The surface area of the cylinder is given by

$$A = 2\pi r^2 + 2\pi rh$$

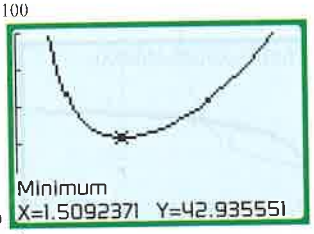
$$A = 2\pi r^2 + 2\pi r\left(\frac{21.6}{\pi r^2}\right) \quad \text{• Substitute for } h.$$

$$A = 2\pi r^2 + \frac{2(21.6)}{r} \quad \text{• Simplify.}$$

$$A = \frac{2\pi r^3 + 43.2}{r} \qquad (1)$$

b. Use Equation (1) with $y = A$ and $x = r$ and a graphing utility to determine that A is a minimum when $r \approx 1.5$ inches. See Figure 3.40.

100



0 0 4

$y = \dfrac{2\pi x^3 + 43.2}{x}$

Figure 3.40

▶ **Try Exercise 88, page 322**

Answer graphs to Exercises 33 to 54, 65 to 82, 87c, and 91a are on pages AA13–AA14.

EXERCISE SET 3.5

Concept Check

In Exercises 1 and 2, determine which of the given functions are rational functions.

1. $F(x) = \dfrac{3x - 5}{x^2 + 3}$ $G(x) = \dfrac{\sqrt{x}}{x^3 - 2}$ $H(x) = \dfrac{1}{2x - 3}$ F, H

2. $J(x) = \dfrac{4}{|x|}$ $K(x) = \dfrac{x^{0.5}}{x^2 - 3}$ $L(x) = \dfrac{2x}{3x^3 - x^2 - 1}$ L

In Exercises 3 to 12, determine the domain of the rational function.

3. $F(x) = \dfrac{1}{x}$
All real numbers except 0

4. $F(x) = \dfrac{2}{x - 3}$
All real numbers except 3

5. $F(x) = \dfrac{x^2 - 3}{x^2 + 1}$
All real numbers

6. $F(x) = \dfrac{x^3 + 4}{x^2 - 25}$
All real numbers except 5 and −5

7. $F(x) = \dfrac{2x - 1}{2x^2 - 15x + 18}$
All real numbers except $\frac{3}{2}$ and 6

8. $F(x) = \dfrac{3x - 2}{4x^2 - 27x + 18}$
All real numbers except $\frac{3}{4}$ and 6

9. $F(x) = \dfrac{2x^2}{x^3 - 4x^2 - 12x}$
All real numbers except 0, −2, and 6

10. $F(x) = \dfrac{3x^2}{x^2 - 5}$
All real numbers except $\sqrt{5}$ and $-\sqrt{5}$

11. $F(x) = \dfrac{x^2 + 1}{x^3 - 8}$
All real numbers except 2

12. $F(x) = \dfrac{x^3 + x + 5}{x^4 - 8x^2 - 9}$
All real numbers except 3 and −3

In Exercises 13 to 22, find all vertical asymptotes of each rational function.

13. $F(x) = \dfrac{2x - 1}{x^2 + 3x}$
$x = 0, x = -3$

14. $F(x) = \dfrac{3x^2 + 5}{x^2 - 4}$
$x = 2, x = -2$

15. $F(x) = \dfrac{x^2 + 11}{6x^2 - 5x - 4}$
$x = -\frac{1}{2}, x = \frac{4}{3}$

16. $F(x) = \dfrac{3x - 5}{x^3 - 8}$
$x = 2$

17. $F(x) = \dfrac{5x^2 - 3}{4x^3 - 25x^2 + 6x}$
$x = 0, x = \frac{1}{4}, x = 6$

18. $F(x) = \dfrac{5x}{x^4 - 81}$
$x = 3, x = -3$

19. $F(x) = \dfrac{3x - 5}{x^2 - 2}$
$x = \sqrt{2}, x = -\sqrt{2}$

20. $F(x) = \dfrac{4x^2 + 1}{2x^3 - 3x^2 - 20x}$
$x = 0, x = -\frac{5}{2}, x = 4$

21. $F(x) = \dfrac{2x}{3x^2 + 5}$
No vertical asymptote

22. $F(x) = \dfrac{4x^3 + 1}{5x^2}$
$x = 0$

In Exercises 23 to 32, find the horizontal asymptote of each rational function.

