3.5 WS 3

KEY

Determine the vertical and horizontal asymptotes of each rational function.

1. $F(x)=\dfrac{2x-x^2}{x^2+4x-5}$

$5 \cdot 1$

$\boxed{x=-5,\ x=1,\ \text{and}\ y=1}$

2. $F(x)=\dfrac{4x+4x^2+x^3}{x^2-1}$

$\boxed{x=-1,\ x=1,\ \text{and No Horizontal asymptotes}}$

3. $G(x)=\dfrac{-2x}{x^3+2x^2-4x-8}$

$x^2(x+2)-4(x+2)$

$(x^2-4)(x+2)$

$\boxed{x=2,\ x=-2,\ \text{and}\ y=0}$

4. $H(x)=\dfrac{4x-21}{5x^2+10}$

$\boxed{\text{No Vertical asymptotes}\ y=0}$

Find the slant asymptote of each rational function.

5. $F(x)=\dfrac{-5x^2+4x}{5x-4}$

$$\begin{array}{r l}
 & -x \\
5x-4 \,) & -5x^2+4x \\
 & +5x^2 \mp 4x \\
\hline
 & 0
\end{array}$$

$y=-x$

6. $H(x)=\dfrac{-x^3+3x^2-x-3}{x^2+2x-3}$

$$\begin{array}{r l}
 & -x+5 \\
x^2+2x-3 \,) & -x^3+3x^2-x-3 \\
 & +x^3+2x^2 \mp 3x \\
\hline
 & 5x^2-4x-3 \\
 & -5x^2 \mp 10x \pm 15 \\
\hline
 & -14x+12
\end{array}$$

$\boxed{y=-x+5}$

7. $P(x)=\dfrac{6-7x+x^3}{2+x^2}$

$$\begin{array}{r l}
 & x \\
x^2+2 \,) & x^3+0x^2-7x+6 \\
 & -x^3+0x^2 \mp 2x \\
\hline
 & -9x+6
\end{array}$$

$\boxed{y=x}$

Use Descartes' Rule of Signs to state the number of possible positive and negative real zeros of each polynomial function.

8. $f(x)=x^4-2x^3+4x-8$ 1 2 3

$f(-x)=x^4+2x^3-4x-8$ 1

$\boxed{\text{3 or 1 positive zeros, 1 negative zeros}}$

9. $h(x)=3x^4-8x^3-13x-24$ 1

$h(-x)=3x^4+8x^3+13x-24$ 1

$\boxed{\text{1 positive zero, 1 negative zero}}$

10. $g(x)=x^5+x^4-3x^3+5x+2$ 1 2

$g(-x)=-x^5+x^4+3x^3-5x+2$ 1 2 3

$\boxed{\text{2 or 0 positive zeros, 3 or 1 negative zeros}}$

Use the Rational Zero Theorem to list possible rational zeros for each polynomial function.

11. $h(x) = 3x^4 - 8x^3 - 13x - 24$

p: ±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24

q: ±1, ±3

$\boxed{\frac{p}{q}: \pm 1, \pm\frac{1}{3}, \pm 2, \pm\frac{2}{3}, \pm 3, \pm 4, \pm\frac{4}{3}, \pm 6, \pm 8, \pm\frac{8}{3}, \pm 12, \pm 24}$

12. $f(x) = 2x^4 - 9x^3 + 4x^2 + 21x - 18$

p: ±1, ±2, ±3, ±6, ±9, ±18

q: ±1, ±2

$\boxed{\frac{p}{q}: \pm 1, \pm\frac{1}{2}, \pm 2, \pm 3, \pm\frac{3}{2}, \pm 6, \pm 9, \pm\frac{9}{2}, \pm 18}$

Use the given zero to find the remaining zeros of each polynomial function.

13. $s(x) = 2x^4 - x^3 + 17x^2 - 9x - 9$; $-3i$ & $3i$

$(x-3i)(x+3i)$

x^2+9

$$\begin{array}{r} 2x^2 - x - 1 \\ x^2+9 \overline{)\,2x^4 - x^3 + 17x^2 - 9x - 9} \\ -2x^4 \quad \mp 18x^2 \\ \hline -x^3 - x^2 - 9x - 9 \\ +x^3 \quad +9x \\ \hline -x^2 - 9 \\ +x^2 + 9 \\ \hline 0 \end{array}$$

$2x^2 - x - 1 = 0$ $\quad -2$

$\frac{-2}{2}$ $\frac{1}{2}$

-1

$\boxed{x = 3i, 1, \frac{-1}{2}}$

14. $P(x) = x^4 + 2x^2 + 8x + 5$; $1+2i$ & $1-2i$

$(x - 1 + 2i)(x - 1 - 2i)$

$x^2 - 2x + 1 - 4i^2$

$x^2 - 2x + 5$

$$\begin{array}{r} x^2 + 2x + 1 \\ x^2-2x+5 \overline{)\,x^4 + 2x^2 + 8x + 5} \\ -x^4 + 2x^3 \mp 5x^2 \\ \hline 2x^3 - 3x^2 + 8x + 5 \\ -2x^3 + 4x^2 \mp 10x \\ \hline x^2 - 2x + 5 \\ -x^2 + 2x \mp 5 \\ \hline 0 \end{array}$$

$x^2 + 2x + 1 = 0$

$\boxed{x = -1 \text{ (mult. 2)}, 1-2i}$

Find the polynomial function *P*, with real coefficients, that has the indicated zeros and satisfies the given conditions.

15. Zeros: 3, $2i$; degree: 3

$P(x) = (x-3)(x-2i)(x+2i)$

$P(x) = (x-3)(x^2+4)$

$P(x) = x^3 + 4x - 3x^2 - 12$

$\boxed{P(x) = x^3 - 3x^2 + 4x - 12}$

16. Zeros: ± 1, $1-3i$; degree: 4

$P(x) = (x-1)(x+1)(x-1-3i)(x-1+3i)$

$P(x) = (x^2-1)(x^2-2x+10)$

$P(x) = x^4 - 2x^3 + 10x^2 - x^2 + 2x - 10$

$\boxed{P(x) = x^4 - 2x^3 + 9x^2 + 2x - 10}$