONE  
 HA:  $y=0$

Find the slant asymptote of each rational function.

12.  $F(x) = \frac{3x^2 + 5x - 1}{x + 4}$

$$\begin{array}{r}
 x+4 \overline{) 3x^2 + 5x - 1} \\
 \underline{-3x^2 - 12x} \phantom{-1} \\
 7x - 1 \\
 \underline{-7x + 28} \\
 27
 \end{array}$$

SA:  $y = 3x - 7$

13.  $H(x) = \frac{x^3 - 2x^2 + 3x + 4}{x^2 - 3x + 5}$

$$\begin{array}{r}
 x^2 - 3x + 5 \overline{) x^3 - 2x^2 + 3x + 4} \\
 \underline{-x^3 + 3x^2 - 5x} \phantom{+4} \\
 5x^2 - 2x + 4 \\
 \underline{-5x^2 + 15x - 25} \\
 x - 1
 \end{array}$$

SA:  $y = x + 1$

14.  $P(x) = \frac{x^3 - 1}{x^2}$

$$\begin{array}{r}
 x^2 \overline{) x^3 - 1} \\
 \underline{-x^3} \\
 -1
 \end{array}$$

SA:  $y = x$

Use the given zero to find the remaining zeros of each polynomial function.

15.  $P(x) = x^3 + 3x^2 + x + 3$ ;  $-i$   $i$

$(x+i)(x-i)$

$x^2 - i^2$

$x^2 + 1$

$$\begin{array}{r}
 x^2 + 1 \overline{) x^3 + 3x^2 + x + 3} \\
 \underline{-x^3} \phantom{+3} \\
 3x^2 + x + 3 \\
 \underline{-3x^2 + 3} \\
 x + 3 \\
 \underline{-x - 3} \\
 0
 \end{array}$$

$x + 3 = 0$

$x = -3$

$x = i, -3$

16.  $H(x) = x^5 - 6x^4 + 22x^3 - 64x^2 + 117x - 90$ ;  $3i$   $-3i$

$(x-3i)(x+3i)$

$x^2 - 9i^2$

$x^2 + 9$

$$\begin{array}{r}
 x^2 + 9 \overline{) x^5 - 6x^4 + 22x^3 - 64x^2 + 117x - 90} \\
 \underline{-x^5} \phantom{+9x^4} \\
 9x^4 + 22x^3 - 64x^2 + 117x - 90 \\
 \underline{-9x^4} \phantom{+22x^3} \\
 13x^3 - 10x^2 + 117x - 90 \\
 \underline{-13x^3} \phantom{+10x^2} \\
 -10x^2 - 90 \\
 \underline{+10x^2 + 90} \\
 0
 \end{array}$$

$x^3 - 6x^2 + 13x - 10 = 0$

$$\begin{array}{r}
 2 \overline{) 1 \quad -6 \quad 13 \quad -10} \\
 \underline{2 \quad -8} \phantom{10} \\
 1 \quad -4 \quad 5 \quad 0
 \end{array}$$

$x^2 - 4x + 5 = 0$   
 $x = \frac{4 \pm \sqrt{16 - 4(1)(5)}}{2}$

$x = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm 2i}{2}$

$x = 2 \pm i$

$x = 2 \pm i, 2, -3i$