23. $F(x) = \dfrac{4x^2 + 1}{x^2 + x + 1}$ $y = 4$

24. $F(x) = \dfrac{3x^3 - 27x^2 + 5x - 11}{x^5 - 2x^3 + 7}$ $y = 0$

25. $F(x) = \dfrac{15{,}000x^3 + 500x - 2000}{700 + 500x^3}$ $y = 30$

26. $F(x) = 6000\left(1 - \dfrac{25}{(x + 5)^2}\right)$ $y = 6000$

27. $F(x) = \dfrac{4x^2 - 11x + 6}{4 - x + \frac{1}{3}x^2}$ $y = 12$

28. $F(x) = \dfrac{(2x - 3)(3x + 4)}{(1 - 2x)(3 - 5x)}$ $y = \frac{3}{5}$

Indicates Try It Exercises

29. $F(x) = \dfrac{2x^2 + x + 7}{\frac{1}{3}x^2 + 5x - 4}$ $y = 6$

30. $F(x) = \dfrac{3x^3 - 5x - 1}{2x^2 - 3x + 11}$ No horizontal asymptote

31. $F(x) = \dfrac{-3x^2 + 8}{x^3 + x^2 - 5x + 2}$ $y = 0$

32. $F(x) = \dfrac{1}{x}$ $y = 0$

In Exercises 33 to 54, determine the vertical and horizontal asymptotes and sketch the graph of the rational function *F*. Label all intercepts and asymptotes.

33. $F(x) = \dfrac{1}{x + 4}$ $x = -4, y = 0$

34. $F(x) = \dfrac{1}{x - 2}$ $x = 2, y = 0$

35. $F(x) = \dfrac{-4}{x - 3}$ $x = 3, y = 0$

36. $F(x) = \dfrac{-3}{x + 2}$ $x = -2, y = 0$

37. $F(x) = \dfrac{4}{x}$ $x = 0, y = 0$

38. $F(x) = \dfrac{-4}{x}$ $x = 0, y = 0$

39. $F(x) = \dfrac{x}{x + 4}$ $x = -4, y = 1$

40. $F(x) = \dfrac{x}{x - 2}$ $x = 2, y = 1$

41. $F(x) = \dfrac{x + 4}{2 - x}$ $x = 2, y = -1$

42. $F(x) = \dfrac{x + 3}{1 - x}$ $x = 1, y = -1$

43. $F(x) = \dfrac{1}{x^2 - 9}$ $x = 3, x = -3, y = 0$

44. $F(x) = \dfrac{-2}{x^2 - 4}$ $x = 2, x = -2, y = 0$

45. $F(x) = \dfrac{1}{x^2 + 2x - 3}$ $x = -3, x = 1, y = 0$

46. $F(x) = \dfrac{1}{x^2 - 2x - 8}$ $x = 4, x = -2, y = 0$

47. $F(x) = \dfrac{x^2}{x^2 + 4x + 4}$ $x = -2, y = 1$

48. $F(x) = \dfrac{2x^2}{x^2 - 1}$ $x = -1, x = 1, y = 2$

49. $F(x) = \dfrac{10}{x^2 + 2}$ No vertical asymptote; $y = 0$

50. $F(x) = \dfrac{x^2}{x^2 - 6x + 9}$ $x = 3, y = 1$

51. $F(x) = \dfrac{2x^2 - 2}{x^2 - 9}$ $x = 3, x = -3, y = 2$

52. $F(x) = \dfrac{6x^2 - 5}{2x^2 + 6}$ No vertical asymptote; $y = 3$

53. $F(x) = \dfrac{x^2 + x + 4}{x^2 + 2x - 1}$ $x = -1 + \sqrt{2}, x = -1 - \sqrt{2}, y = 1$

54. $F(x) = \dfrac{2x^2 - 14}{x^2 - 6x + 5}$ $x = 5, x = 1, y = 2$

In Exercises 55 to 64, find the slant asymptote of each rational function.

55. $F(x) = \dfrac{3x^2 + 5x - 1}{x + 4}$ $y = 3x - 7$

56. $F(x) = \dfrac{x^3 - 2x^2 + 3x + 4}{x^2 - 3x + 5}$ $y = x + 1$

57. $F(x) = \dfrac{x^3 - 1}{x^2}$ $y = x$

58. $F(x) = \dfrac{4000 + 20x + 0.0001x^2}{x}$ $y = 0.0001x + 20$

59. $F(x) = \dfrac{-4x^2 + 15x + 18}{x - 5}$ $y = -4x - 5$

60. $F(x) = \dfrac{-x^4 - 2x^3 - 3x^2 + 4x - 1}{x^3 - 1}$ $y = -x - 2$

61. $F(x) = \dfrac{x^5 - 2}{x^4 + 1}$ $y = x$

62. $F(x) = \dfrac{6x^3 - 2x^2 + 7x - 11}{2x^2 + x + 1}$ $y = 3x - \dfrac{5}{2}$

63. $F(x) = \dfrac{\frac{1}{3}x^2 - 1}{2x + 3}$ $y = \dfrac{1}{6}x - \dfrac{1}{4}$

64. $F(x) = \dfrac{3 - 2x - 5x^2}{6 + x}$ $y = -5x + 28$

In Exercises 65 to 74, determine the vertical and slant asymptotes and sketch the graph of the rational function *F*.

65. $F(x) = \dfrac{x^2 - 4}{x}$ $x = 0, y = x$

66. $F(x) = \dfrac{x^2 + 10}{2x}$ $x = 0, y = \dfrac{1}{2}x$

67. $F(x) = \dfrac{x^2 - 3x - 4}{x + 3}$ $x = -3, y = x - 6$

68. $F(x) = \dfrac{x^2 - 4x - 5}{2x + 5}$ $x = -\dfrac{5}{2}, y = \dfrac{1}{2}x - \dfrac{13}{4}$

69. $F(x) = \dfrac{2x^2 + 5x + 3}{x - 4}$ $x = 4, y = 2x + 13$

70. $F(x) = \dfrac{4x^2 - 9}{x + 3}$ $x = -3, y = 4x - 12$

71. $F(x) = \dfrac{x^2 - x}{x + 2}$ $x = -2, y = x - 3$

72. $F(x) = \dfrac{x^2 + x}{x - 1}$ $x = 1, y = x + 2$

73. $F(x) = \dfrac{x^3 + 1}{x^2 - 4}$ $x = 2, x = -2, y = x$

74. $F(x) = \dfrac{x^3 - 1}{3x^2}$ $x = 0, y = \dfrac{1}{3}x$

In Exercises 75 to 82, sketch the graph of the rational function *F*.

75. $F(x) = \dfrac{x^2 + x}{x + 1}$

76. $F(x) = \dfrac{x^2 - 3x}{x - 3}$

77. $F(x) = \dfrac{2x^3 + 4x^2}{2x + 4}$

78. $F(x) = \dfrac{x^2 - x - 12}{x^2 - 2x - 8}$

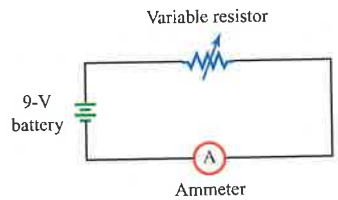
79. $F(x) = \dfrac{-2x^3 + 6x}{2x^2 - 6x}$

80. $F(x) = \dfrac{x^3 + 3x^2}{x(x + 3)(x - 1)}$

81. $F(x) = \dfrac{x^2 - 3x - 10}{x^2 + 4x + 4}$

82. $F(x) = \dfrac{2x^2 + x - 3}{x^2 - 2x + 1}$

83. **Electrical Current** A variable resistor, an ammeter, and a 9-volt battery are connected as shown in the following diagram.



The internal resistance of the ammeter is 4.5 ohms. The current I, in amperes, through the ammeter is given by

$$I(x) = \frac{9}{x + 4.5}$$

where x is the resistance, in ohms, provided by the variable resistor.

a. Find the current through the ammeter when the variable resistor has a resistance of 3 ohms. 1.2 amps

b. Determine the resistance of the variable resistor when the current through the ammeter is 0.24 ampere. 33 ohms

c. Determine the horizontal asymptote of the graph of I, and explain the meaning of the horizontal asymptote in the context of this application. $y = 0$. The current approaches 0 amps as the resistance of the variable resistor increases without bound.

84. **Average Speed** During the first 30 miles of city driving, you average 40 miles per hour. For the remainder of the trip, you drive on a highway at a constant rate of 70 miles per hour. Your average speed for the entire trip is given by

$$r(x) = \frac{30 + x}{\frac{3}{4} + \frac{1}{70}x}$$

where x is the number of miles you have driven on the highway.

a. How far will you need to drive on the highway to bring your average speed up to 60 miles per hour? 105 mi

b. Determine the horizontal asymptote of the graph of r, and write a sentence that explains the meaning of the horizontal asymptote in the context of this application. $y = 70$. The further you drive at 70 mph, the closer your average speed will be to 70 mph.

85. **Average Cost of Golf Balls** The cost, in dollars, of producing x golf balls is given by

$$C(x) = 0.43x + 76{,}000$$

The average cost per golf ball is given by

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{0.43x + 76{,}000}{x}$$

a. Find the average cost per golf ball of producing 1000, 10,000, and 100,000 golf balls. \$76.43, \$8.03, \$1.19

b. What is the equation of the horizontal asymptote of the graph of $\overline{C}$? Explain the significance of the horizontal asymptote as it relates to this application. $y = 0.43$. As the number of golf balls produced increases, the average cost per ball approaches \$0.43.

86. **Average Cost of Blu-ray Players** The cost, in dollars, of producing x Blu-ray players is given by

$$C(x) = 0.001x^2 + 54x + 175{,}000$$

The average cost per Blu-ray player is given by

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{0.001x^2 + 54x + 175{,}000}{x}$$

a. Find the average cost per Blu-ray player of producing 1000, 10,000, and 100,000 Blu-ray players. \$230, \$81.50, \$155.75

b. What is the minimum average cost per Blu-ray player? How many Blu-ray players should be produced to minimize the average cost per Blu-ray player? ≈\$80.46; ≈13,229 Blu-ray players

87. **Desalinization** The cost C, in dollars, to remove $p\%$ of the salt in a tank of seawater is given by

$$C(p) = \frac{2000p}{100 - p}, \quad 0 \le p < 100$$

a. Find the cost of removing 40% of the salt. \$1333.33

b. Find the cost of removing 80% of the salt. \$8000

c. Sketch the graph of C.

88. **Production Costs** The cost, in dollars, of producing x cell phones is given by

$$C(x) = 0.0006x^2 + 9x + 401{,}000$$

The average cost per cell phone is

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{0.0006x^2 + 9x + 401{,}000}{x}$$

a. Find the average cost per cell phone when 1000, 10,000, and 100,000 phones are produced. \$410.60, \$55.10, \$73.01

b. What is the minimum average cost per cell phone? How many cell phones should be produced to minimize the average cost per phone? \$40.02; 25,852 cell phones

89. **Average Wait Time** Rational functions can be used to model the expected average time a customer will wait in line, dependent on conditions such as the customer arrival rate, the average time it takes to serve a customer, and the number of service lanes that are open. For instance, the rational function

$$W(x) = \frac{7.5}{x(x - 15)}, \text{ where } 15 < x \le 40$$

models the average expected wait time, in hours, for customers of a convenience store, under the following conditions.

- The store has one cashier.
- The customers arrive at a rate of 15 people per hour.
- The cashier can serve x customers per hour, where $x > 15$.

a. Use W to determine the average expected wait time for a cashier who can serve 25 customers per hour. State your answer in hours and then convert that result to minutes. 0.03 hour = 1.8 minutes

b. How many customers must the cashier serve per hour to reduce the average expected wait time to 0.0125 hour (45 seconds)? (*Hint:* Replace $W(x)$ with 0.0125 and solve the resulting equation for x.) Round your answer to the nearest tenth. 33.1 customers per hour

c. Explain, in the context of this application, why W decreases as x increases. If more customers can be served per hour, then the average wait time will decrease.

90. **Medication Model** The rational function

$$M(t) = \frac{0.5t + 400}{0.04t^2 + 1}$$

models the number of milligrams of medication in the bloodstream of a patient t hours after 400 milligrams of the medication have been injected into the patient's bloodstream.

a. Find $M(5)$ and $M(10)$. Round to the nearest milligram. 201 mg, 81 mg

b. What will M approach as $t \to \infty$? 0 mg

91. **Minimizing Surface Area** A cylindrical soft drink can is to be made so that it will have a volume of 354 milliliters. If r is the radius of the can in centimeters, then the total surface area A in square centimeters of the can is given by the rational function

$$A(r) = \frac{2\pi r^3 + 708}{r}$$

a. Graph A and use the graph to estimate (to the nearest tenth of a centimeter) the value of r that produces the minimum value of A. 3.8 cm

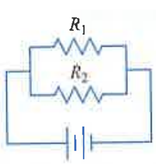
b. Does the graph of A have a slant asymptote? No

c. Explain the meaning of the following statement as it applies to the graph of A.

As $r \to \infty, A \to 2\pi r^2$.

As the radius r increases without bound, the surface area approaches twice the area of a circle with radius r.

92. **Resistors in Parallel** The electronic circuit below shows two resistors connected in parallel.



One resistor has a resistance of R_1 ohms, and the other has a resistance of R_2 ohms. The total resistance for the circuit, measured in ohms, is given by the formula

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Assume that R_1 has a fixed resistance of 10 ohms. $\frac{5}{3}$ ohms, $\frac{20}{3}$ ohms

a. Compute R_T for $R_2 = 2$ ohms and for $R_2 = 20$ ohms.

b. Find R_2 when $R_T = 6$ ohms. 15 ohms

c. What happens to R_T as $R_2 \to \infty$? $R_T \to 10$ ohms

93. Determine the point at which the graph of

$$F(x) = \frac{2x^2 + 3x + 4}{x^2 + 4x + 7}$$

intersects its horizontal asymptote. $(-2, 2)$

Enrichment Exercises

94. **Parabolic Asymptotes** It can be shown that the rational function $F(x) = R(x)/S(x)$, where $R(x)$ and $S(x)$ have no common factors, has a parabolic asymptote provided the degree of $R(x)$ is 2 *greater than* the degree of $S(x)$. For instance, the rational function

$$F(x) = \frac{x^3 + 2}{x + 1}$$

has a parabolic asymptote given by $y = x^2 - x + 1$.

a. Use a graphing utility to graph F and the parabola given by $y = x^2 - x + 1$ in the same viewing window. Does the parabola appear to be an asymptote for the graph of F? Explain. Answers will vary.

b. Write a paragraph that explains how to determine the equation of the parabolic asymptote for a rational function $F(x) = R(x)/S(x)$, where $R(x)$ and $S(x)$ have no common factors and the degree of $R(x)$ is 2 greater than the degree of $S(x)$. Answers will vary.

c. What is the equation of the parabolic asymptote for the rational function $G(x) = \dfrac{x^4 + x^2 + 2}{x^2 - 1}$? Use a graphing utility to graph G and the parabolic asymptote in the same viewing window. Does the parabola appear to be an asymptote for the graph of G? $y = x^2 + 2$; Answers will vary.

d. Create a rational function that has $y = x^2 + x + 2$ as its parabolic asymptote. Explain the procedure you used to create your rational function. Answers will vary.

In Exercises 95 and 96, create a rational function whose graph has the given characteristics.

95. Is symmetric to the y-axis, has vertical asymptotes at $x = 3$ and $x = -3$, has a horizontal asymptote at $y = 2$, and passes through the origin
One possible function: $f(x) = \dfrac{2x^2}{x^2 - 9}$

96. Has a vertical asymptote at $x = 5$, has $y = x - 3$ as a slant asymptote, and intersects the x-axis at $(4, 0)$
One possible function: $f(x) = \dfrac{x^2 - 8x + 16}{x - 5}$